What we need to know about δ_{NS}

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Purpose of this talk

1. Set the stage for ab-initio calculations of δ_{NS}
2. Provide a quick yet accurate reference to the most important steps
3. Stimulate further thoughts and discussions

References:

CYS, Gorchtein and Ramsey-Musolf, PRD 100 (2019) 1, 013001;
Gorchtein, PRL 123 (2019) 4, 042503;
CYS and Gorchtein, PRC 107 (2023) 3, 035503;
CYS, Particles 4 (2021) 4, 397;
Michael Gennari's talk, INT Program INT-23-1B

Basic notations and conventions

Isospin currents:
$$V^{\mu}_{Im_I} = \bar{\psi}\gamma^{\mu}\Gamma_{Im_I}\psi, \quad A^{\mu}_{Im_I} = \bar{\psi}\gamma^{\mu}\gamma_5\Gamma_{Im_I}\psi$$

$$\psi = (u, d)^{\mathrm{T}} \qquad \Gamma_{00} = \mathbb{I}_2, \quad \Gamma_{10} = \tau_3, \quad \Gamma_{1\pm 1} = \mp\frac{\tau_1 \pm i\tau_2}{\sqrt{2}}.$$

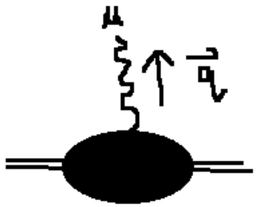
Electroweak currents:
$$J^{\mu}_{\mathrm{em}} = J^{(0)\mu}_{\mathrm{em}} + J^{(1)\mu}_{\mathrm{em}} = \frac{1}{6}V^{\mu}_{00} + \frac{1}{2}V^{\mu}_{10},$$

my convention

$$J^{\mu}_{W} = (J^{\mu}_{W})_V + (J^{\mu}_{W})_A = -\frac{1}{\sqrt{2}}V^{\mu}_{1+1} + \frac{1}{\sqrt{2}}A^{\mu}_{1+1},$$

$$J^{\mu}_{Z} = (J^{\mu}_{Z})_V + (J^{\mu}_{Z})_A$$
$$= (1 - 2\sin^2\theta_W)V^{\mu}_{10} - \frac{2}{3}\sin^2\theta_W V^{\mu}_{00} - A^{\mu}_{10},$$

(Spatial) Fourier transform of a current operator:



$$J^{\mu}(\vec{q}) \equiv \int d^3x e^{-i\vec{q}\cdot\vec{x}} J^{\mu}(t=0, \vec{x})$$

Basic notations and conventions

"Plane-wave" states: $|\phi(\vec{p})\rangle$,

$$\langle\phi(\vec{p})|\phi(\vec{p})\rangle = (2\pi)^3 2E(\vec{p})\delta^3(\vec{p}-\vec{p}) \to 2E(\vec{p})V$$

"Quantum-mechanical" states: $|\phi,\vec{p}) \equiv \dfrac{1}{\sqrt{2E(\vec{p})V}}|\phi(\vec{p})\rangle$,

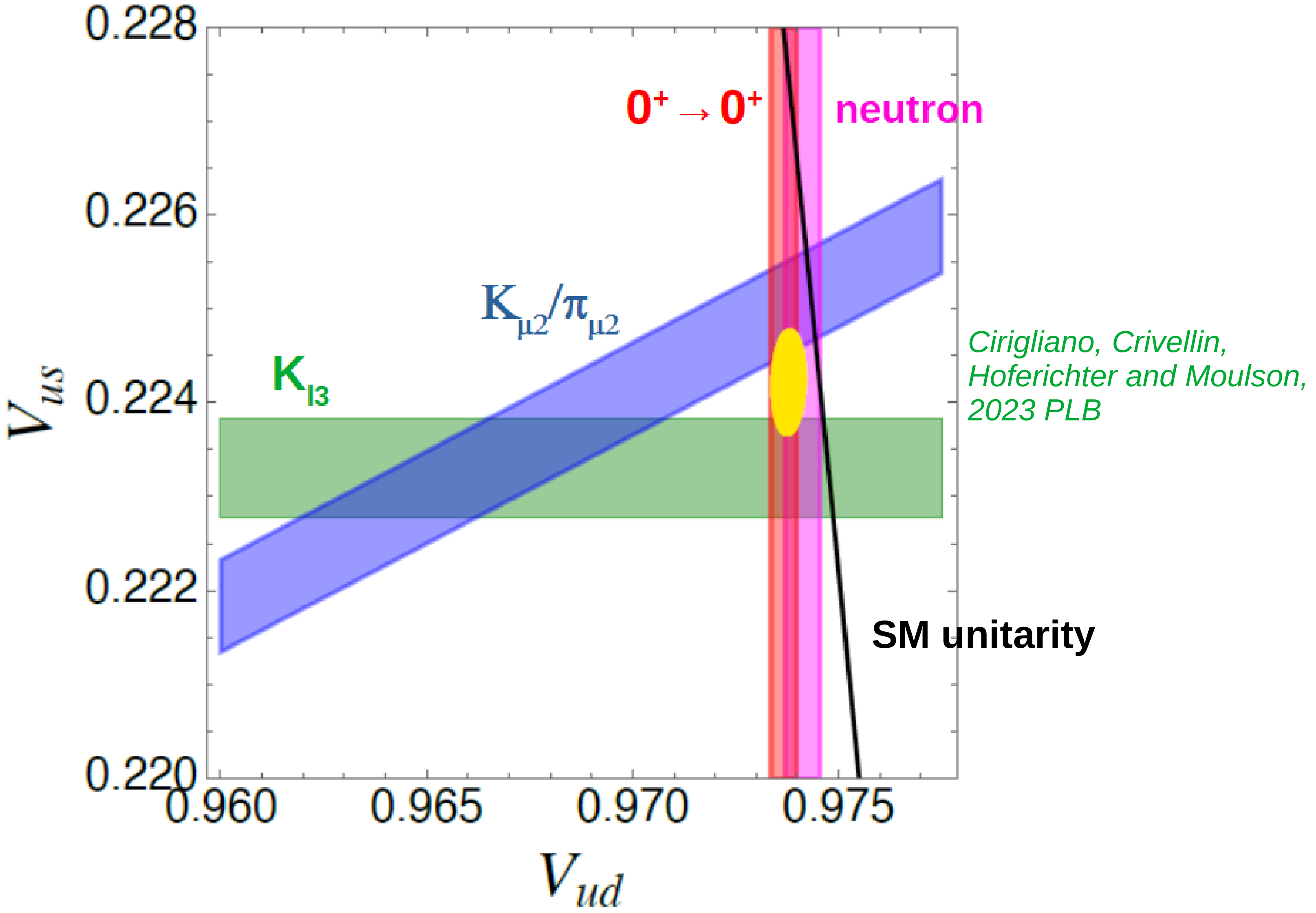
$$(\phi,\vec{p}|\phi,\vec{p}) = 1$$

Isospin symmetry is assumed in the pure-QCD system (at the amplitude level):

$$M_i = M_f = M$$

Isospin convention: $(I, m_I) = (½, +½)$ for proton (Particle physics's convention)

Background and Basic Setup

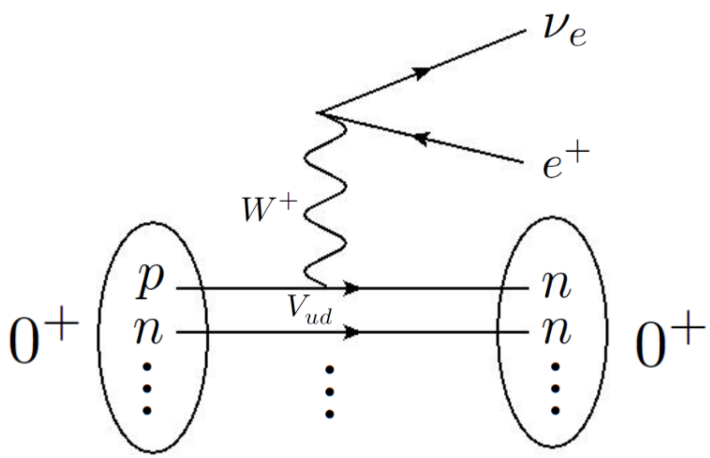


See Misha Gorshteyn's INT talk for a nice review of beta dacay, V_{ud} and CKM unitarity:

https://www.int.washington.edu/program/schedule/1208/2

“Superallowed” beta decays of I=1, $J^p=0^+$ nuclei

$$i(0^+) \to f(0^+) + e^+ + \nu_e$$



Provides the **best measurement of V_{ud} :**

- **23** measured transitions
- **15** with ft-precision better than **0.23%**

Hardy and Towner, 2020 PRC

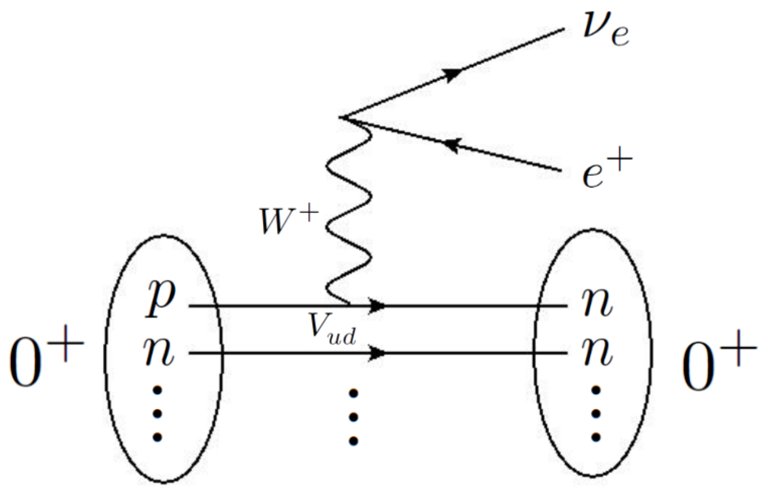
$m_I=+1 \to m_I=0$
${}^{10}_{6}\mathrm{C} \to {}^{10}_{5}\mathrm{B}$
${}^{14}_{8}\mathrm{O} \to {}^{14}_{7}\mathrm{N}$
${}^{18}_{10}\mathrm{Ne} \to {}^{18}_{9}\mathrm{F}$
${}^{22}_{12}\mathrm{Mg} \to {}^{22}_{11}\mathrm{Na}$
${}^{26}_{14}\mathrm{Si} \to {}^{26}_{13}\mathrm{Al}$
${}^{30}_{16}\mathrm{S} \to {}^{30}_{15}\mathrm{P}$
${}^{34}_{18}\mathrm{Ar} \to {}^{34}_{17}\mathrm{Cl}$
${}^{38}_{20}\mathrm{Ca} \to {}^{38}_{19}\mathrm{K}$
${}^{42}_{22}\mathrm{Ti} \to {}^{42}_{21}\mathrm{Sc}$
${}^{46}_{24}\mathrm{Cr} \to {}^{46}_{23}\mathrm{V}$
${}^{50}_{26}\mathrm{Fe} \to {}^{50}_{25}\mathrm{Mn}$
${}^{54}_{28}\mathrm{Ni} \to {}^{54}_{27}\mathrm{Co}$

$m_I=0 \to m_I=-1$
${}^{26m}_{13}\mathrm{Al} \to {}^{26}_{12}\mathrm{Mg}$
${}^{34}_{17}\mathrm{Cl} \to {}^{34}_{16}\mathrm{S}$
${}^{38m}_{19}\mathrm{K} \to {}^{38}_{18}\mathrm{Ar}$
${}^{42}_{21}\mathrm{Sc} \to {}^{42}_{20}\mathrm{Ca}$
${}^{46}_{23}\mathrm{V} \to {}^{46}_{22}\mathrm{Ti}$
${}^{50}_{25}\mathrm{Mn} \to {}^{50}_{24}\mathrm{Cr}$
${}^{54}_{27}\mathrm{Co} \to {}^{54}_{26}\mathrm{Fe}$
${}^{62}_{31}\mathrm{Ga} \to {}^{62}_{30}\mathrm{Zn}$
${}^{66}_{33}\mathrm{As} \to {}^{66}_{32}\mathrm{Ge}$
${}^{70}_{35}\mathrm{Br} \to {}^{70}_{34}\mathrm{Se}$
${}^{74}_{37}\mathrm{Rb} \to {}^{74}_{36}\mathrm{Kr}$

: ft-precision better than τ_n in UCNτ

"Superallowed" beta decays of I=1, $J^p=0^+$ nuclei

$$i(0^+) \to f(0^+) + e^+ + \nu_e$$



Tree-level amplitude:

$$\boxed{\begin{aligned} \mathfrak{M}_0 &= -\frac{G_F}{\sqrt{2}} V_{ud} \bar{u}_\nu \gamma_\mu (1-\gamma_5) v_e \langle \phi_f(\vec{p}\,') | J_W^{\dagger\mu}(0) | \phi_i(\vec{p}) \rangle \\ &\approx -\frac{G_F}{\sqrt{2}} V_{ud} \bar{u}_\nu \gamma_0 (1-\gamma_5) v_e \cdot 2M f_+(0) \end{aligned}}$$

Including radiative corrections:

$$\frac{d\Gamma}{dE_e} \to F(E_e)\left(1 + \frac{\alpha}{2\pi} g(E_e) + \delta_{\text{inner}}\right)\frac{d\Gamma}{dE_e}$$

Fermi's function   Sirlin's function   "inner correction"

Fermi's function: Coulomb interaction between outgoing positron and final nucleus

Fermi, Z.Phys, 88, 161 (1934)

Sirlin's function: Point-charge QED corrections, excluding Fermi's function

Sirlin, Phys.Rev. 164, 1767 (1967)

Including radiative corrections:

$$\frac{d\Gamma}{dE_e} \to F(E_e)\left(1 + \frac{\alpha}{2\pi} g(E_e) + \delta_{\text{inner}}\right)\frac{d\Gamma}{dE_e}$$

Fermi's function | Sirlin's function | "inner correction"

It was long believed that the "inner correction" is E_e-independent, but recent work suggested possibility of non-trivial E_e-dependence in nuclear system

Gorchtein, PRL 123 (2019) 4, 042503

We split the inner correction, after averaging over phase space, into "single-nucleon" and "nucleus-dependent" piece:

$$\boxed{\langle \delta_{\text{inner}}(E_e)\rangle = \underline{\Delta_R^V} + \underline{\delta_{\text{NS}}}}$$

where $$\left\langle \hat{O}(E_e)\right\rangle \equiv \frac{\int_{m_e}^{E_0} dE_e |\vec{p}_e| E_e (E_0 - E_e)^2 F(E_e) \hat{O}(E_e)}{\int_{m_e}^{E_0} dE_e |\vec{p}_e| E_e (E_0 - E_e)^2 F(E_e)}$$

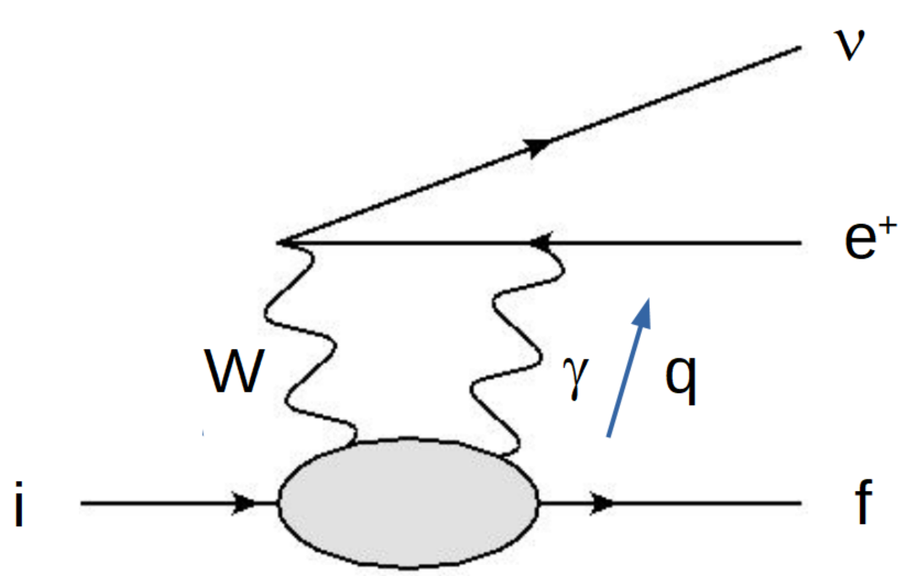
Formula: Average over phase space

$$E_0 = (M_i^2 - M_f^2 + m_e^2)/(2M_i)$$

The only inner correction that distinguishes nucleon and nucleus comes from the **γW-box diagram**, with an ε-tensor from the lepton structure:

Sirlin, Rev.Mod.Phys.50 (1978) 573

$$\delta\mathfrak{M}^b_{\gamma W} = -\sqrt{2} i G_F V_{ud}\, e^2 \bar{u}_\nu \gamma_\lambda (1-\gamma_5) v_e \cdot \int \frac{d^4q}{(2\pi)^4} \frac{M_W^2}{M_W^2 - q^2} \frac{\epsilon^{\mu\nu\alpha\lambda} q_\alpha T_{\mu\nu}(p,q)}{[(p_e-q)^2 - m_e^2 + i\varepsilon](q^2+i\varepsilon)}$$



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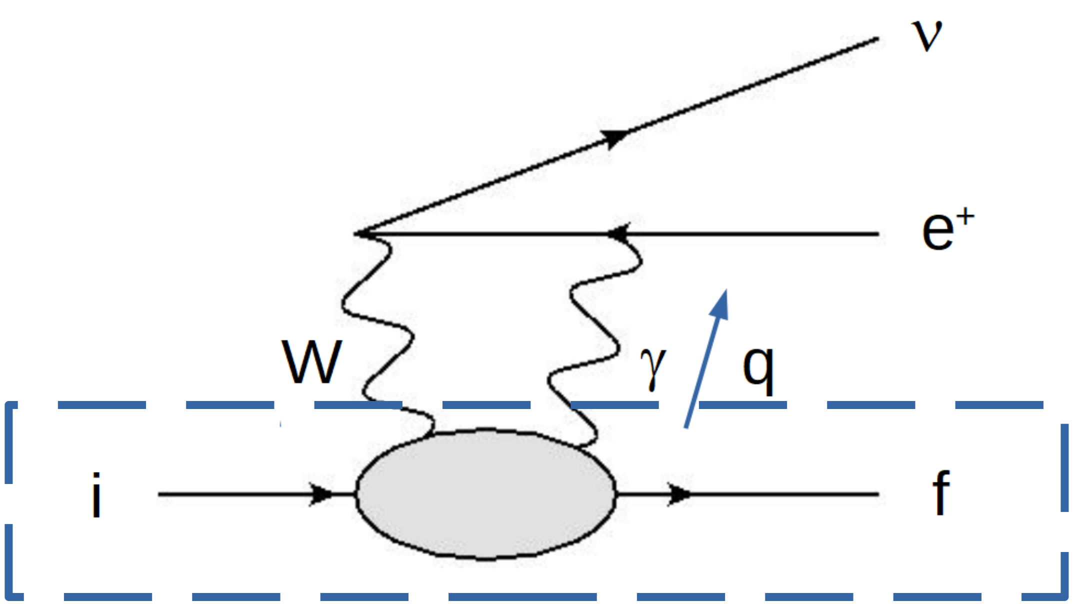
$$\boxed{\begin{aligned} \delta\mathfrak{M}^b_{\gamma W} \;&=\; -\sqrt{2} i G_F V_{ud}\, e^2 \bar{u}_\nu \gamma_\lambda (1-\gamma_5) v_e \cdot \\ &\int \frac{d^4 q}{(2\pi)^4} \frac{M_W^2}{M_W^2 - q^2} \frac{\epsilon^{\mu\nu\alpha\lambda} q_\alpha T_{\mu\nu}(p,q)}{[(p_e-q)^2 - m_e^2 + i\varepsilon](q^2+i\varepsilon)} \end{aligned}}$$

$T_{\mu\nu}(p,q)$



“Generalized forward Compton tensor”:

$$T^{\mu\nu}(p,q) = \frac{1}{2}\int d^4x e^{iq\cdot x}\langle\phi_f(p)|T[J^\mu_{\mathrm{em}}(x)\left(J^{\dagger\nu}_W(0)\right)_A]|\phi_i(p)\rangle$$

$$= \frac{i\epsilon^{\mu\nu\alpha\beta}p_\alpha q_\beta}{2M\nu}T_3(\nu,Q^2)$$

$\epsilon^{0123} = -1 \qquad \nu = \frac{p\cdot q}{M}\,,\; Q^2 = -q^2$

We follow the normalization in *Seng and Gorchtein, PRC 107 (2023) 3, 035503*

Setting $\vec{p} = \vec{0},\;\; \vec{q} = \mathbf{q}\hat{z}\;\;,$

one can write the **invariant amplitude** in terms of quantum-mechanical states:

$$\boxed{T_3(\nu,Q^2) = -\frac{2M\nu}{\mathbf{q}}\sum_X\left\{\frac{(\phi_f,\vec{0}|J^x_{\mathrm{em}}(\vec{q})|X)(X|\left(J^{\dagger y}_W(-\vec{q})\right)_A|\phi_i,\vec{0})}{\nu_X-\nu-i\varepsilon} + \frac{(\phi_f,\vec{0}|\left(J^{\dagger y}_W(-\vec{q})\right)_A|X)(X|J^x_{\mathrm{em}}(\vec{q})|\phi_i,\vec{0})}{\nu_X+\nu-i\varepsilon}\right\}}$$

where $\quad \nu_X \equiv E_X - M$

Neglecting terms suppressed by m_e, this RC is proportional to the tree-level amplitude:

$$\delta\mathfrak{M} = \Box^b_{\gamma W}(E_e)\mathfrak{M}_0$$

where

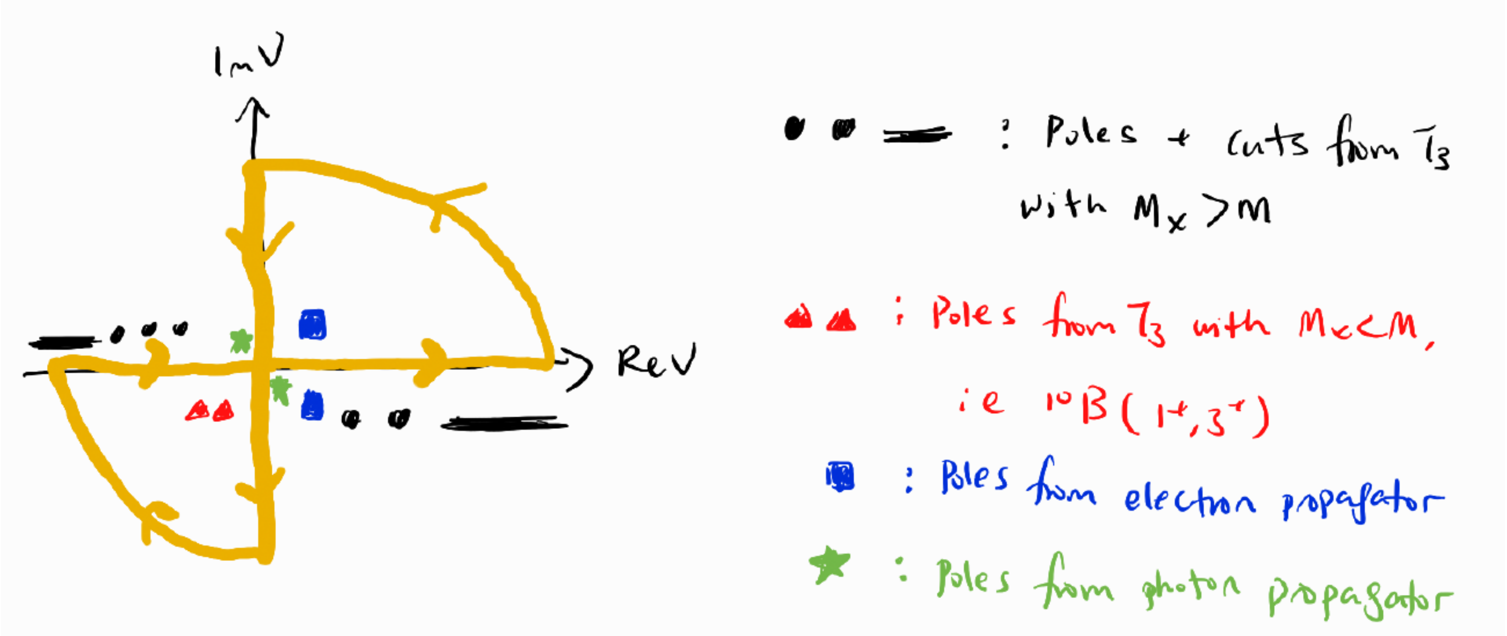
$$\boxed{\Box^b_{\gamma W}(E_e) = e^2 \mathfrak{Re} \int \frac{d^4q}{(2\pi)^4} \frac{M_W^2}{M_W^2 - q^2} \frac{1}{[(p_e - q)^2 - m_e^2 + i\varepsilon]} \frac{1}{q^2 + i\varepsilon} \cdot \frac{Q^2 + M\nu \frac{p_e \cdot q}{p \cdot p_e}}{M\nu} \frac{T_3(\nu, Q^2)}{f_+(0)}}$$

which is the **starting point** of all further analysis.

Only the **REAL PART** is retained because that's what matter in the interference with tree-level amplitude.

A case-study with $^{10}C \rightarrow\ ^{10}B$

First, perform the ν-integral using **Wick rotation**



$$\boxed{\square^b_{\gamma W} = \underbrace{(\square^b_{\gamma W})_{\mathrm{Wick}} + (\square^b_{\gamma W})_{\mathrm{res},e}}_{\text{Regular to } E_e} + \underbrace{(\square^b_{\gamma W})_{\mathrm{res},T_3}}_{\text{Singular to } E_e}}$$

We expand the "regular" pieces to $O(E_e)$, but retain the full E_e-dependece in the "singular" piece.

Dispersion relation of the invariant amplitude

"**Even**" and "**odd**" components of the invariant amplitude:

$$T_{3,\pm}(\nu, Q^2) \equiv \frac{1}{2}\left[T_3(\nu, Q^2) \pm T_3(-\nu, Q^2)\right]$$

They satisfy different **dispersion relations**:

$$\begin{aligned} iT_{3,-}(\nu, Q^2) &= 4\nu \int_{\nu_{\rm thr}}^{\infty} d\nu' \frac{F_{3,-}(\nu', Q^2)}{\nu'^2 - \nu^2} \\ iT_{3,+}(\nu, Q^2) &= 4\nu^2 \int_{\nu_{\rm thr}}^{\infty} d\nu' \frac{F_{3,+}(\nu', Q^2)}{\nu'(\nu'^2 - \nu^2)} \end{aligned}$$

where the **structure function/response function** reads:

$$\boxed{\begin{aligned} F_{3,\pm}(\nu, Q^2) &= -\frac{iM\nu}{2\mathbf{q}} \sum_X \delta(E_X - M - \nu) \times \\ & \Big\{ ({}^{10}\mathrm{B}(0^+), \vec{0}| J^x_{\rm em}(\vec{q})|X)(X|(J^{\dagger y}_W(-\vec{q}))_A|{}^{10}\mathrm{C}(0^+), \vec{0}) \\ & \mp ({}^{10}\mathrm{B}(0^+), \vec{0}|(J^{\dagger y}_W(-\vec{q}))_A|X)(X|J^x_{\rm em}(\vec{q})|{}^{10}\mathrm{C}(0^+), \vec{0}) \Big\} \end{aligned}}$$

$$\nu_{\rm thr} = (M_X)_{\rm min} - M$$

Expansion of the “regular” terms, neglecting electron mass:

Detail: *Seng and Gorchtein, PRC 107 (2023) 3, 035503, Section V*

$$\boxed{(\Box^b_{\gamma W})_{\mathrm{Wick}} + (\Box^b_{\gamma W})_{\mathrm{res},e} = \boxminus_0 + \boxminus_1 E_e + \mathcal{O}(E_e^2)}$$

The coefficients can be computed either in **dispersive** or **non-dispersive** way.

Dispersive representation:

$$\boxminus_0 = \frac{\alpha}{\pi}\int_0^\infty dQ^2 \frac{M_W^2}{M_W^2+Q^2}\int_{\nu_{\rm thr}}^\infty \frac{d\nu}{\nu}\frac{\nu+2\sqrt{\nu^2+Q^2}}{(\nu+\sqrt{\nu^2+Q^2})^2}\frac{F_{3,-}(\nu,Q^2)}{Mf_+(0)}$$

$$\boxminus_1 = \frac{2\alpha}{3\pi}\int_0^\infty dQ^2 \int_{\nu_{\rm thr}}^\infty \frac{d\nu}{\nu}\frac{\nu+3\sqrt{\nu^2+Q^2}}{(\nu+\sqrt{\nu^2+Q^2})^3}\frac{F_{3,+}(\nu,Q^2)}{Mf_+(0)}$$

What needs to be computed using ab-initio methods are the **response functions**:

$$\begin{aligned} F_{3,\pm}(\nu,Q^2) \quad = \quad & -\frac{iM\nu}{2\mathbf{q}}\sum_X \delta(E_X - M - \nu)\times \\ & \Big\{ ({}^{10}\mathrm{B}(0^+),\vec{0}|J^x_{\rm em}(\vec{q})|X)(X|(J^{\dagger y}_W(-\vec{q}))_A|{}^{10}\mathrm{C}(0^+),\vec{0}) \\ & \mp ({}^{10}\mathrm{B}(0^+),\vec{0}|(J^{\dagger y}_W(-\vec{q}))_A|X)(X|J^x_{\rm em}(\vec{q})|{}^{10}\mathrm{C}(0^+),\vec{0}) \Big\} \end{aligned}$$

Non-dispersive representation:

$$\boxminus_0 = e^2 \mathfrak{Re} \int \frac{d^4 q_E}{(2\pi)^4} \frac{M_W^2}{M_W^2 + Q^2} \frac{1}{(Q^2)^2} \frac{Q^2 - \nu_E^2}{\nu_E} \frac{T_3(i\nu_E, Q^2)}{M f_+(0)}$$

$$\boxminus_1 = -\frac{8}{3} e^2 \mathfrak{Re} \int \frac{d^4 q_E}{(2\pi)^4} \frac{Q^2 - \nu_E^2}{(Q^2)^3} \frac{i T_3(i\nu_E, Q^2)}{M f_+(0)}$$

$$q_E^\mu = (\vec{q}, \nu_E) \qquad Q^2 = \mathbf{q}^2 + \nu_E^2$$

What needs to be computed using ab-initio methods is $T_3(i\nu_E, Q^2)$

$$T_3(i\nu_E, Q^2) = -\frac{2iM\nu_E}{\mathbf{q}} \sum_X \left\{ \frac{(\phi_f, \vec{0}|J^x_{\mathrm{em}}(\vec{q})|X)(X|\left(J_W^{\dagger y}(-\vec{q})\right)_A|\phi_i, \vec{0})}{\nu_X - i\nu_E} + \frac{(\phi_f, \vec{0}|\left(J_W^{\dagger y}(-\vec{q})\right)_A|X)(X|J^x_{\mathrm{em}}(\vec{q})|\phi_i, \vec{0})}{\nu_X + i\nu_E} \right\}$$

No singularity in the loop integral, thanks to the **imaginary argument**

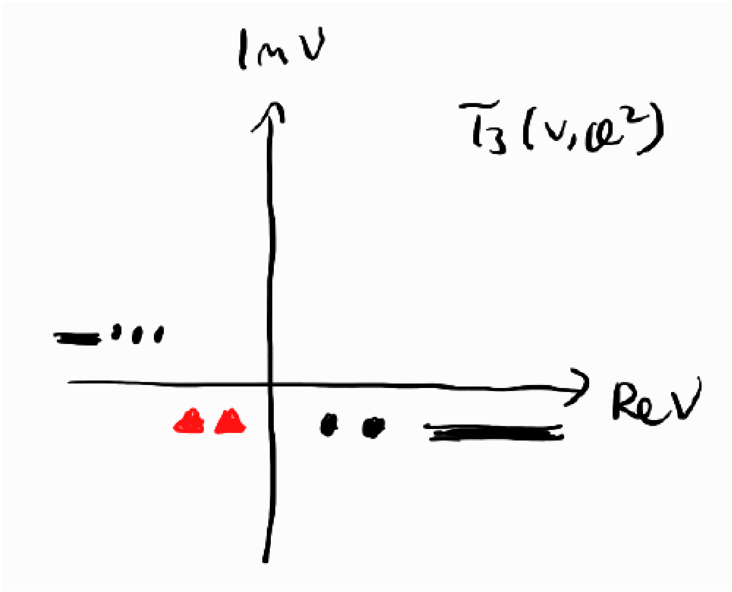
Residue contribution

(First pointed out by Michael Gennari, TRIUMF)

For A=10,14,18,22,26,30 and 38, the m_I=0, $J^p=0^+$ state is an excited state.

m_I=0 initial nucleus → extra poles of T_3 in the first quadrant;
m_I=0 final nucleus → extra poles of T_3 in the third quadrant

For ^{10}C → ^{10}B :



$$\boxed{\nu_k = \sqrt{M_k^2+\mathbf{q}^2} - M - i\varepsilon}$$

In third quadrant if $\mathbf{q}^2 < M^2 - M_k^2$

Excited Nuclear States for B-10 (Boron)

Energy levels

E^* [keV]	J^π	T	ℓ_p
0.0	3^+	0	1
718.35(4)	1^+	0	1
1740.2(2)	0^+	1	1
2154.3(5)	1^+	0	1
3587.1(5)	2^+	0	1
4774.0(5)	3^+	0	1

$$\left(\Box^b_{\gamma W}\right)_{\mathrm{res},T_3} = -\frac{e^2}{M}\sum_k \Re\mathfrak{e}\int_0^{\mathbf{q}_{\max}} \frac{d\mathbf{q}\mathbf{q}^2}{(2\pi)^2}\frac{1}{\mathbf{q}^2-\nu_k^2}\frac{i\mathfrak{Res}T_3(\nu_k,\mathbf{q})}{f_+(0)} \times\left\{\frac{2E_e\mathbf{q}^2+\nu_k A}{4\nu_k E_e|\vec{p}_e|\mathbf{q}}\ln\left|\frac{A+2|\vec{p}_e|\mathbf{q}}{A-2|\vec{p}_e|\mathbf{q}}\right| - \frac{1}{E_e}\right\}$$

$$A=\nu_k^2-2E_e\nu_k-\mathbf{q}^2\ ,\quad \mathbf{q}_{\max}=\sqrt{M^2-M_k^2}$$

Singular when E_e, $p_e \to 0$ (regular in reality, of course)

Principal-valued integration

Residue of T_3 due to a low-lying m_I=0 state k:

$$\mathfrak{Res}T_3(\nu_k,\mathbf{q}) = \frac{2M\nu_k}{\mathbf{q}}\left({}^{10}\mathrm{B}(0^+),\vec{0}\right|J^x_{\mathrm{em}}(\vec{q})\left|k,\vec{q}\right)\cdot \left(k,\vec{q}\right|\left(J^{\dagger y}_W(-\vec{q})\right)_A\left|{}^{10}\mathrm{C}(0^+),\vec{0}\right)$$

Involves only **ONE intermediate state** (instead of an inclusive sum). Easy to compute!

Preliminary calculation shows that this is the dominant piece in δ_{NS} ($^{10}C \to {}^{10}B$).

Michael Gennari's talk, INT Program INT-23-1B

So, what do we need to do?

1. Write down the electroweak currents in terms of nucleon DOFs
2. Compute the residue contribution which is easier. It serves as a benchmark to cross-check different methods
3. Compute either the response function $F_3(\nu,Q^2)$ or the invariant amplitude $T_3(i\nu_E,q)$, whichever is more convenient to you
4. If possible, perform the (ν,Q^2) integral for F_3 or the (ν_E,q) integral for T_3. Otherwise, provide at least a series of discrete values so that we can fit them to a specific parameterization

Crossing symmetry of T_3 under $\nu \to -\nu$

Detail: *Seng and Gorchtein, PRC 107 (2023) 3, 035503, Section IV*

If we split the electromagnetic current into **isoscalar** and **isovector** piece:

$$J^{\mu}_{\rm em} = J^{(0)\mu}_{\rm em} + J^{(1)\mu}_{\rm em}$$

and correspondingly: $T_3(\nu,Q^2) = T_3^{(0)}(\nu,Q^2) + T_3^{(1)}(\nu,Q^2)$

The isoscalar piece has a definite crossing symmetry: $T_3^{(0)}(-\nu,Q^2) = -T_3^{(0)}(\nu,Q^2)$

while the isovector piece is more complicated:

- For pion system, $T_3^{(1)}$ vanishes;
- For I=1/2 system, $T_3^{(1)}$ is even;
- For I=1 nuclear system, $T_3^{(1)}$ has no definite crossing symmetry

So,

$$\boxed{\begin{aligned} T_{3,-}(\nu,Q^2) &= T_3^{(0)}(\nu,Q^2) + \frac{1}{2}\left[T_3^{(1)}(\nu,Q^2) - T_3^{(1)}(-\nu,Q^2)\right] \\ T_{3,+}(\nu,Q^2) &= \frac{1}{2}\left[T_3^{(1)}(\nu,Q^2) + T_3^{(1)}(-\nu,Q^2)\right] \end{aligned}}$$

Prefer flavor-diagonal matrix elements? Use isospin rotation!

Detail: *Seng and Gorchtein, PRC 107 (2023) 3, 035503, Section VII*

$$\langle 1,0|V^{\mu}_{00}\otimes A^{\nu}_{1-1}|1,1\rangle = \langle 1,-1|V^{\mu}_{00}\otimes A^{\nu}_{1-1}|1,0\rangle$$
$$= -3\left[\langle 1,1|J^{\mu}_{\mathrm{em}}\otimes \left(J^{\nu}_{Z}\right)_A|1,1\rangle - \langle 1,-1|J^{\mu}_{\mathrm{em}}\otimes \left(J^{\nu}_{Z}\right)_A|1,-1\rangle\right].$$

$$\langle 1,-1|V^{\mu}_{10}\otimes A^{\nu}_{1-1}|1,0\rangle = 2\left[\langle 1,0|\left(J^{\mu}_{W}\right)_V\otimes \left(J^{\dagger\nu}_{W}\right)_A|1,0\rangle - \langle 1,-1|\left(J^{\mu}_{W}\right)_V\otimes \left(J^{\dagger\nu}_{W}\right)_A|1,-1\rangle\right],$$
$$\langle 1,0|V^{\mu}_{10}\otimes A^{\nu}_{1-1}|1,1\rangle = 2\left[\langle 1,1|\left(J^{\mu}_{W}\right)_V\otimes \left(J^{\dagger\nu}_{W}\right)_A|1,1\rangle - \langle 1,0|\left(J^{\mu}_{W}\right)_V\otimes (J^{\dagger\nu}_{W})_A|1,0\rangle\right]$$

(1,0) : $^{10}B(0^+)$
(1,1) : $^{10}C(0^+)$
(1,-1): $^{10}Be(0^+)$

δ_{NS} from the nuclear box diagram

$$\delta_{\mathrm{NS}} = 2\langle\Box^{b,\mathrm{nucl}}_{\gamma W}(E_e)\rangle - 2\Box^{b,n}_{\gamma W}$$
$$\approx 2\langle(\Box^{b,\mathrm{nucl}}_{\gamma W}(E_e))_{\mathrm{res},T_3}\rangle + 2\,\underline{\left(\boxminus^{\mathrm{nucl}}_0 - \boxminus^{n}_0\right)} + 2\,\boxminus^{\mathrm{nucl}}_1\,\langle E_e\rangle$$

Both terms contain the same physics at large Q^2, which cancel out in the difference

where $\Box^{b,n}_{\gamma W} = \boxminus^{n}_0$

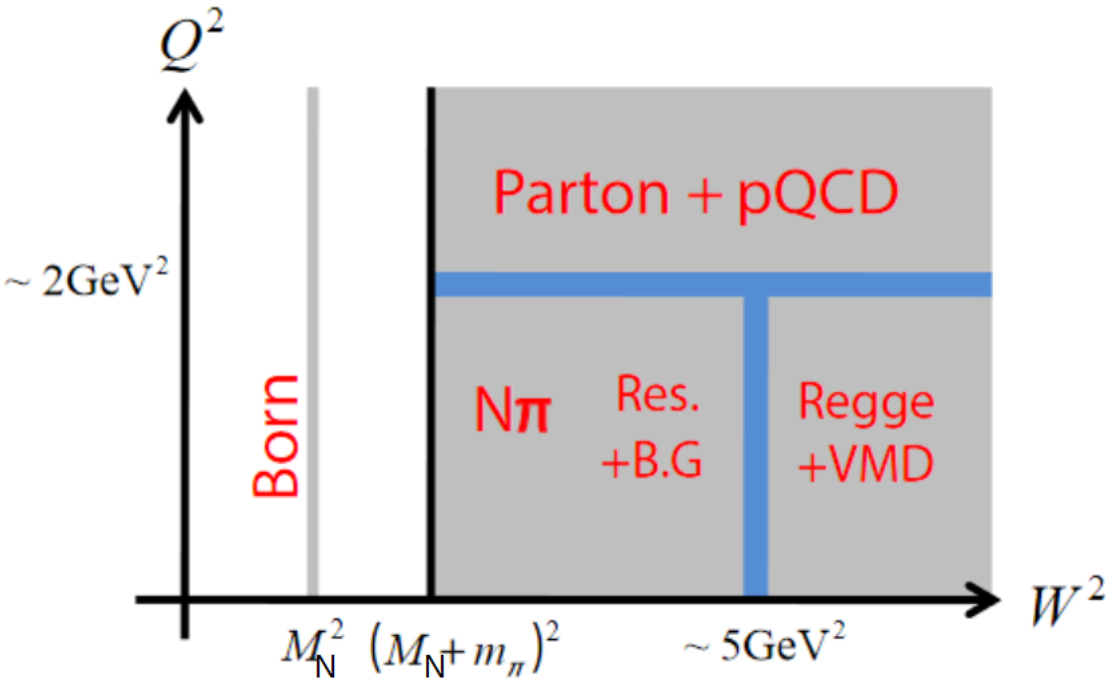
But nuclear ab-initio calculations of response functions **DO NOT** include intermediate states that survive at **large-Q^2**, e.g.

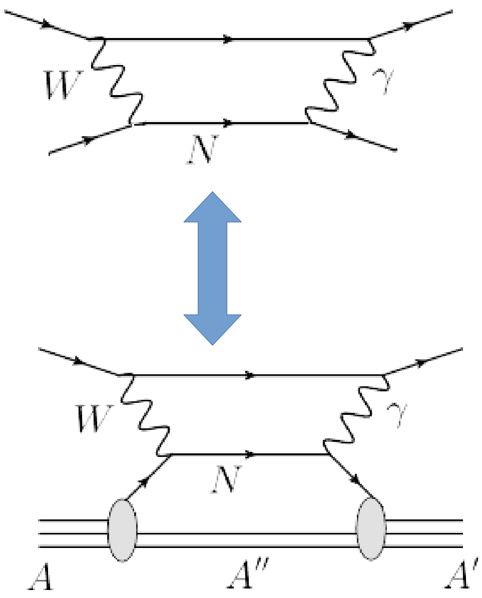
$$|X\rangle = |\mathrm{Nucl.} + (N\pi\ ,\ \mathrm{resonance}\ ,\ \mathrm{multi\text{-}hadron}\ ,\ \mathrm{parton}\ \ldots)\rangle$$

Question: How to do the subtraction in a self-consistent way?

One possible strategy: Subtract only the part in the single-nucleon box diagram that has a **nuclear physics analogy** in ab-initio calculations!

$$\Box^{b,n}_{\gamma W} = (\Box^{b,n}_{\gamma W})_{\text{elastic}} + (\Box^{b,n}_{\gamma W})_{N\pi+\text{res}} + (\Box^{b,n}_{\gamma W})_{\text{Regge}} + (\Box^{b,n}_{\gamma W})_{\text{DIS}}$$





Only the **"elastic" piece** (magnetic*axial), $(\Box^{b,n}_{\gamma W})_{\text{elastic}} = 1.06(6) \times 10^{-3}$

CYS, Gorchtein and Ramsey-Musolf, PRD 100 (2019) 1, 013001

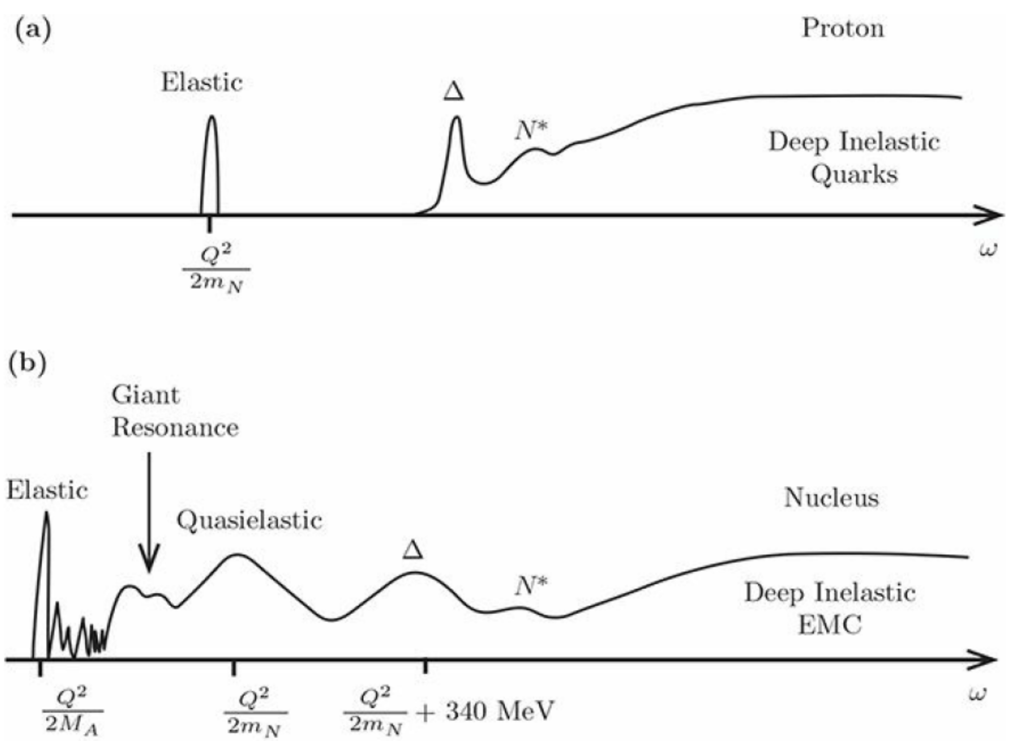
has an analogy in ab-initio nuclear box diagram calculations!

This means, we rephrase δ_{NS} as:

$$\delta_{\mathrm{NS}} \approx 2\left\langle \Box^{b,\mathrm{nucl}}_{\gamma W,\mathrm{ab\text{-}initio}}(E_e)\right\rangle_{\mathrm{res},T_3} + 2\left(\boxminus^{\mathrm{nucl}}_{0,\mathrm{ab\text{-}initio}} - \boxminus^{n}_{0,\mathrm{elastic}}\right) + 2\,\boxminus^{\mathrm{nucl}}_{1,\mathrm{ab\text{-}initio}}\,\langle E_e\rangle$$

This prescription assumes that all other contributions (Nπ, resonance, Regge, DIS…) are the same for single-nucleon and nucleus!

Good as a first step, but may require further scrutiny in more future analysis.



Donnelly, Formaggio, Holstein, Milner and Surrow, "Foundations of Nuclear and Particle Physics"

Something to think about: δ_{NS} in EFT framework

The formalism above is based on Sirlin's current algebra framework, which deals with nuclear matrix elements of SM electroweak operators (DOF: quarks & gauge bosons)

Sirlin, Rev.Mod.Phys.50 (1978) 573

Another starting point is the **effective field theory (EFT) approach**:

SM electroweak theory

↓

LEFT

↓

ChPT

↓

Pionless EFT

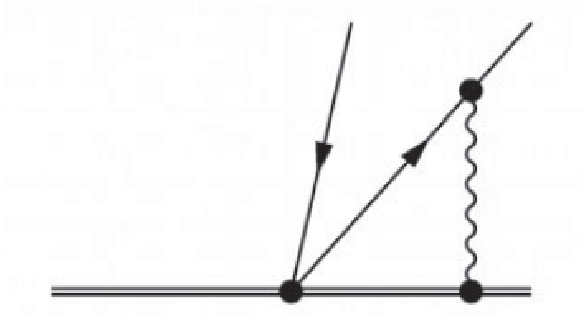
See Emanuele's talk

The pionless EFT Lagrangian:

$$\mathcal{L}_{\not\pi} = -\sqrt{2}G_F V_{ud}\Big\{\bar{e}\gamma_\mu P_L \nu_e\Big[\bar{N}(g_V v^\mu - 2g_A S^\mu)\tau^+ N + \frac{i}{2m_N}\bar{N}(v^\mu v^\nu - g^{\mu\nu} - 2g_A v^\mu S^\nu)(\overleftarrow{\partial} - \overrightarrow{\partial})_\nu \tau^+ N\Big] + \frac{ic_T m_e}{m_N}\bar{N}(S^\mu v^\nu - S^\nu v^\mu)\tau^+ N(\bar{e}\sigma_{\mu\nu}P_L\nu) + \frac{i\mu_{\text{weak}}}{m_N}\bar{N}[S^\mu, S^\nu]\tau^+ N\partial_\nu(\bar{e}\gamma_\mu P_L \nu)\Big\} + \cdots,$$

Cirigliano et al, Phys.Rev.Lett, 129, 121801 (2022)

Fact: In the single-nucleon level, the photon loop diagram in pionless EFT will **NOT** reproduce the “elastic” contribution to the box diagram that we discussed before; it is contained in the LEC reabsorbed into g_V



Question:

If we apply the pionless EFT to ab-initio calculations, do the loop diagrams give rise to the “nuclear analogy of the elastic contribution” that we discussed before?

If no, then where does this piece of physics hide? In a new LEC?

If yes, how do we reconcile it with the fact that at the single-nucleon level such contribution doesn’t appear in EFT loops?

The answer to the question above is crucial to understand which part from the single-nucleon piece that one needs to subtract in order to obtain δ_{NS}, if we operate with EFT.

Summary

- I hope these slides are useful
- Please kindly let me know if you detect any errors

Thanks for your attention!