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Lepton-nucleus Scattering

Noemi Rocco

NTNP Meeting @ INT
June 1st and 2nd, 2023

Addressing Neutrino-Oscillation Physics

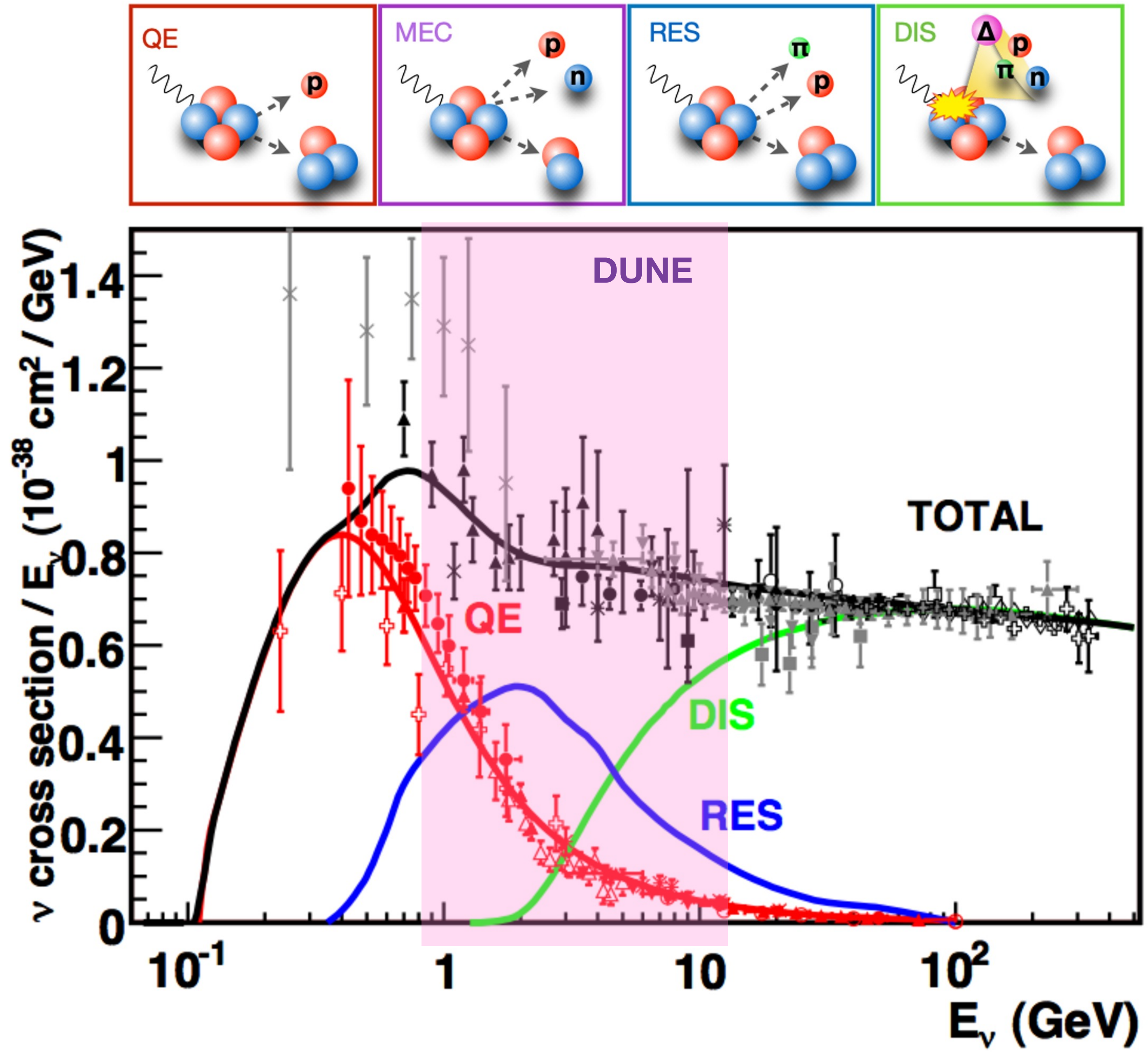
Unprecedented accuracy in the determination of **neutrino-argon cross section** is required to achieve design sensitivity to CP violation at DUNE

Nuclei are **complicated quantum many-body systems**;

More than 60% of the interactions at DUNE are non-quasielastic

Theoretical tools for neutrino scattering, Contribution to: 2022 Snowmass Summer Study

J.A. Formaggio and G.P. Zeller, Rev. Mod. Phys. 84 (2012)



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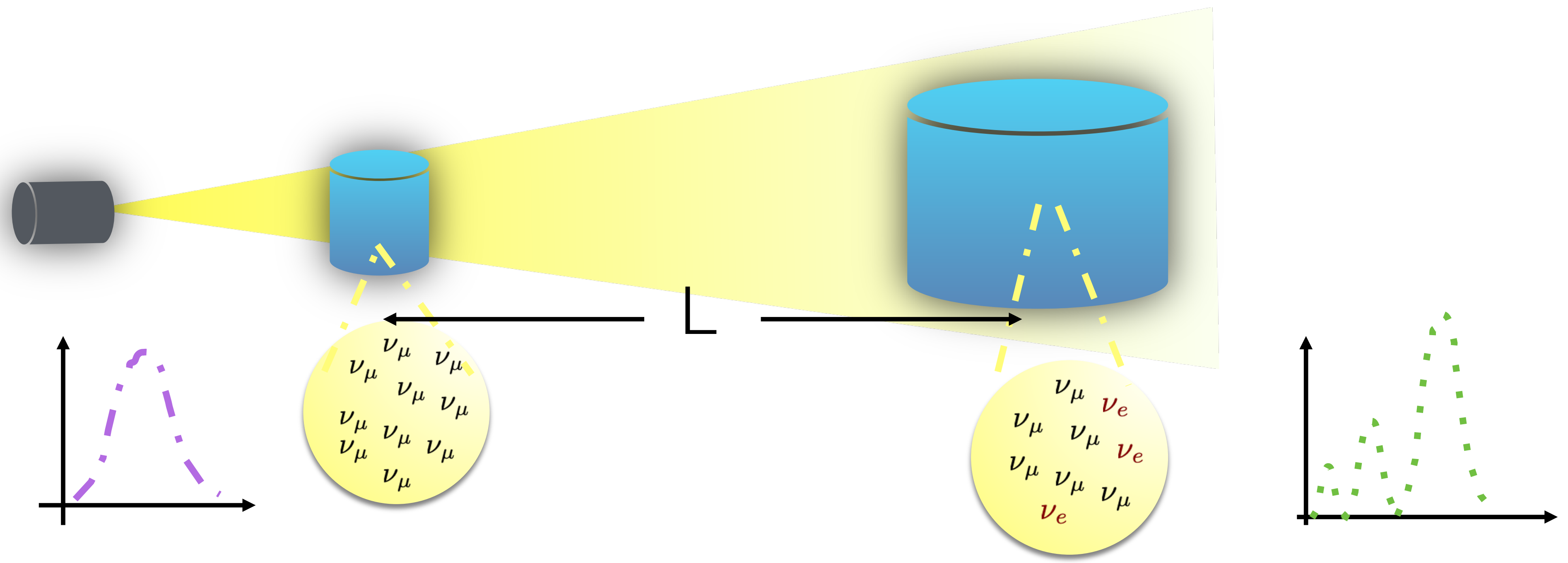
Why do we need more precision?

$$P(\nu_\mu \to \nu_e, E_\nu, L) = \frac{\Phi(E_\nu, L)}{\Phi_\mu(E_\nu, 0)} = \frac{N_e(E_\nu, L)/\sigma_e(E_\nu)}{N_\mu(E_\nu, L)/\sigma_\mu(E_\nu)}$$

Detectors measure the **neutrino interaction rate:**



A precise determination of σ(E) is crucial to extract ν oscillation parameters. Nuclear effects at near and far detector **do not** cancel

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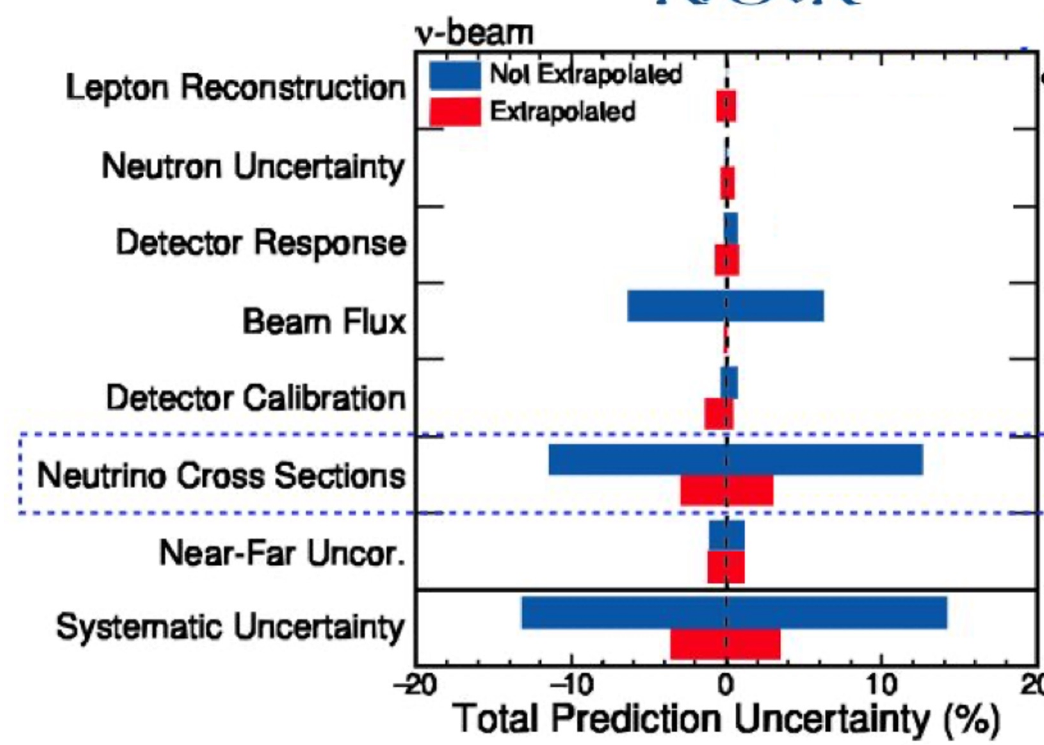
Neutrino-nucleus cross section systematics

Current oscillation experiments report **large systematic uncertainties** associated with neutrino- nucleus interaction models.

Error source	ν_e FHC	$\bar{\nu}_e$ RHC	$\nu_e / \bar{\nu}_e$ FHC/RHC
Flux and (ND unconstrained)	15.1	12.2	1.2
cross section (ND constrained)	3.2	3.1	2.7
SK detector	2.8	3.8	1.5
SK FSI + SI + PN	3.0	2.3	1.6
Nucleon removal energy	7.1	3.7	3.6
$\sigma(\nu_e)/\sigma(\bar{\nu}_e)$	2.6	1.5	3.0
NC1γ	1.1	2.6	1.5
NC other	0.2	0.3	0.2
$\sin^2\theta_{23}$ and Δm^2_{21}	0.5	0.3	2.0
$\sin^2\theta_{13}$ PDG2018	2.6	2.4	1.1
All systematics	8.8	7.1	6.0



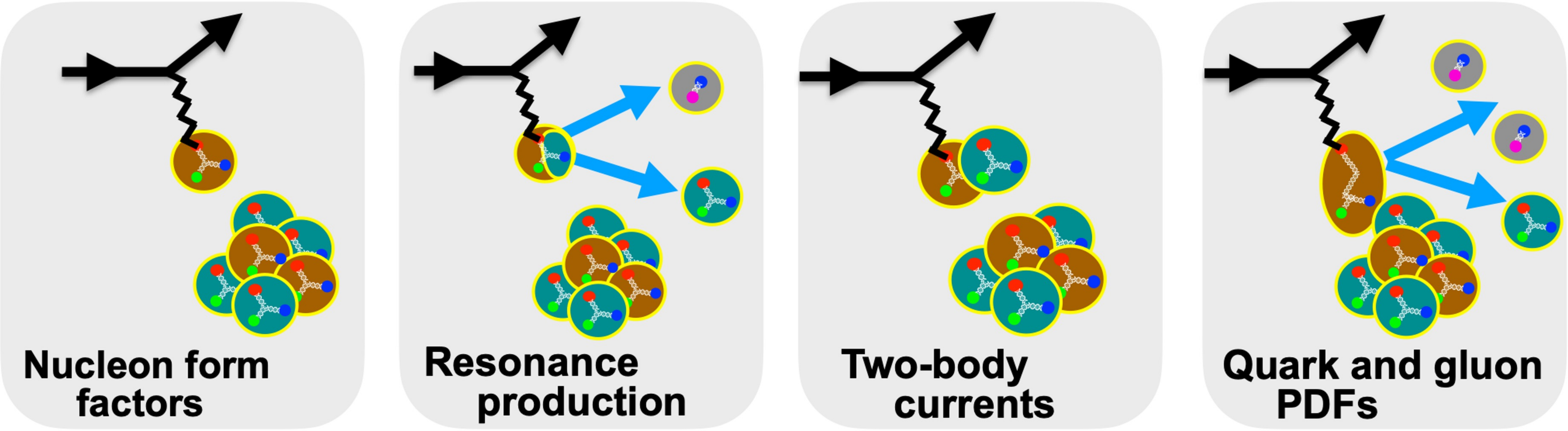
T2K, Phys. Rev. D 103, 112008 (2021)

Input parameters and their precision

There is no EFT that coverages over all of DUNE kinematics

The first steps towards getting few-% cross-section uncertainties are understanding what input parameters we will need and what precision we will need them at.

Lattice QCD can provide inputs to be included in EFTs and nuclear many-body methods



Courtesy of M. Wagman

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NTNP objectives in the lepton-nucleus scattering thrust

- XSEC1: Utilize a variational operator basis to reduce the excited state contamination in LQCD calculations of the nucleon form factors
- XSEC2: Utilize a variational operator basis to compute the $N \rightarrow \Delta$ transition amplitudes induced by electromagnetic and axial currents in LQCD.
- XSEC3: Compute the two-nucleon electroweak matrix elements, determining the short distance contribution to the two-nucleon currents.

- XSEC4: Use LQCD form factors to compute inclusive electroweak transitions for nuclei with A=4, and 12 within GFMC, SF and STA. Use AFDMC to reach A=16 and 40.
- XSEC5: Investigate exclusive reactions and relativistic effects using the SF and STA adopting the nucleonic electroweak transition amplitudes obtained from LQCD. Assessing theory uncertainty varying the adopted computational method, many-nucleon interactions, and nucleonic inputs.

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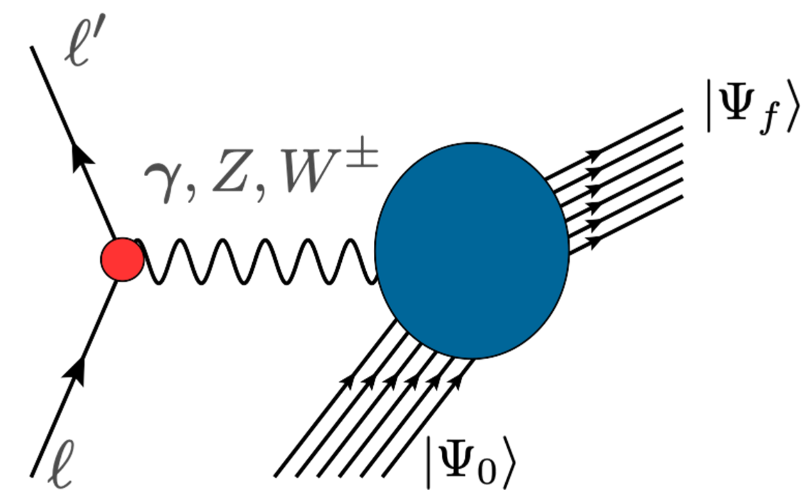
Theory of lepton-nucleus scattering

- The cross section of the process in which a lepton scatters off a nucleus is given by

$$d\sigma \propto L^{\alpha\beta} R_{\alpha\beta}$$



Leptonic Tensor: can include new physics models

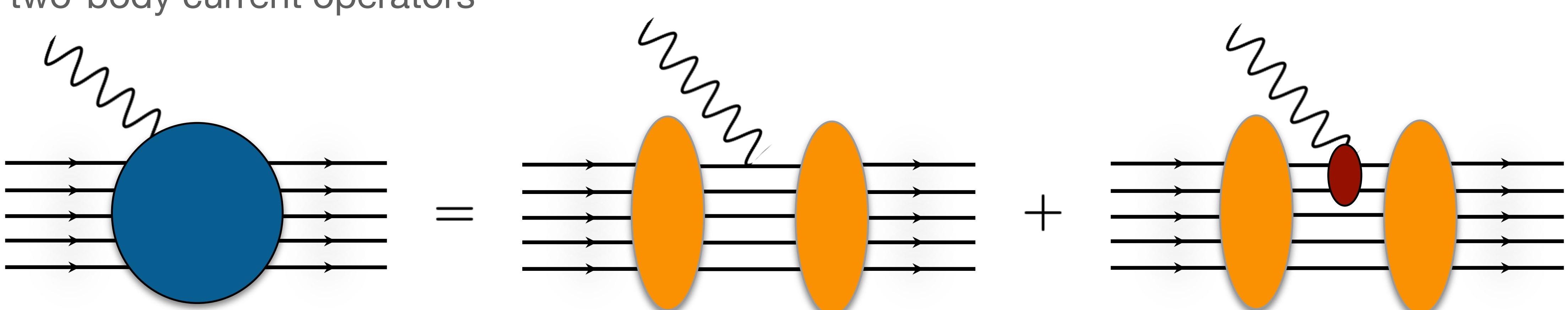
Hadronic Tensor: nuclear response function

$$R_{\alpha\beta}(\omega, \mathbf{q}) = \sum_f \langle 0|J_\alpha^\dagger(\mathbf{q})|f\rangle\langle f|J_\beta(\mathbf{q})|0\rangle\delta(\omega - E_f + E_0)$$

The initial and final wave functions describe many-body states:

$$|0\rangle = |\Psi_0^A\rangle \;, |f\rangle = |\Psi_f^A\rangle, |\psi_p^N, \Psi_f^{A-1}\rangle, |\psi_k^\pi, \psi_p^N, \Psi_f^{A-1}\rangle \ldots$$

One and two-body current operators



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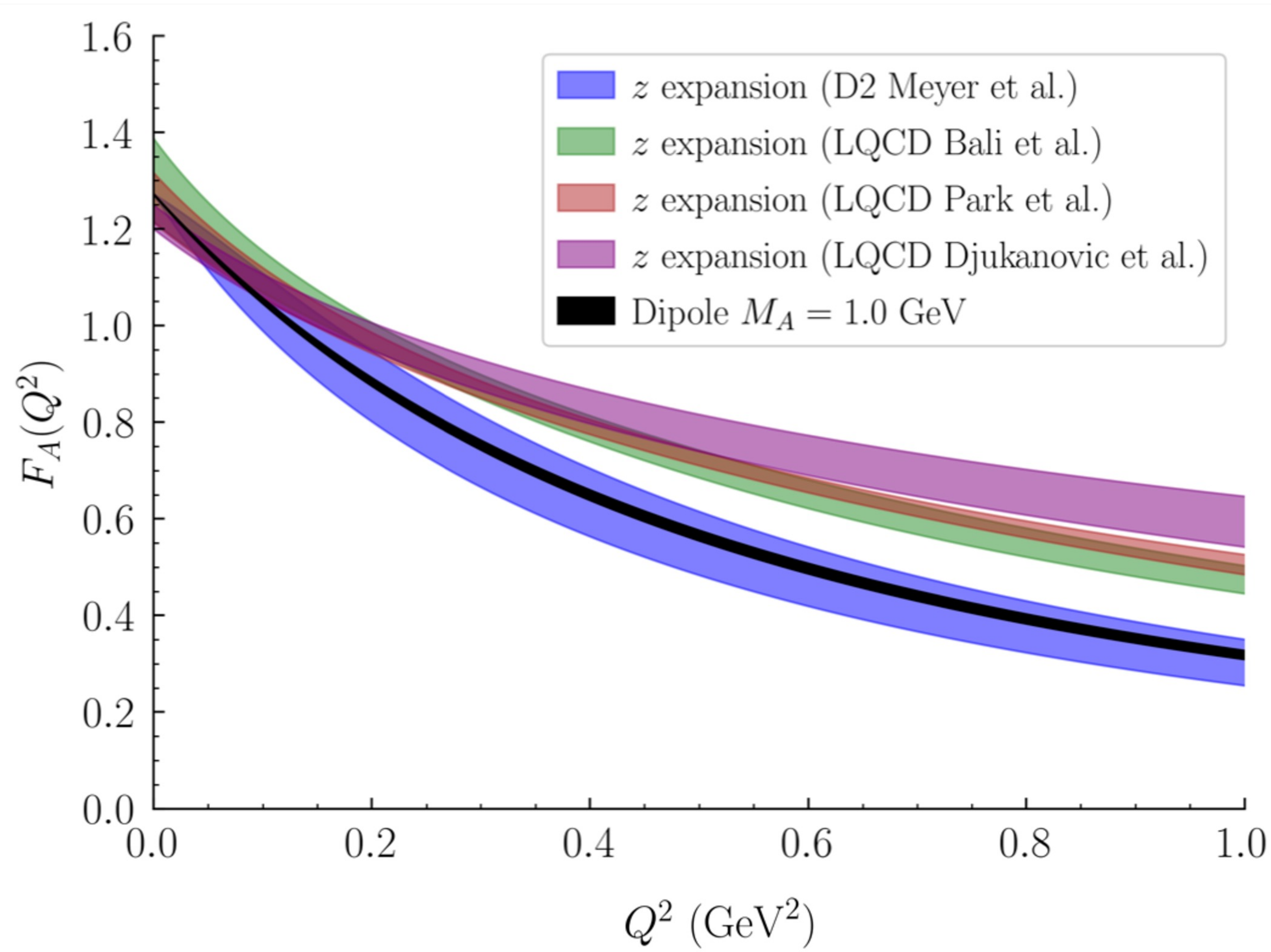
Elementary Input: Form Factors



Different parametrization of the axial form factor:

Dipole:
$$F_A(Q^2) = \frac{g_A}{(1 + Q^2/M_A^2)^2},$$

Different determinations of nucleon axial form factor using the z-expansion

$$F_A(Q^2) = \sum_{k=0}^{\infty} a_k\, z(Q^2)^k \approx \sum_{k=0}^{k_{\max}} a_k\, z(Q^2)^k,$$

UQ independent on assumptions about the shape of the axial form factor.

D2 Meyer et al: fits to neutrino-deuteron scattering data

LQCD result: general agreement between the different calculations, results are 2-3σ larger than D2 Meyer ones for $Q^2 > 0.3$ GeV^2

- XSEC1: Utilize a variational operator basis to reduce the excited state contamination in LQCD calculations of the nucleon form factors

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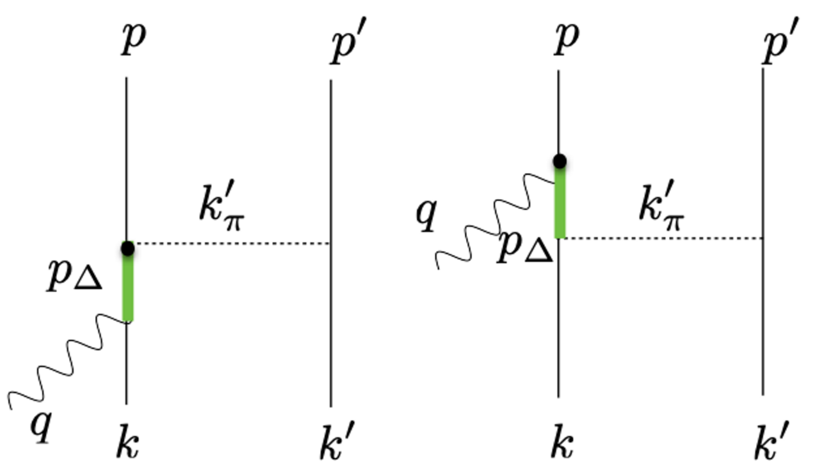
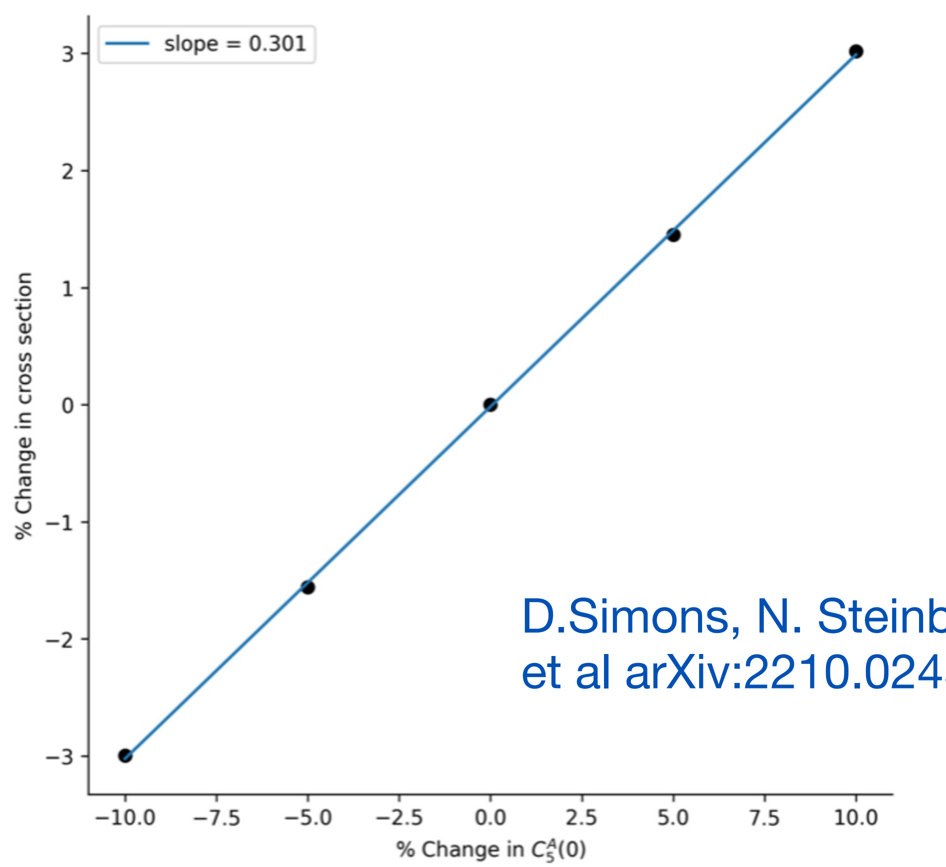
Resonance Uncertainty needs

The largest contributions to two-body currents arise from N→Δ transitions yielding pion production





D.Simons, N. Steinberg et al arXiv:2210.02455

The normalization of the dominant N→Δ transition form factor needs be known to 3% precision to achieve 1% cross-section precision for MiniBooNE kinematics

State-of-the-art determinations of this form factor from experimental data on pion electroproduction achieve 10-15% precision (under some assumptions)

Hernandez et al, PRD 81 (2010)

XSEC2: Further constraints on N→Δ transition relevant for two-body currents and π production will be necessary to achieve few-percent cross-section precision

XSEC3: Compute the two-nucleon electroweak matrix elements, determining the short distance contribution to the two-nucleon currents.

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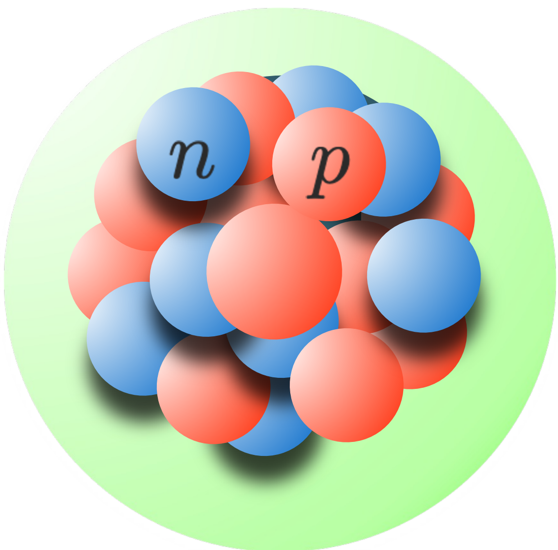
The basic model of nuclear theory

At low energy, the effective degrees of freedom are pions and nucleons:



$$H = \sum_i \frac{\mathbf{p}_i^2}{2m} + \sum_{i<j} v_{ij} + \sum_{i<j<k} V_{ijk} + \ldots$$

1-body **2-body** **3-body**

Different 2- and 3-body interactions can be used:

Semi-phenomenological: av18+IL7

Chiral EFT potentials: non-local interaction needs to be used in QMC based approaches; Δ-less and Δ-full potential

Bayesian techniques applied to constrain LECS, estimate EFT truncation errors

The electromagnetic current is constrained by the Hamiltonian through the **continuity equation**

$$\boldsymbol{\nabla} \cdot \mathbf{J}_{\rm EM} + i[H, J^0_{\rm EM}] = 0 \qquad [v_{ij}, j_i^0] \neq 0$$

The above equation implies that the current operator includes one and two-body contributions

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Green's Function Monte Carlo approach

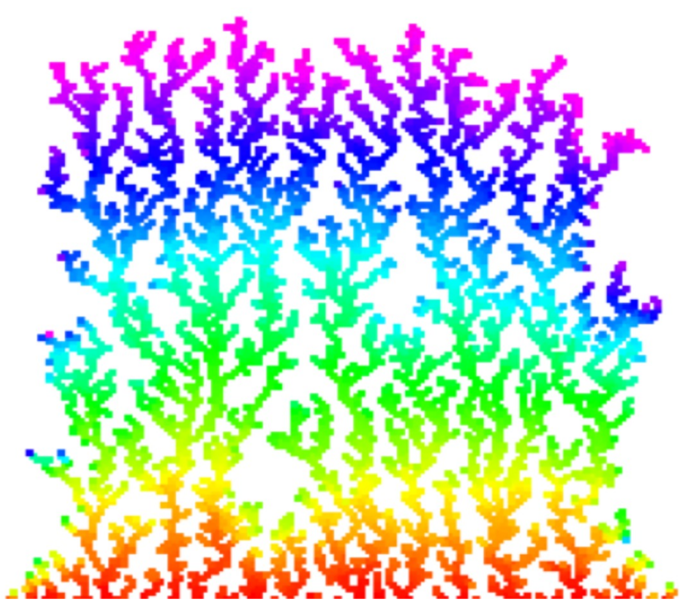
We want to solve the Schrödinger equation

$$H\Psi(\mathbf{R}; s_1 \dots s_A, \tau_1 \dots \tau_A) = E\Psi(\mathbf{R}; s_1 \dots s_A, \tau_1 \dots \tau_A)$$

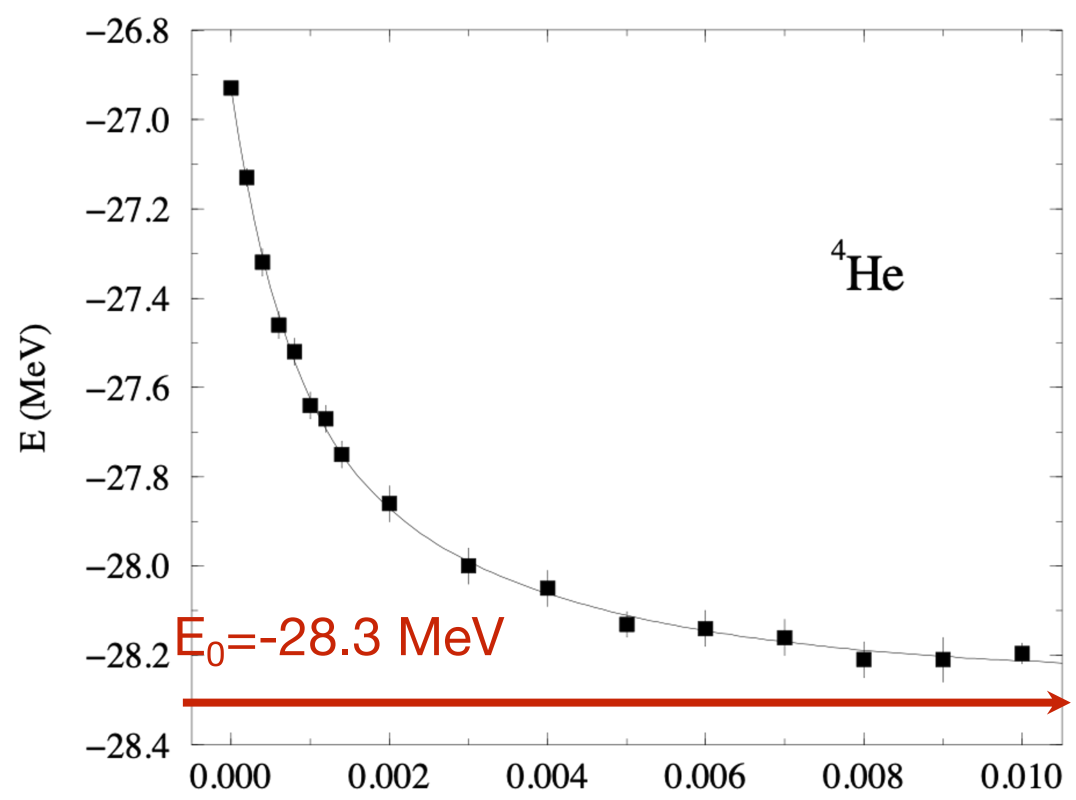
Any trial wave function can be expanded in the complete set of eigenstates of the the Hamiltonian according to

$$|\Psi_T\rangle = \sum_n c_n |\Psi_n\rangle \qquad\qquad H|\Psi_n\rangle = E_n|\Psi_n\rangle$$

QMC techniques **projects out the exact lowest-energy state:** $e^{-(H-E_0)\tau}|\Psi_T\rangle \to |\Psi_0\rangle$



The system is cooled down by evolving it in time



B. Pudliner et al., PRC **56**, 1720 (1997)

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Cross sections: Green’s Function Monte Carlo

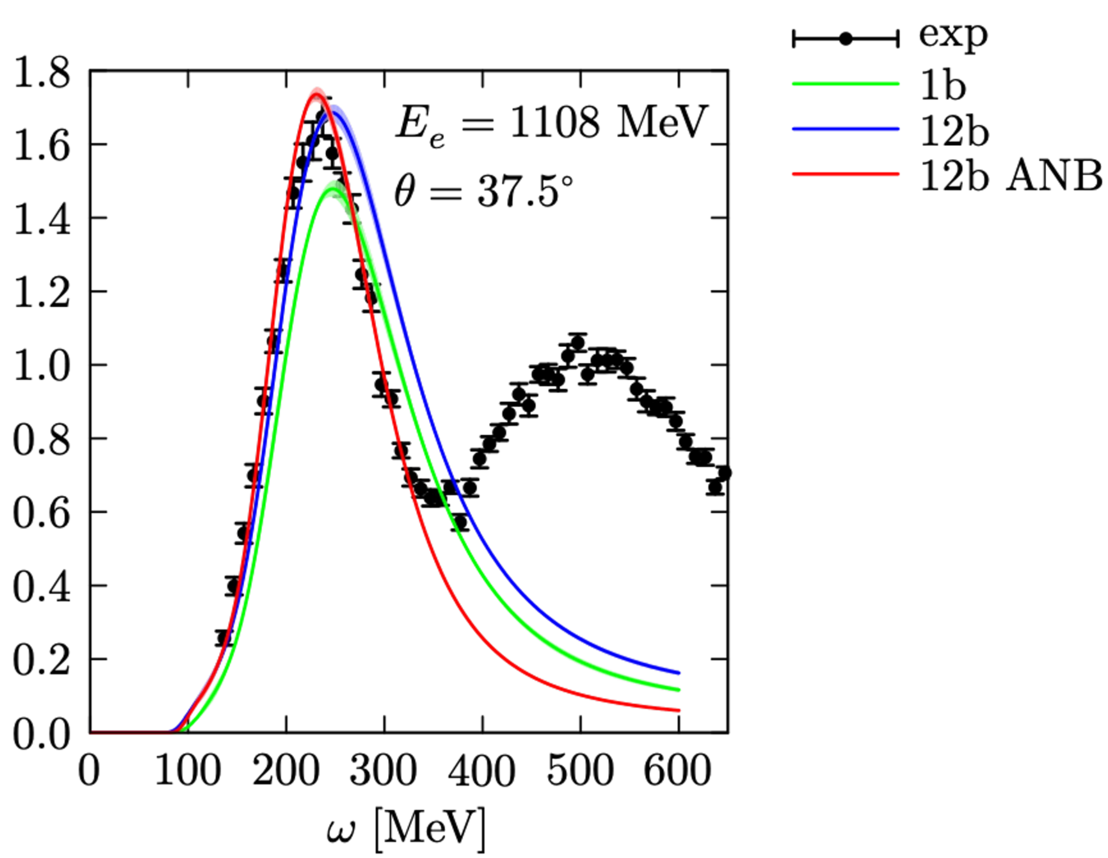
GFMC accurately obtain the properties of nuclei using integral transform techniques

$$E_{\alpha\beta}(\sigma,\mathbf{q}) = \int d\omega K(\sigma,\omega) R_{\alpha\beta}(\omega,\mathbf{q}) = \langle\psi_0|J^\dagger_\alpha(\mathbf{q})K(\sigma,H-E_0)J_\beta(\mathbf{q})|\psi_0\rangle$$

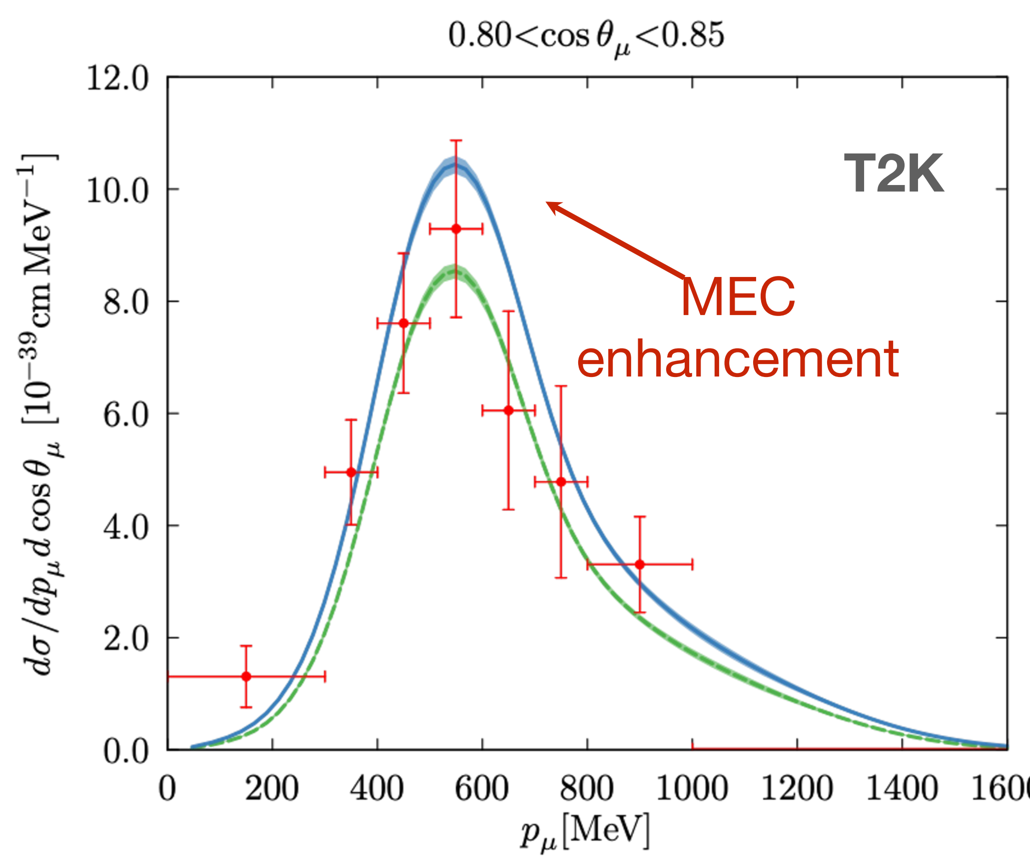
Exact results for ν-cross sections in the **quasi-elastic region** up to moderate values of **q**.

A. Lovato, et al , PRX. 10 (2020) 3, 031068

— electron-^{4}He scattering



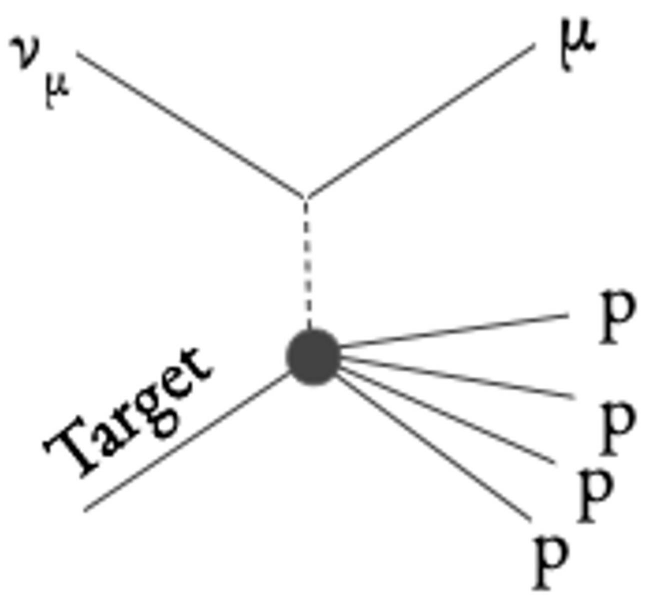
— CC neutrino-^{12}C scattering

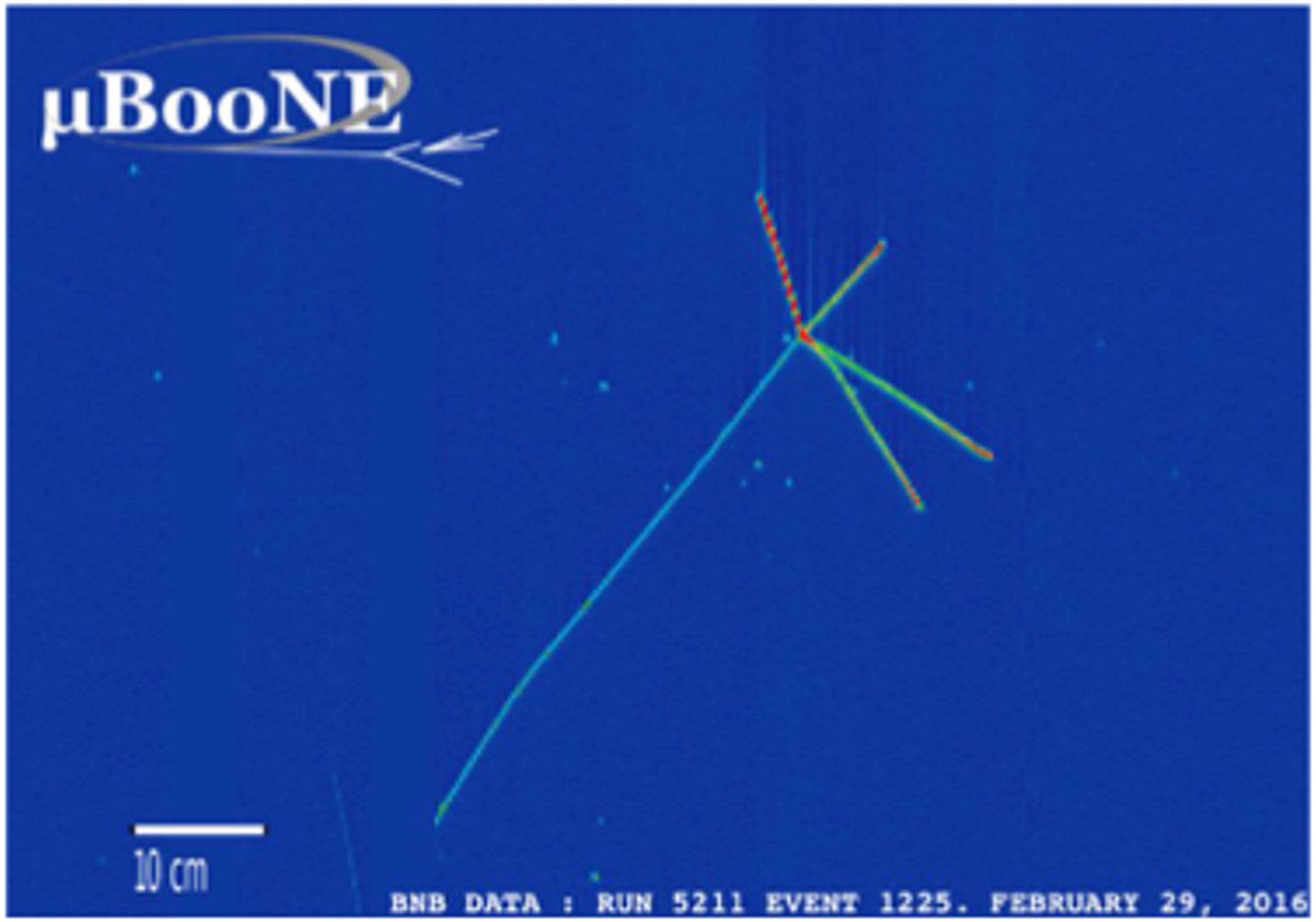


Relies on non-relativistic approximation, inclusive reactions & no explicit pions

Addressing new experimental capabilities

ν_μ CC exclusive topologies: ν_μ CC0πNp with N ≥ 1





E. Gramellini @ Fermilab 54th Annula Users Meeting

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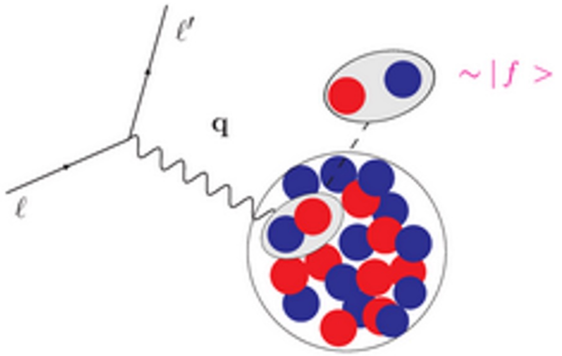
Short-Time Approximation



Short-Time-Approximation:

- Based on Factorization
- **Retains two-body physics**
- Response functions are given by the **scattering from pairs of fully interacting nucleons** that propagate into a correlated pair of nucleons
- Allows to retain both two-body correlations and currents at the vertex
- Provides "more" exclusive information in terms of nucleon-pair kinematics via the Response Densities

Response Functions ∝ Cross Sections

$$R_\alpha(q,\omega) = \sum_f \delta\left(\omega + E_0 - E_f\right) |\langle f|O_\alpha(\mathbf{q})|0\rangle|^2$$

Response ***Densities***

$$R(q,\omega) \sim \int \delta\left(\omega + E_0 - E_f\right)\, dP'\, dp' \mathcal{D}(p', P'; q)$$

P' and *p'* are the CM and relative momenta of the struck nucleon pair

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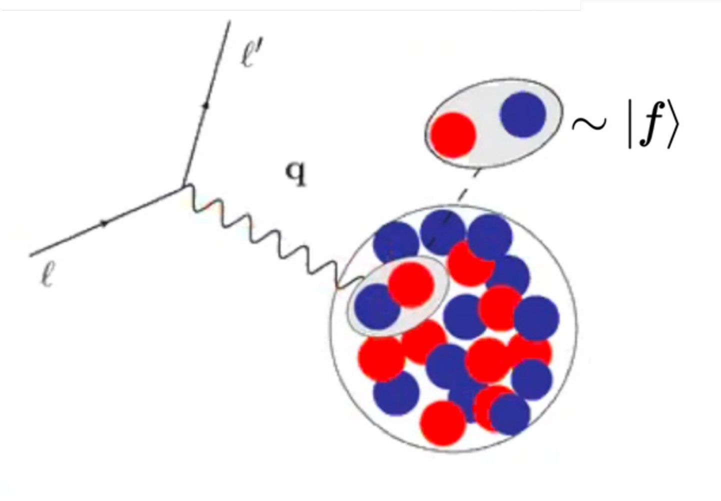
Short-Time Approximation

The sum over all final states is replaced by a two nucleon propagator

$$R_\alpha(q,\omega) = \int_{-\infty}^{\infty} \frac{dt}{2\pi} e^{i(\omega+E_i)t} \langle \Psi_i | O_\alpha^\dagger(\mathbf{q}) e^{-iHt} O_\alpha(\mathbf{q}) | \Psi_i \rangle$$

The STA restrict the propagation to only two active nucleons

$$\begin{aligned} O^\dagger e^{-iHt} O &= \left(\sum_i O_i^\dagger + \sum_{i<j} O_{ij}^\dagger \right) e^{-iHt} \left(\sum_{i'} O_{i'} + \sum_{i'<j'} O_{i'j'} \right) \\ &= \sum_i O_i^\dagger e^{-iHt} O_i + \sum_{i \neq j} O_i^\dagger e^{-iHt} O_j \\ &\quad + \sum_{i \neq j} \Big(O_i^\dagger e^{-iHt} O_{ij} + O_{ij}^\dagger e^{-iHt} O_i \\ &\quad + O_{ij}^\dagger e^{-iHt} O_{ij} \Big) + \ldots \end{aligned}$$



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Validity of the Short-Time Approximation

If one expands the propagator to second order

$$e^{-iHt} \approx 1 - i\left(\sum_i T_i + \sum_{ij} V_{ij}\right)t - \frac{1}{2}\left(\sum_i T_i + \sum_{ij} V_{ij}\right)\left(\sum_{i'} T_{i'} + \sum_{i'j'} V_{i'j'}\right)t^2 + \ldots$$

And notes that in light nuclei $\frac{T}{A} \sim \frac{2|V|}{A(A-1)} \equiv \epsilon_{\rm nuc}$ we have an energy (and thus time) scale associated with including correlations of A nucleons which is ~20 MeV

This gives a way to estimate the impact on the description of the quasi-elastic peak $\omega_{qe} = \sqrt{m^2 + q^2} - m$

In the STA, one drops terms of order $\mathcal{O}\left(\frac{\epsilon_{\rm nuc}^2}{\omega_{qe}^2}\right)$ so it is valid at sufficiently high ω and $|\mathbf{q}|$

Consistent with the notion that there is a high probability for the correlated pair to absorb the entirety of $|\mathbf{q}|$ when the final state mometa are far from the Fermi surface

One may note that the PWIA comes from dropping terms of order $\mathcal{O}\left(\frac{\epsilon_{\rm nuc}}{\omega_{qe}}\right)$

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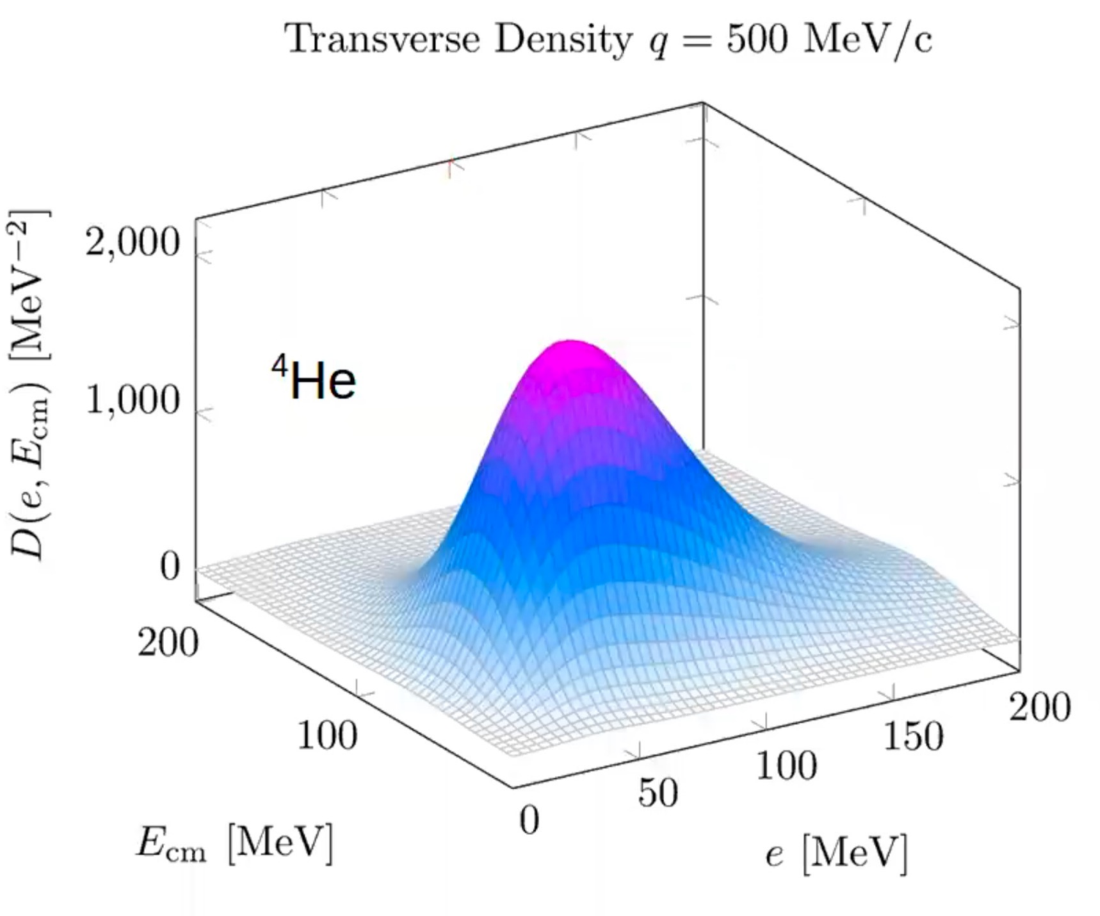
Short-Time Approximation Results

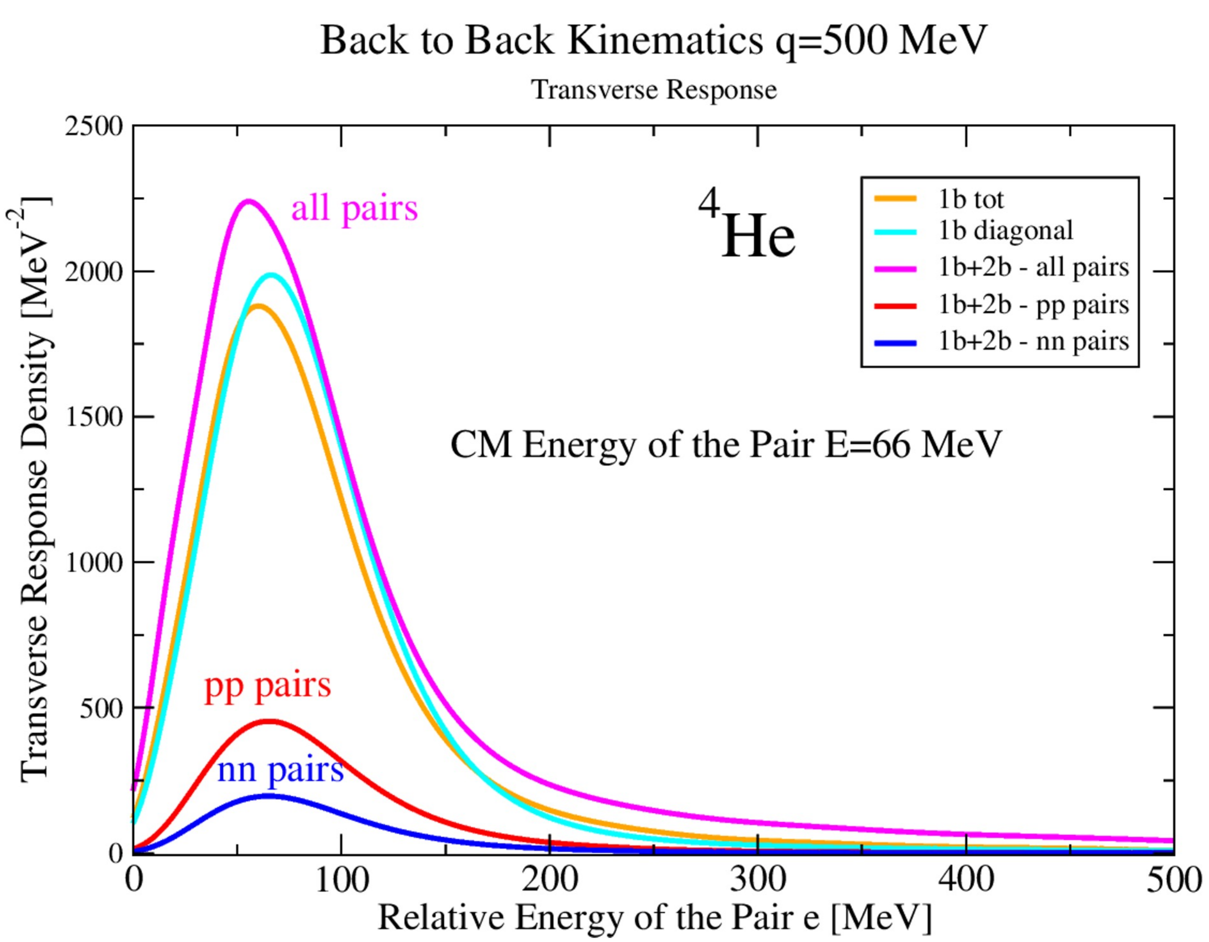


Pastore et al. PRC101(2020)044612

$$R^{\rm STA}(q,\omega) \sim \int \delta(\omega + E_0 - E_f) de\, dE_{cm} \mathcal{D}(e, E_{cm}; q)$$

Electron scattering from ^{4}He:

- Response density as a function of (E,e)
- Give access to particular kinematics for the struck nucleon pair



- pp pairs
- nn pairs
- 1 body tot
- all pairs tot

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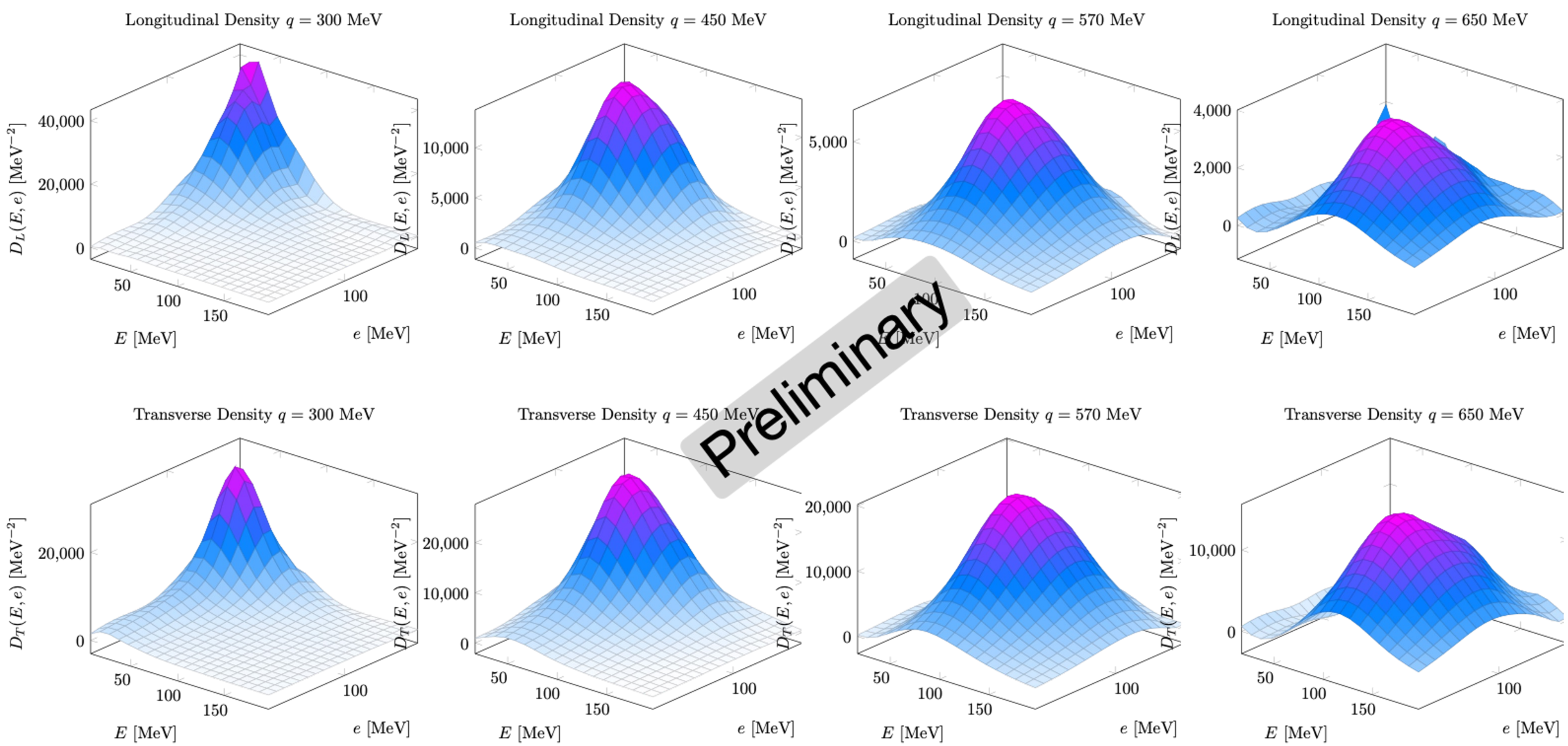
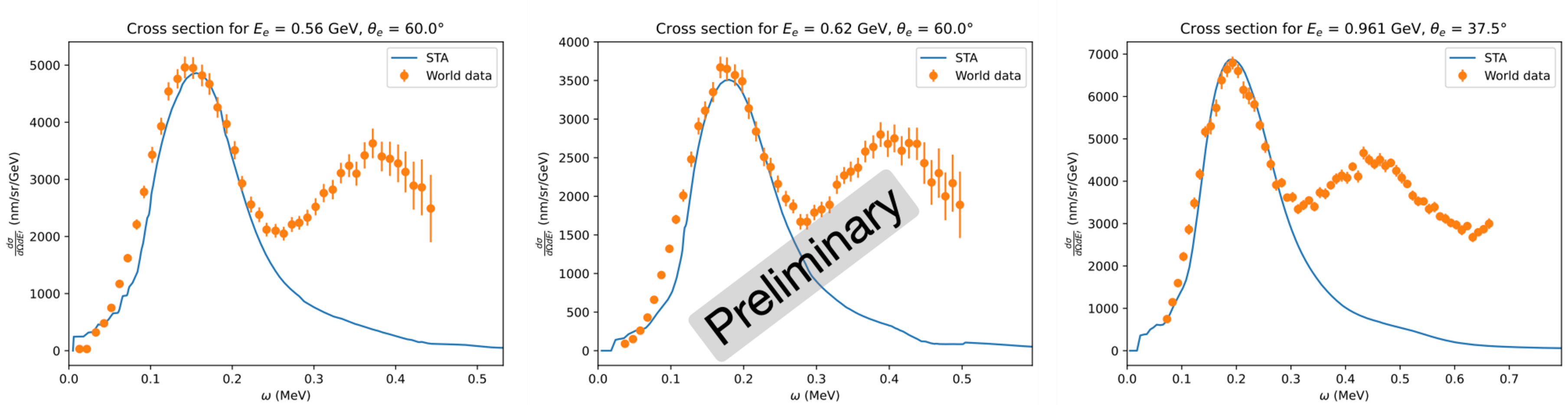
Short-Time Approximation Results

Longitudinal and transverse for ^{12}C for 300 < q < 650 MeV:





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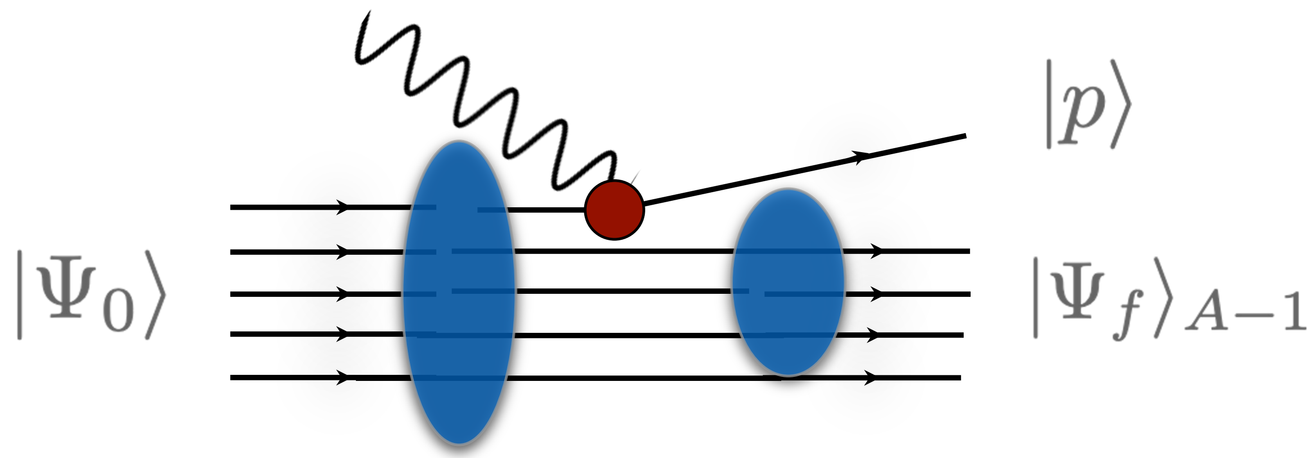
Cross sections: Spectral function approach

For sufficiently large values of |**q**|, the **factorization scheme** can be applied under the assumptions

$$|\Psi_f\rangle \to |p\rangle \otimes |\Psi_f\rangle_{A-1}$$

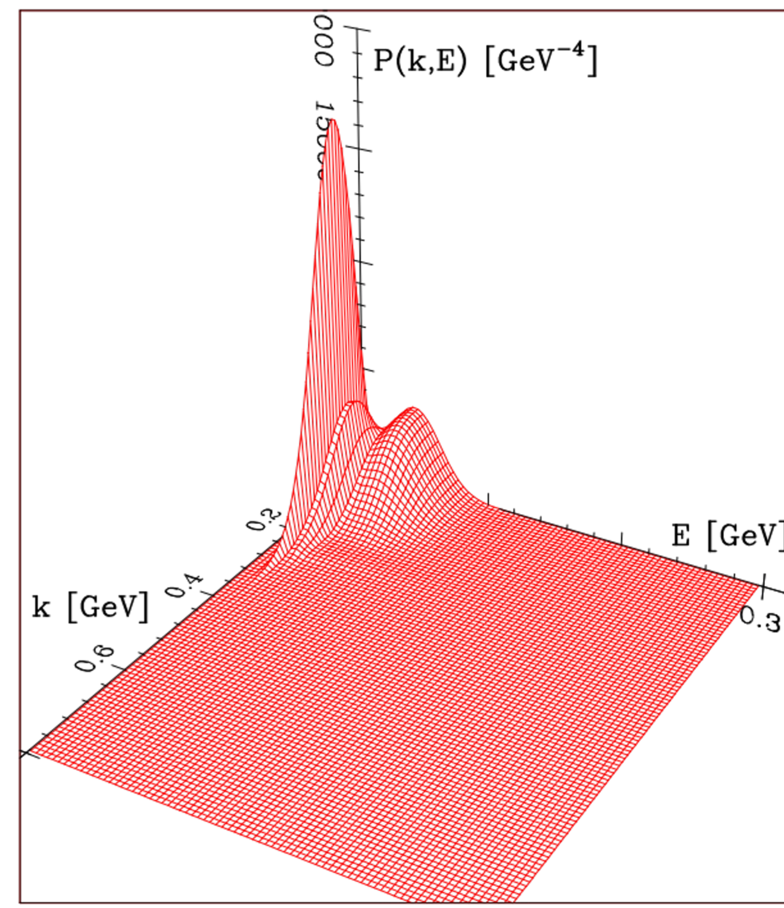
$$J_\alpha = \sum_i j_\alpha^i$$



The nuclear cross section is given in terms of the one describing the interaction with individual bound nucleons

$$d\sigma_A = \int dE d^3k \; d\sigma_N P(\mathbf{k}, E)$$

The intrinsic properties of the nucleus are described by the **Spectral Function**→ EFT and nuclear many-body methods

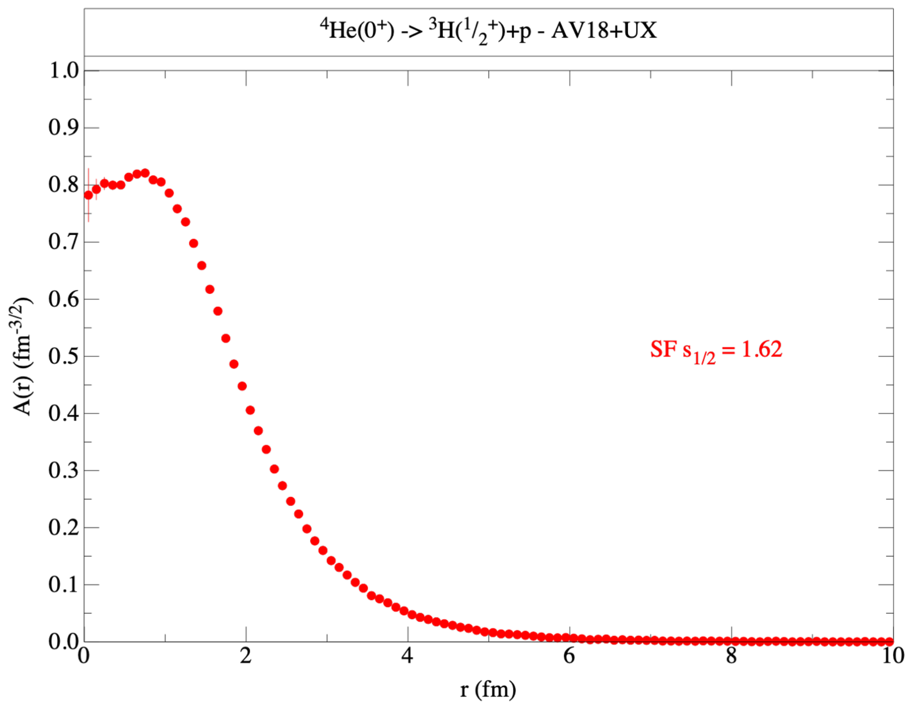


O. Benhar et al, Rev.Mod.Phys. 80 (2008)

QMC Spectral function of light nuclei

- Single-nucleon spectral function:

$$P_{p,n}(\mathbf{k},E)=\sum_n \left|\langle\Psi_0^A|[|k\rangle\otimes|\Psi_n^{A-1}\rangle]\right|^2\delta(E+E_0^A-E_n^{A-1})=P^{MF}(\mathbf{k},E)+P^{\rm corr}(\mathbf{k},E)$$

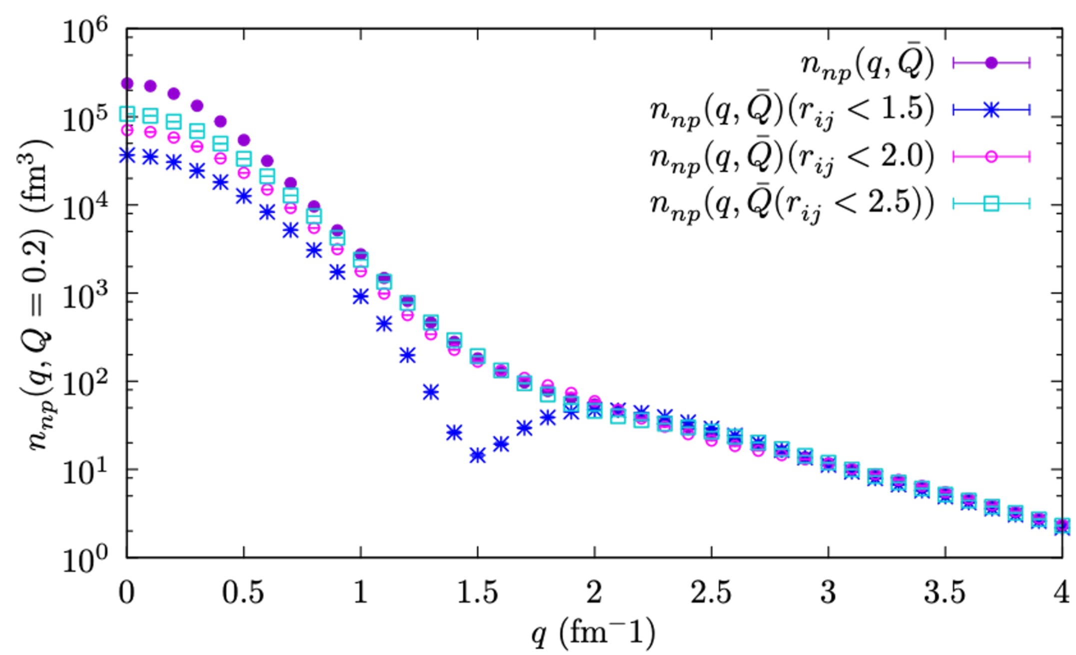


$$P^{MF}(\mathbf{k},E)=\left|\langle\Psi_0^A|[|k\rangle\otimes|\Psi_n^{A-1}\rangle]\right|^2 \times\delta\left(E-B_A+B_{A-1}-\frac{\mathbf{k}^2}{2m_{A-1}}\right)$$

- The single-nucleon overlap has been computed within VMC (center of mass motion fully accounted for)

$$P^{\rm corr}(\mathbf{k},E)=\int d^3k'\left|\langle\Psi_0^A|[|k\rangle|k'\rangle\otimes|\Psi_n^{A-2}\rangle]\right|^2 \times\delta\left(E-B_A-e(\mathbf{k}')+B_{A-2}-\frac{(\mathbf{k}+\mathbf{k}')^2}{2m_{A-2}}\right)$$

- Written in terms of two-body momentum distribution



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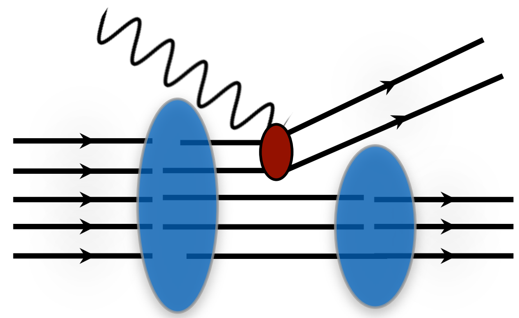
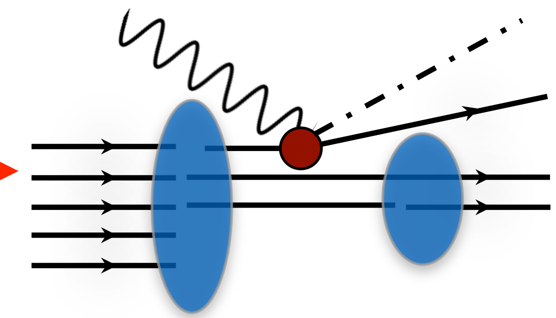
SF: Extended Factorization Scheme

$$|f\rangle \to |pp'\rangle_a \otimes |f_{A-2}\rangle$$



$$|f\rangle \to |p_\pi p\rangle \otimes |f_{A-1}\rangle$$

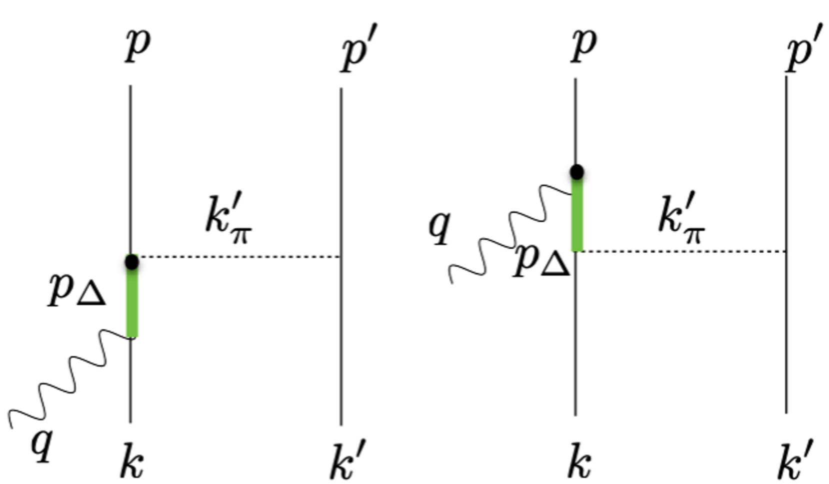


The hadronic tensor for two-body current factorizes as

$$R^{\mu\nu}_{2b}(\mathbf{q},\omega) \propto \int dE d^3k d^3k' P_{2b}(\mathbf{k},\mathbf{k}',E) \times d^3p d^3p' |\langle kk'|j^\mu_{2b}|pp'\rangle|^2$$

Production of real π in the final state

$$R^{\mu\nu}_{1b\pi}(\mathbf{q},\omega) \propto \int dE d^3k P_{1b}(\mathbf{k},E) \times d^3p d^3k_\pi |\langle k|j^\mu|pk_\pi\rangle|^2$$



- Pion production elementary amplitudes currently derived within the extremely sophisticated **Dynamic Couple Chanel approach**;

Delta decay width:

$$\Gamma(p_\Delta) = \frac{(4f_{\pi N\Delta})^2}{12\pi m_\pi^2}\frac{|\mathbf{d}|^3}{\sqrt{s}}(m_N + E_d)R(\mathbf{r}^2)$$

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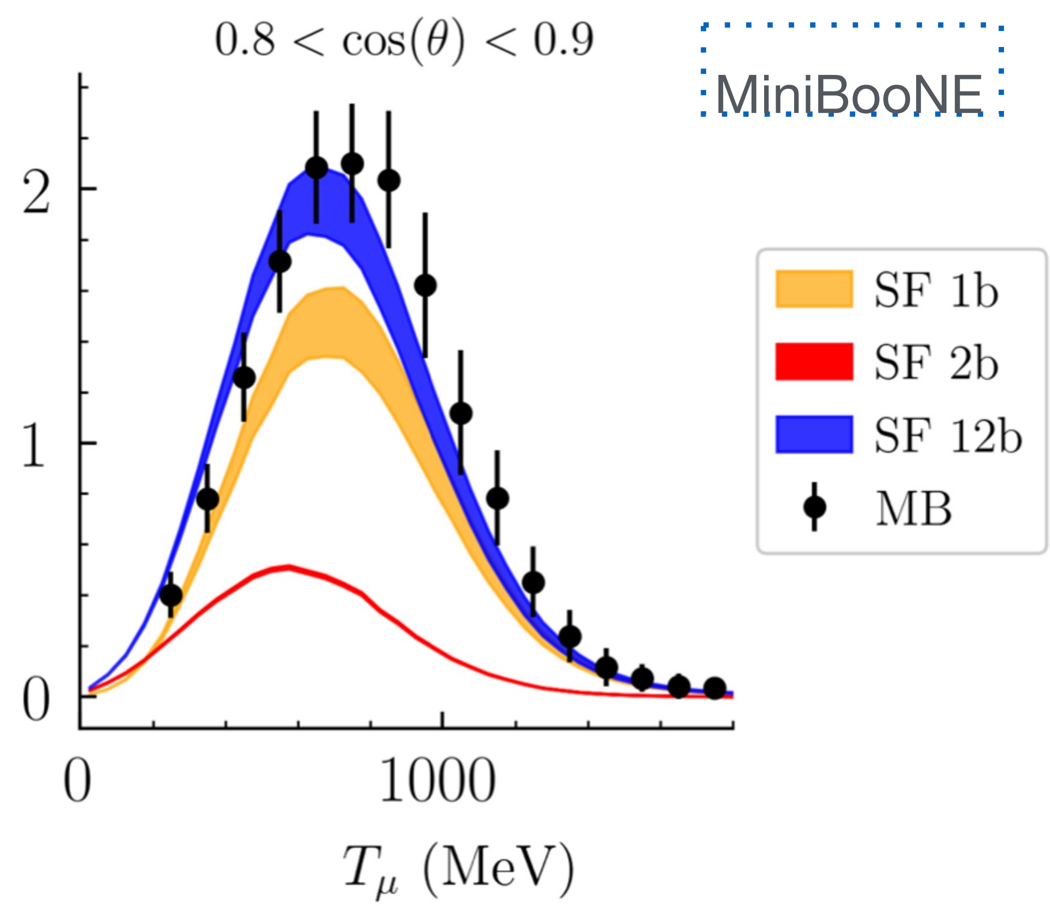
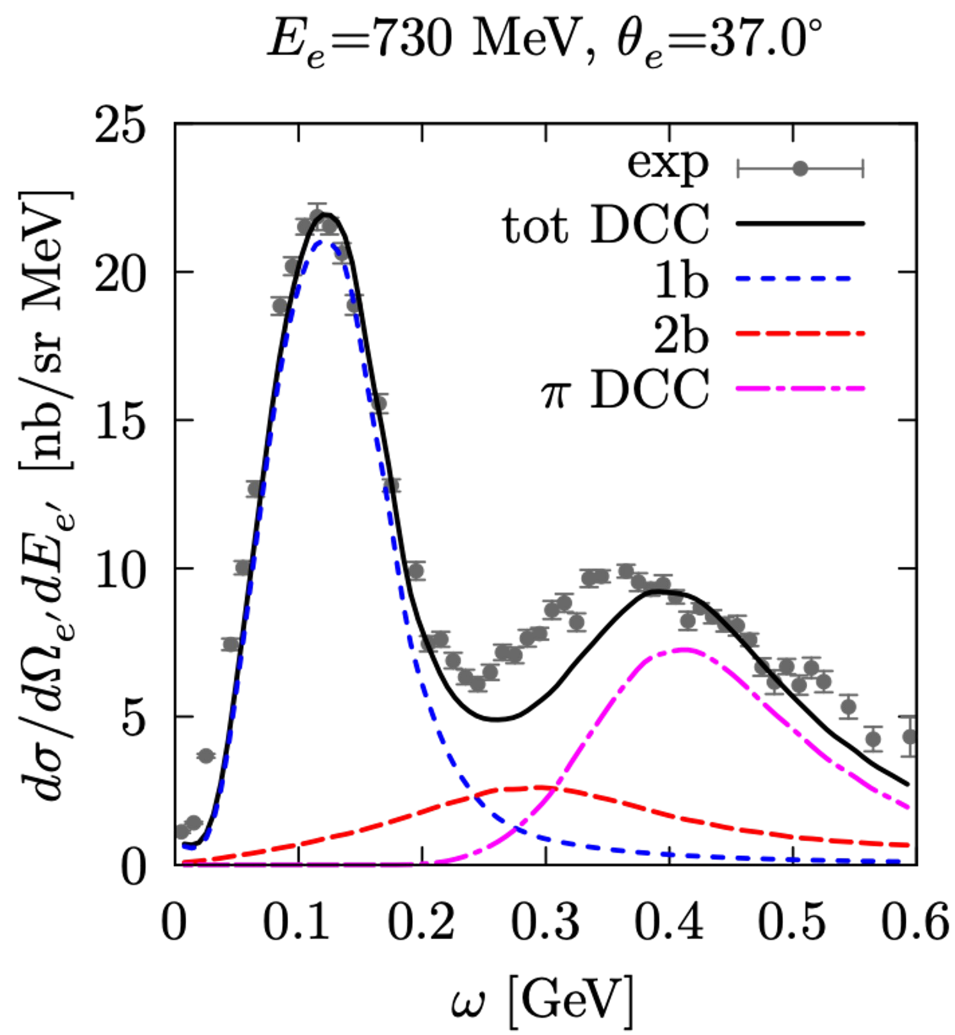
SF: Extended Factorization Scheme

Spectral function formalism: unified framework able to describe the different reaction mechanisms retaining an accurate treatment of nuclear dynamics





Good agreement with inclusive scattering cross section for both electron- and neutrino-nucleus scattering

Suitable to describe fully exclusive processes; results still need to be compared with data

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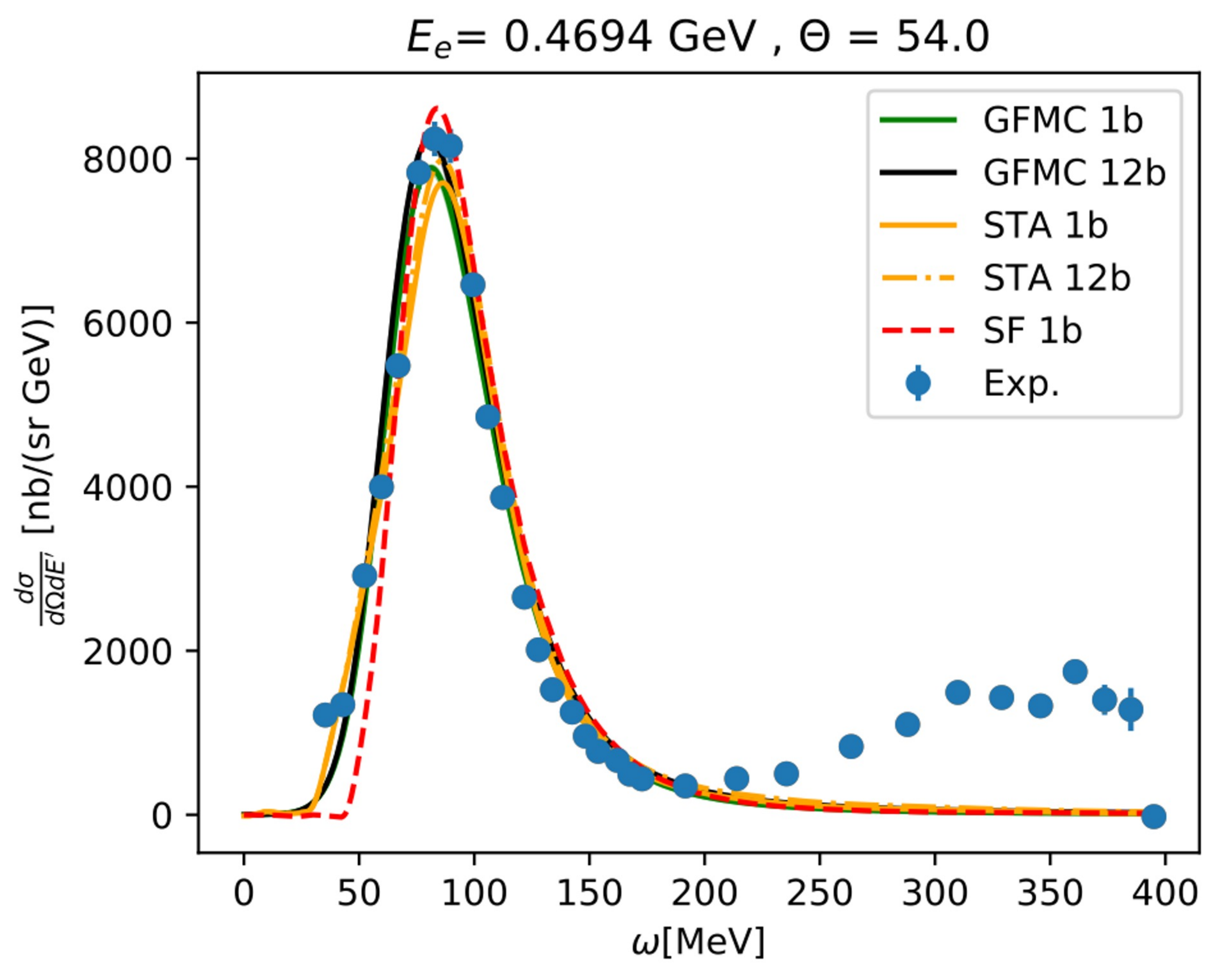
Model selection

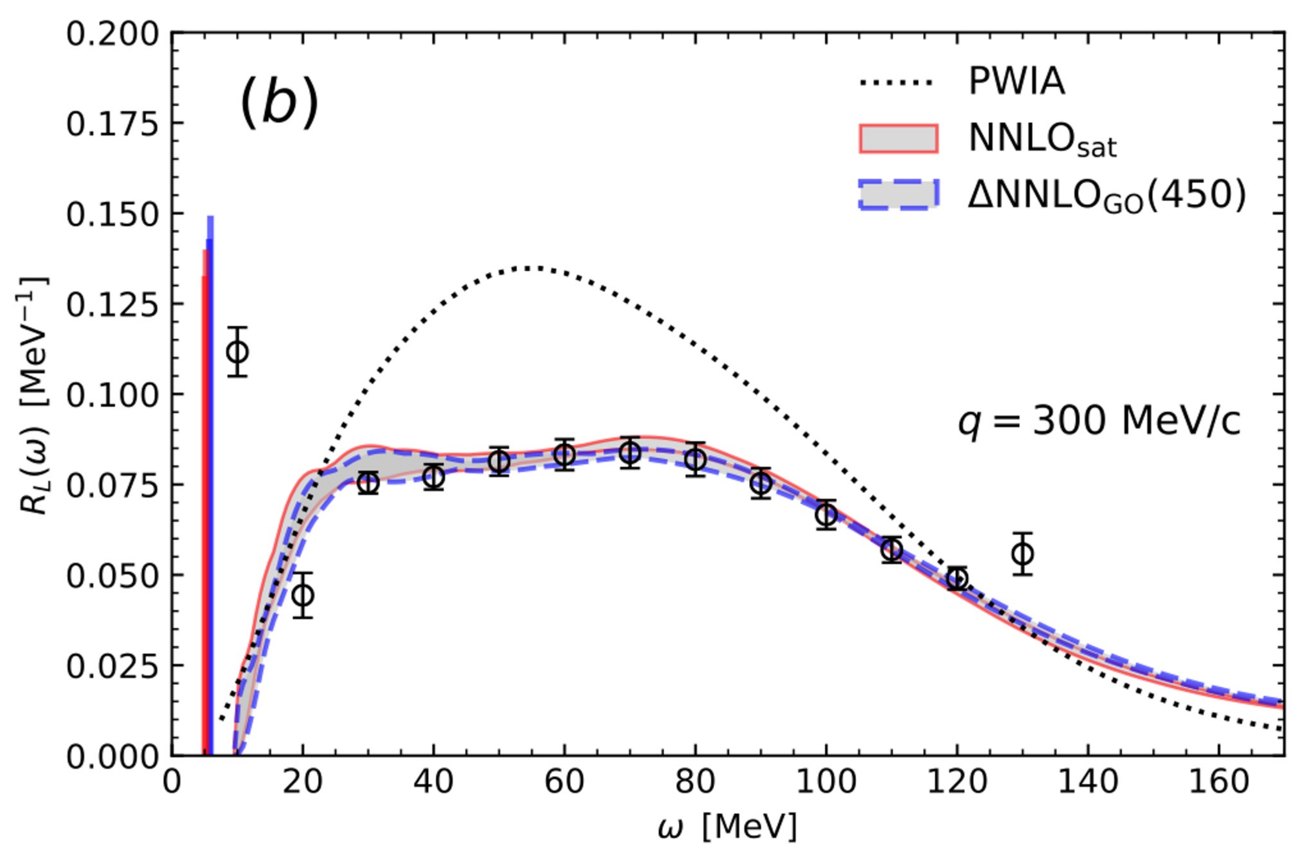
XSEC 4 thrust



Andreoli et al, Phys.Rev.C 105 (2022) 1, 014002

Comparing different nuclear many-body methods is necessary to assess the uncertainty associated with factorization schemes, non-relativistic kinematics

Sobczyk et al, Phys.Rev.Lett. 127 (2021) 7, 072501



Different Chiral EFT interactions and currents can be used as input of the nuclear calculation

An estimate of the truncation error can be provided and incorporated in the cross section predictions

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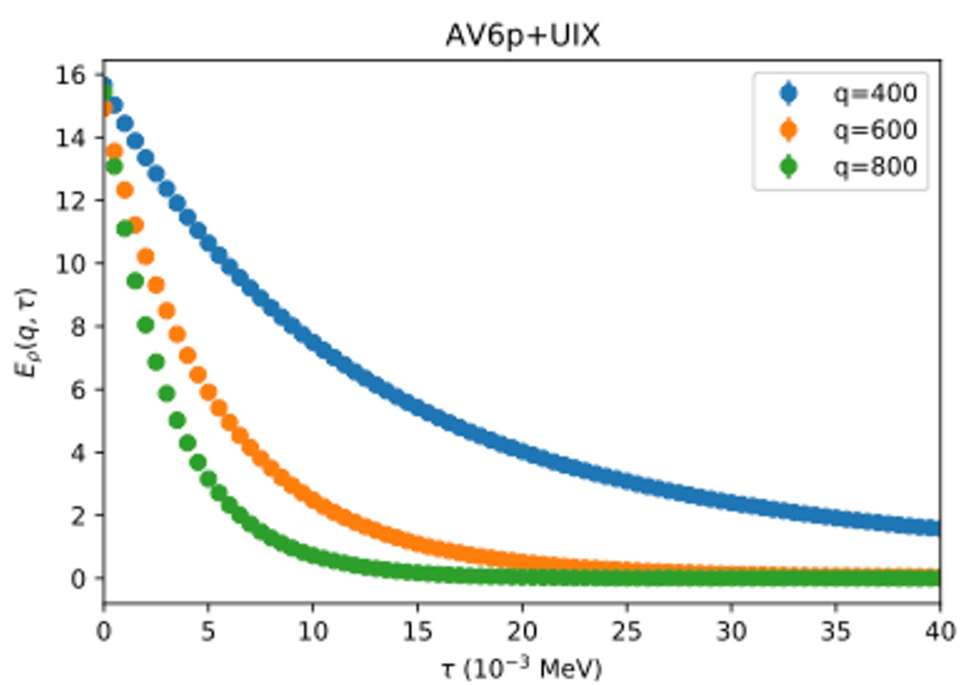
Better constrain our models

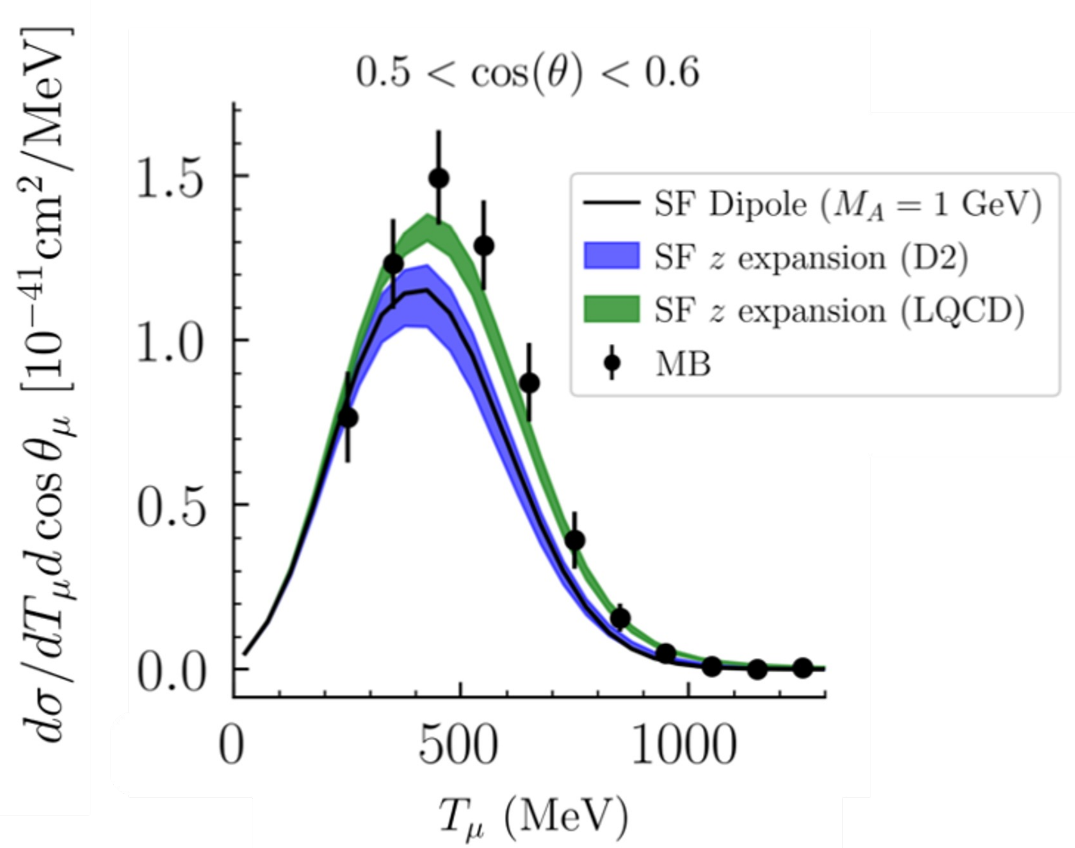
XSEC 4 thrust

– ^{16}O density response function for different q



Use Auxiliary Field Diffusion Monte Carlo to obtain electroweak responses of A=16 and 40 nuclei.
Use different sets of nuclear interactions derived from Chiral EFT

MiniBooNE



D.Simons, N. Steinberg et al arXiv:2210.02455

Different axial form factors leads to ~ 20% difference in the cross section at the peak

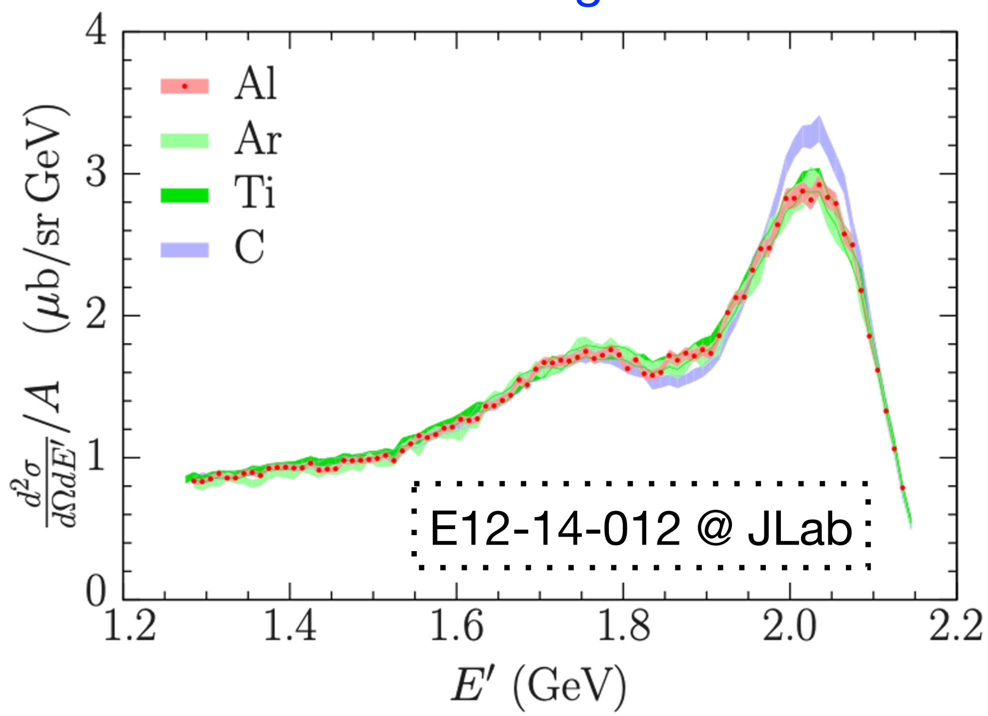
Conduct sensitivity studies of the cross sections for different LQCD inputs computed in the thrusts: XSEC 1, 2 and 3

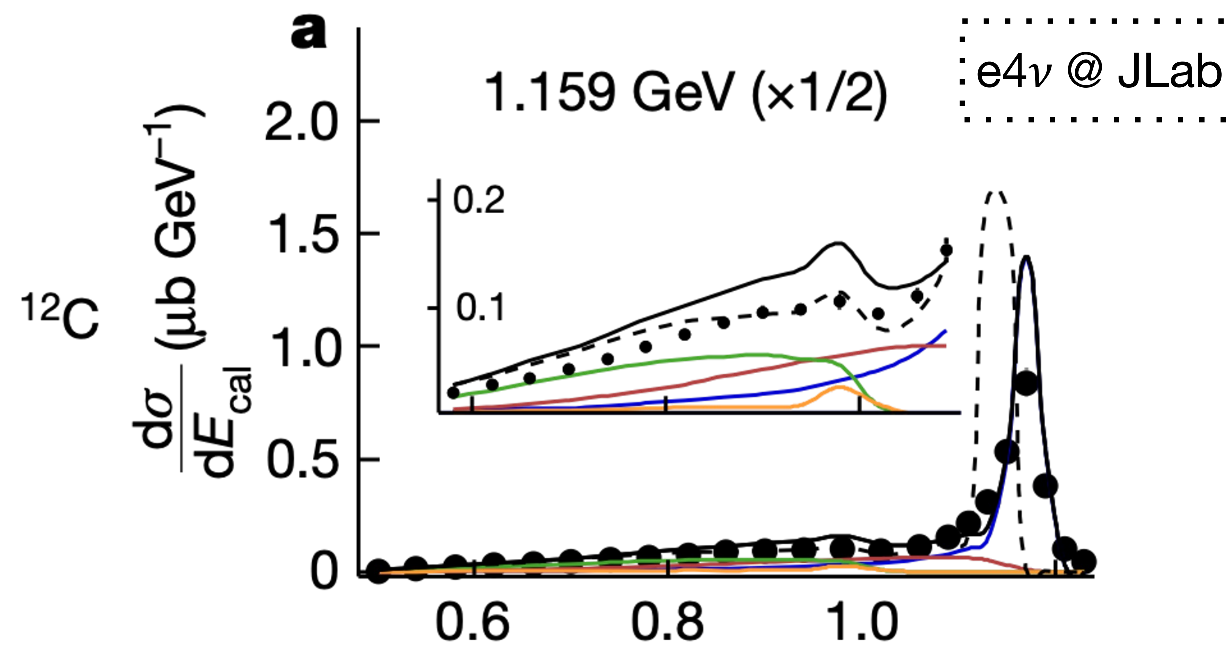
Testing our models

XSEC 5 thrust

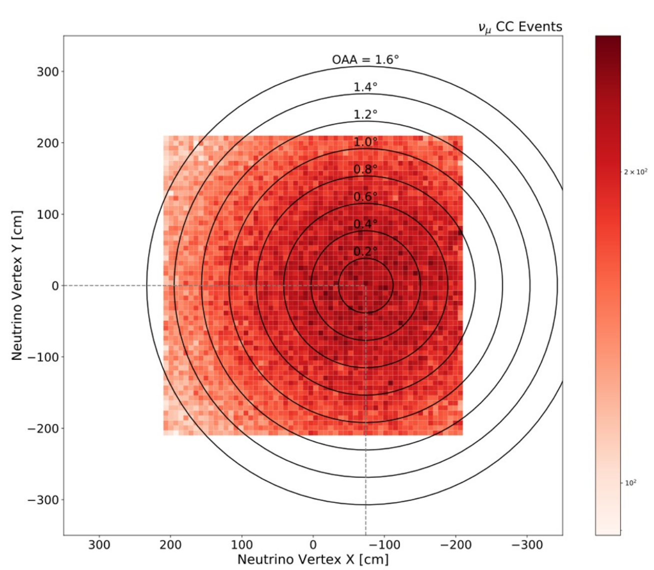
Semi-exclusive electron scattering data provide input and allow to test the accuracy of interaction models and event generators used in oscillation analyses

Electron Scattering and Neutrino Physics, 2022 Snowmass Summer Study





SBND-PRISM



The SBN program will provide an order of magnitude more data of neutrino-Argon interactions than is currently available (test exclusive predictions)

Leverage the PRISM features of SBND to isolate the contribution of different reaction mechanisms and constrain systematic uncertainties

Summary and Outlook

Neutrino physics will have unprecedented precision capabilities. This requires accurate predictions of neutrino-nucleus cross sections + uncertainty

NTNP topical collaboration program will cover different thrusts which include LQCD and nuclear many-body advances which will improve our description of neutrino-nucleus cross sections

Activities	Yr 1	Yr 2	Yr 3	Yr 4	Yr 5
XSEC: *Neutrino-nucleus scattering*					
XSEC-1 Nucleon elastic form factors with sLapH [CM, AN, AS, AWL]					
XSEC-2 $N \rightarrow \Delta$ transitions with sLapH [CM, AN, AS, AWL]					
XSEC-3 NN e.w. matrix elements with LQCD [CM, AN, AS, AWL]					
XSEC-4 Inclusive processes with QMC, STA, SF [JC, BD, SG, AL, SP, MP, NR, RS, IT]					
XSEC-5 Exclusive processes with STA and SF [JC, SG, AL, SP, MP, NR, RS]					

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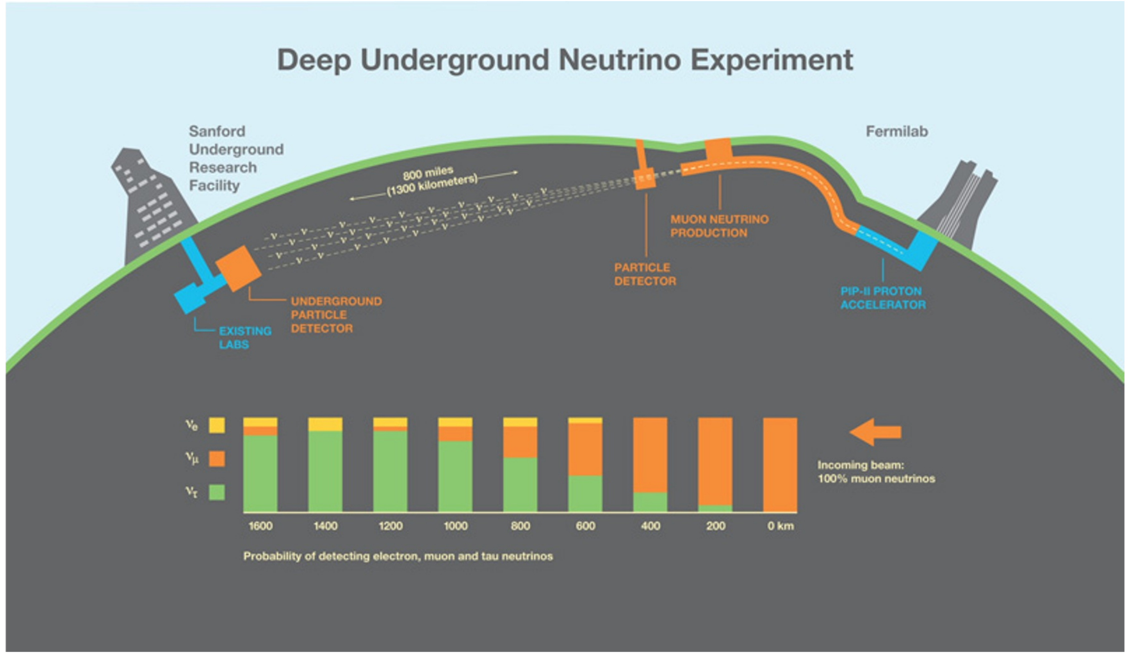
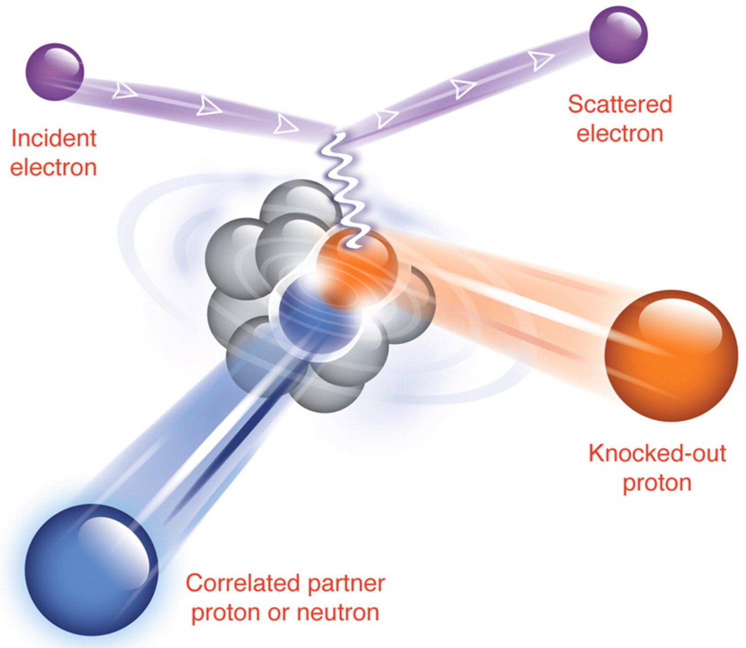
Thank you for your attention!

Short-Time Approximation

Short-Time-Approximation Goals:

- Describe electroweak scattering from A > 12 without losing two-body physics
- Account for exclusive processes
- Incorporate relativistic effects





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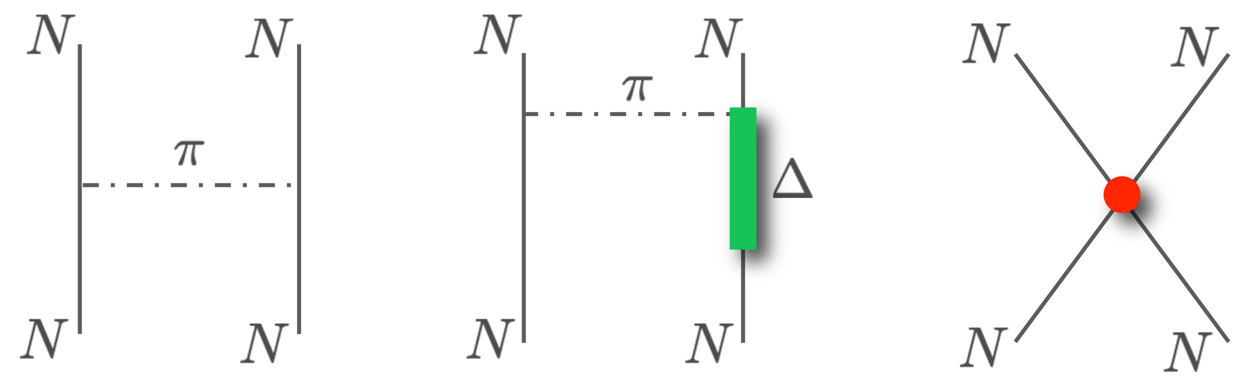
Phenomenological potential: av18 + IL7

Phenomenological potentials explicitly include the **long-range one-pion exchange interaction** and a set of **intermediate- and short-range phenomenological terms**

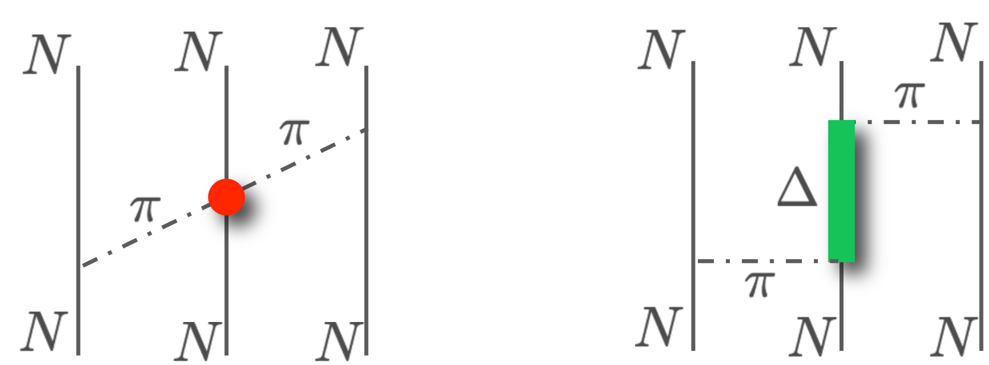
- **Argonne v_{18}** is a finite, local, configuration-space potential controlled by ~4300 np and pp scattering data below 350 MeV of the Nijmegen database

$$v_{ij} = \sum_{p=1}^{18} v^p(r_{ij}) O^p_{ij}$$



- Phenomenological three-nucleon interactions, like the **Illinois 7**, effectively include the lowest nucleon excitation, the Δ(1232) resonance, end other nuclear effects

$$V^{3N}_{ijk} = A^{PW}_{2\pi} O^{2\pi,PW}_{ijk} + A^{SW}_{2\pi} O^{2\pi,SW}_{ijk} + A^{\Delta R}_{3\pi} O^{3\pi,\Delta R}_{ijk} + A^R O^R_{ijk}$$



The parameters of the AV18 + IL7 are fit to properties of **exactly solvable light nuclear systems**.

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Why relativity is important

$$R_{\alpha\beta}(\omega,\mathbf{q}) = \sum_f \langle 0|J^\dagger_\alpha(\mathbf{q})|f\rangle\langle f|J_\beta(\mathbf{q})|0\rangle \delta(\omega - E_f + E_0)$$

Kinematics

Currents

Covariant expression of the e.m. current:

$$j^\mu_{\gamma,S} = \bar{u}(\mathbf{p}')\Big[\frac{G_E^S + \tau G_M^S}{2(1+\tau)}\gamma^\mu + i\frac{\sigma^{\mu\nu}q_\nu}{4m_N}\frac{G_M^S - G_E^S}{1+\tau}\Big]u(\mathbf{p})$$

Nonrelativistic expansion in powers of $\boldsymbol{p}/m_N$

$$j^0_{\gamma,S} = \frac{G_E^S}{2\sqrt{1+Q^2/4m_N^2}} - i\frac{2G_M^S - G_E^S}{8m_N^2}\mathbf{q}\cdot(\boldsymbol{\sigma}\times\mathbf{p})$$

Energy transfer at the quasi-elastic peak:

$$w_{QE} = \sqrt{\mathbf{q}^2 + m_N^2} - m_N \qquad w^{nr}_{QE} = \mathbf{q}^2/(2m_N)$$

Fermilab

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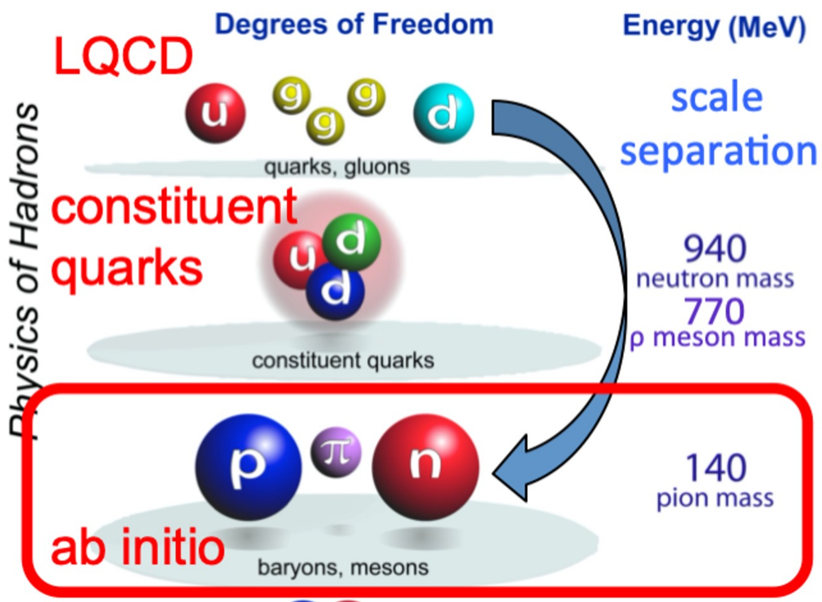
Chiral effective field theory



Wesolowski , et al, PRC 104, 064001 (2021)

T. Djärv, et al, PRC 105, 014005 (2022)

Formulate statistical models for UQ in EFT including Bayesian estimates of EFT truncation errors

Input that can be used in models for neutrino-nucleus interactions

WashU group is using MCMC to optimize determination of LEC and provide UQ for the Δ-full chiral potentials used in QMC calculations

Systematic construction of nuclear forces used to make predictions

Exploits the (approximate) broken chiral symmetry of QCD to construct interactions

Identify the soft and hard scale of the problem:

$$\mathcal{L}^{(n)} \sim \left(\frac{q}{\Lambda_b}\right)^n$$

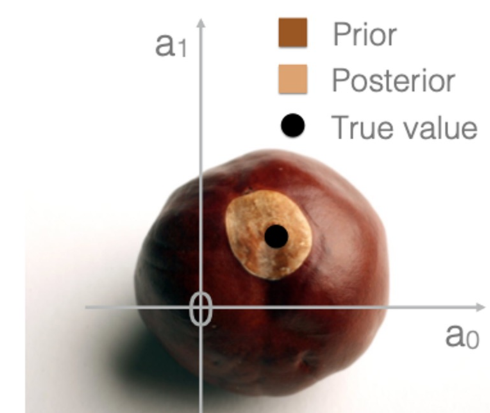
~ 100 MeV soft scale

~ 1 GeV hard scale

Design an organizational scheme to distinguish between more and less important terms

BAND
Bayesian Analysis of Nuclear Dynamics

https://bandframework.github.io/



BUQEYE Collaboration

https://buqeye.github.io/

Fermilab