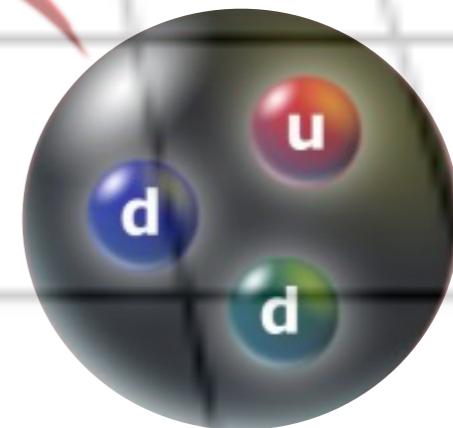


Advances in NN systems



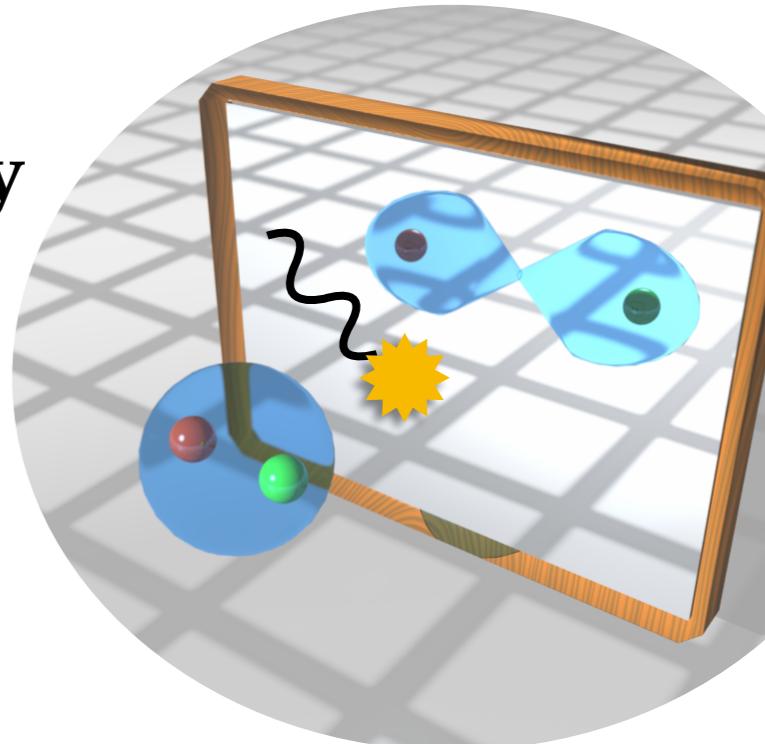
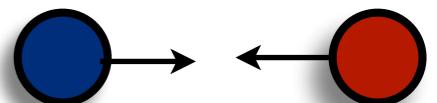
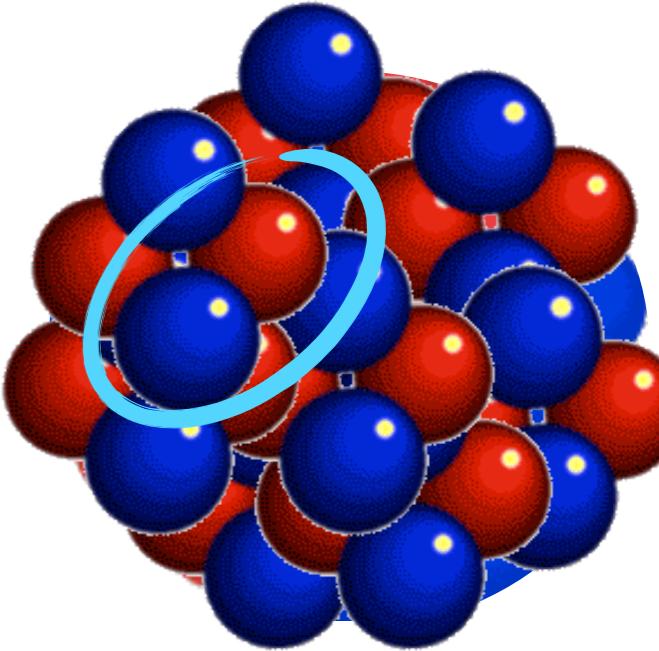
Amy Nicholson
UNC, Chapel Hill

Virtual DBD meeting, May 29, 2020

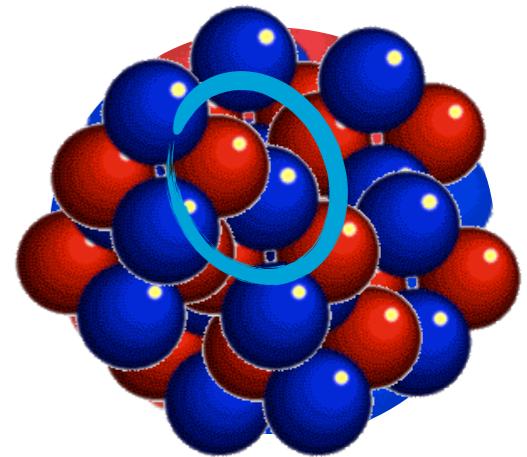


NN systems

- What do we need to get nuclear physics from LQCD?
 - Phase shifts required for infinite volume matching of MEs
 - Must have full control over 2-body systems
 - How do we project onto desired states?
 - How do we disentangle signals from closely spaced energy levels?
 - How do we beat the noise?



Methods for calculating few-body interactions from LQCD:



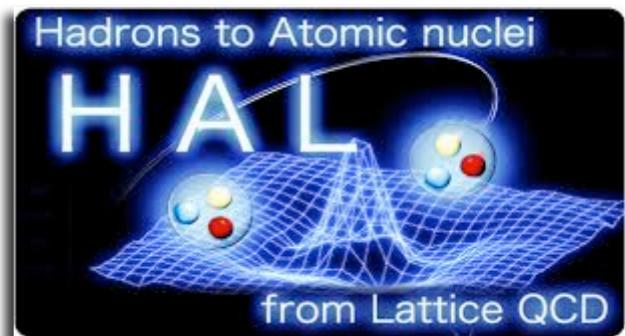
Spectroscopy + Lüscher Method



Yamazaki,
et. al.

Hanlon
et. al.

Potential Method

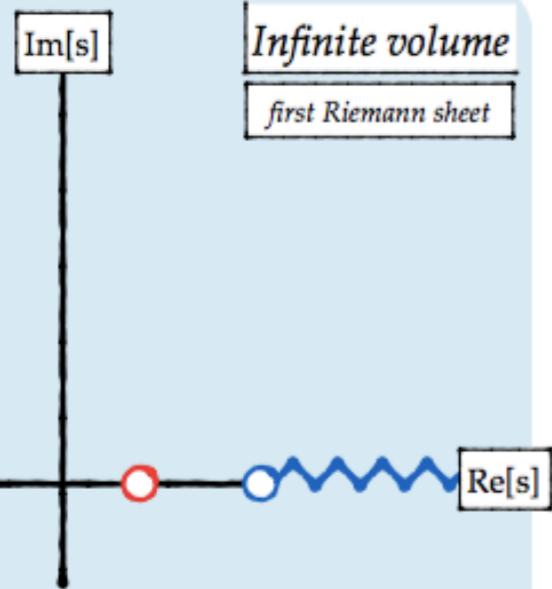


Spectroscopy + HOBET

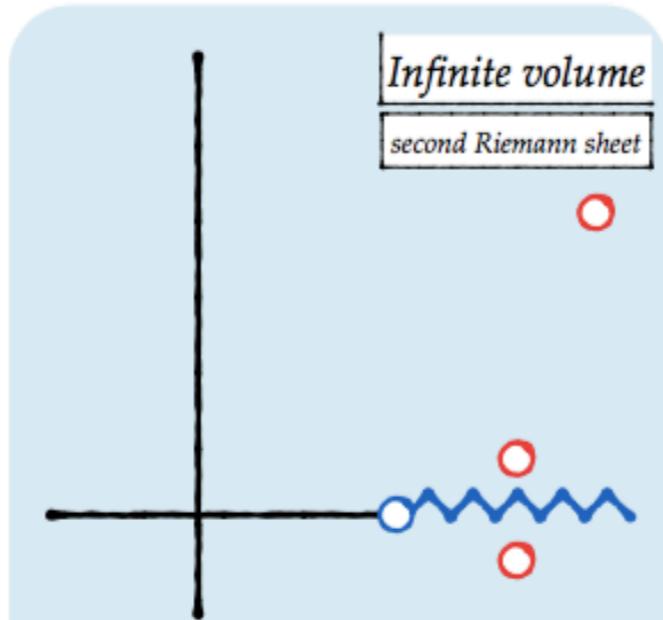


*not an official logo

Spectroscopy



Infinite volume
first Riemann sheet

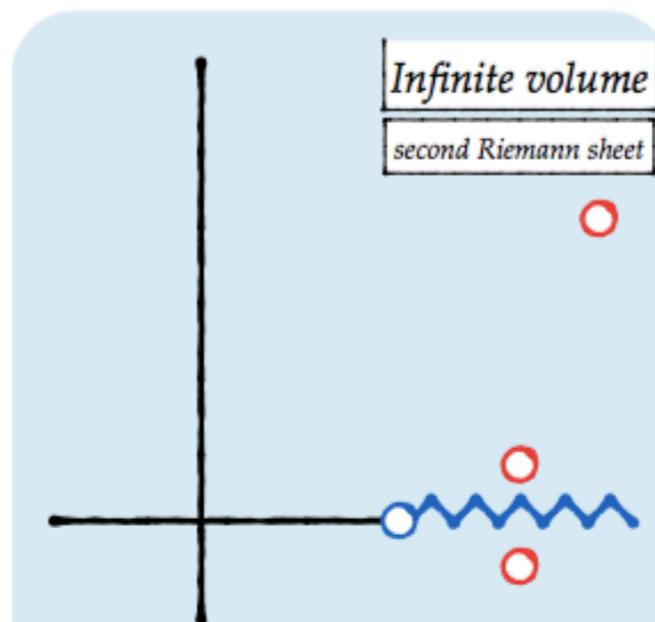
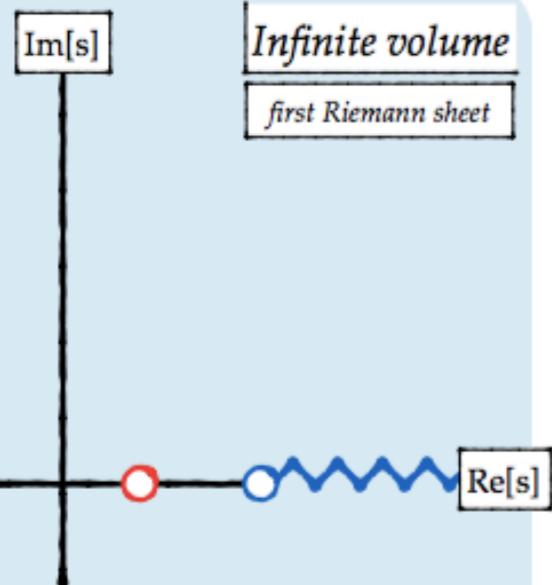


Infinite volume
second Riemann sheet

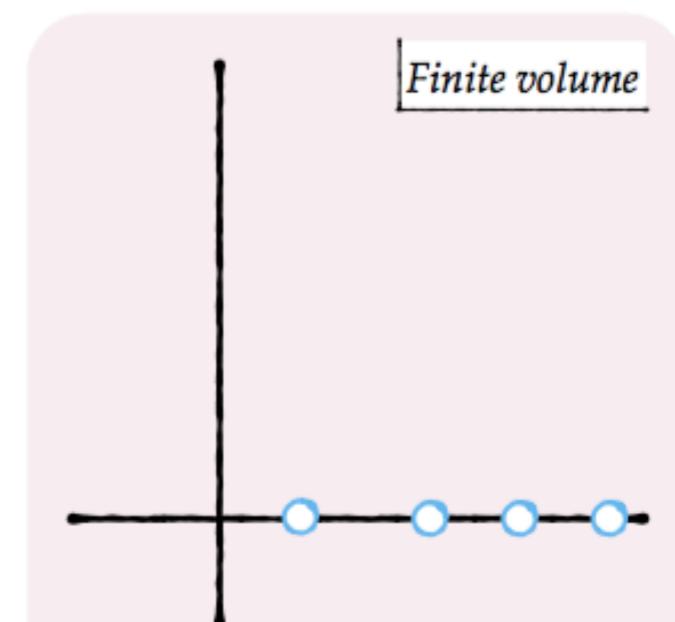
$$s_R = (E_R - \frac{i}{2}\Gamma_R)^2$$

Figures courtesy
R. Briceno

Spectroscopy



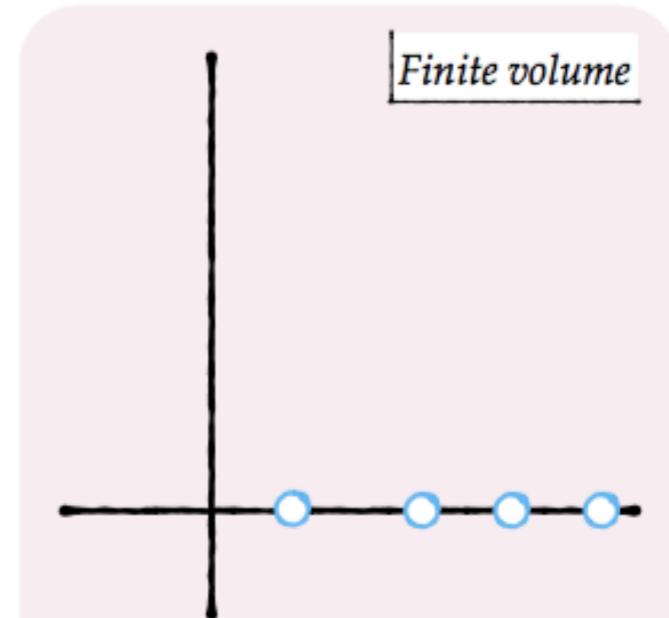
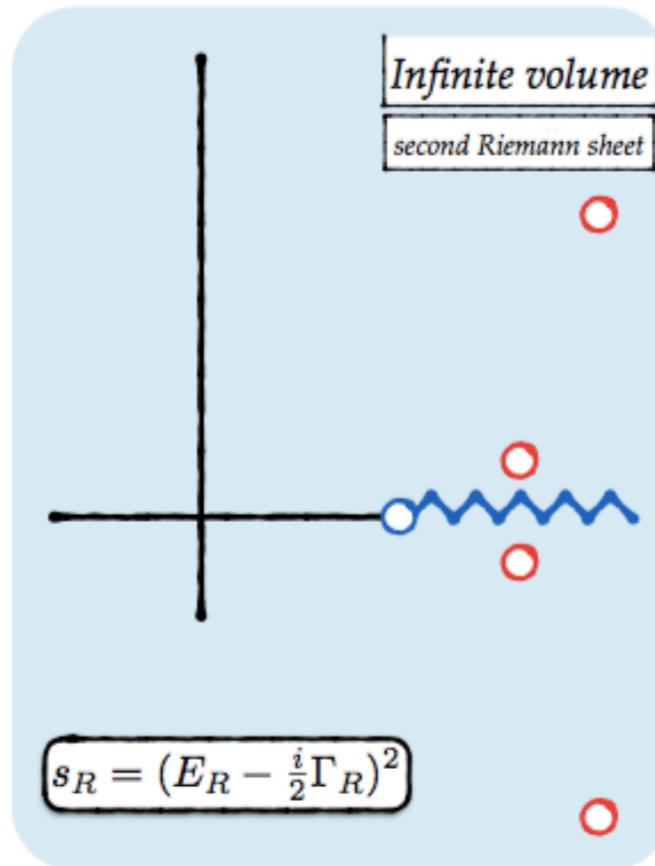
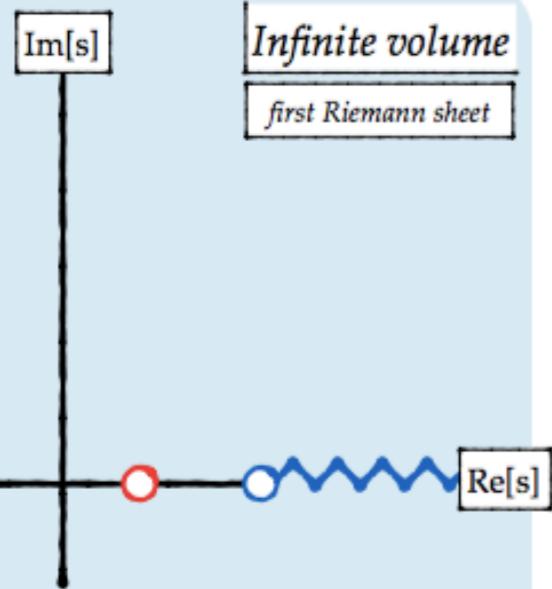
$$s_R = (E_R - \frac{i}{2}\Gamma_R)^2$$



Figures courtesy
R. Briceno

Spectroscopy

$$\langle \mathcal{O}(t)\mathcal{O}^\dagger(0) \rangle = \sum_n |\langle 0|\mathcal{O}|n\rangle|^2 e^{-E_n t}$$
$$\xrightarrow[t \rightarrow \infty]{} \langle 0|\mathcal{O}|E_0\rangle\langle E_0|\mathcal{O}|0\rangle e^{-E_0 t}$$

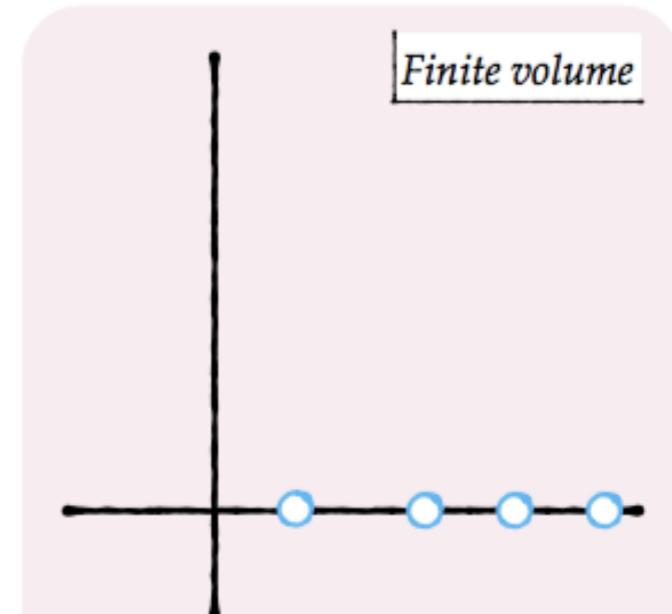
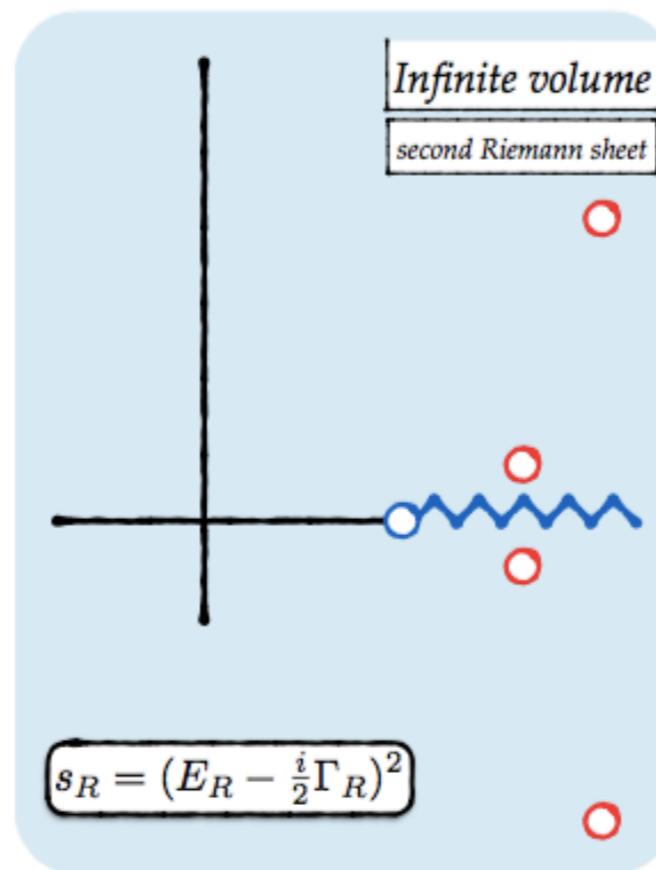
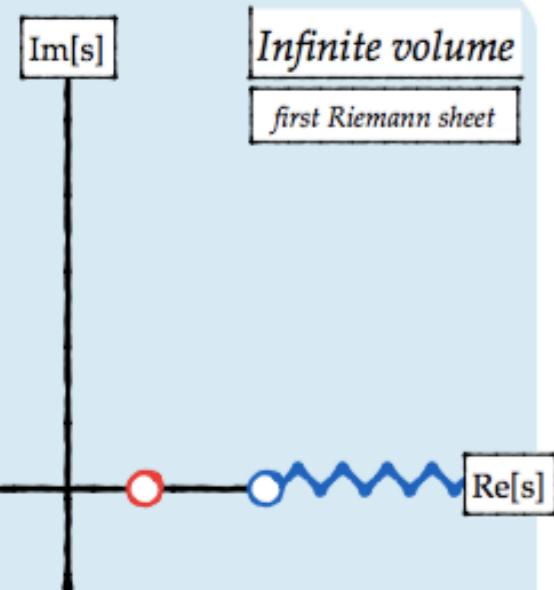


Figures courtesy
R. Briceno

Spectroscopy

- Finite volume energies simple to calculate from correlation functions at large Euclidean time:

$$\langle \mathcal{O}(t)\mathcal{O}^\dagger(0) \rangle = \sum_n |\langle 0|\mathcal{O}|n\rangle|^2 e^{-E_n t}$$
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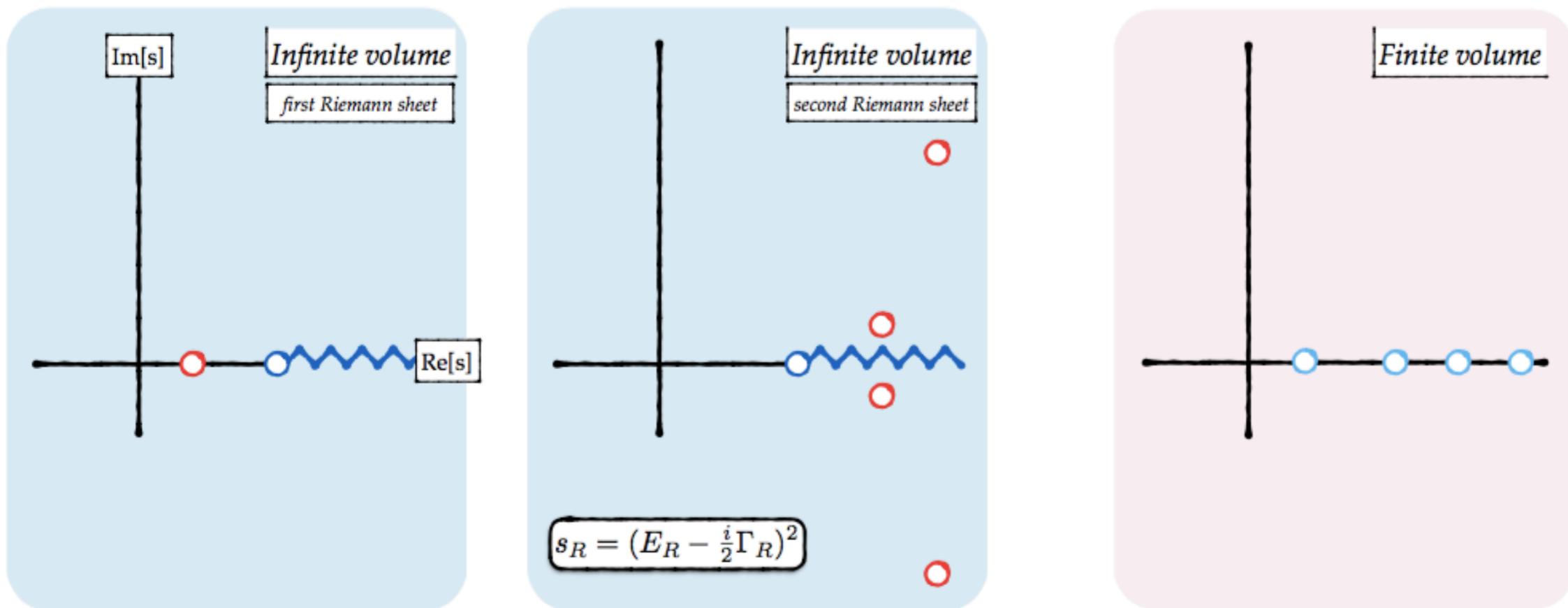
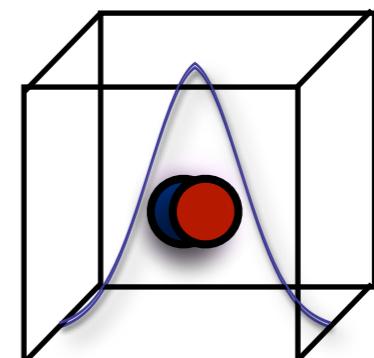
Figures courtesy
R. Briceno

Spectroscopy

- Finite volume energies simple to calculate from correlation functions at large Euclidean time:
- Bound states: infinite volume extrapolation gives binding energies

$$\langle \mathcal{O}(t)\mathcal{O}^\dagger(0) \rangle = \sum_n |\langle 0|\mathcal{O}|n\rangle|^2 e^{-E_n t}$$

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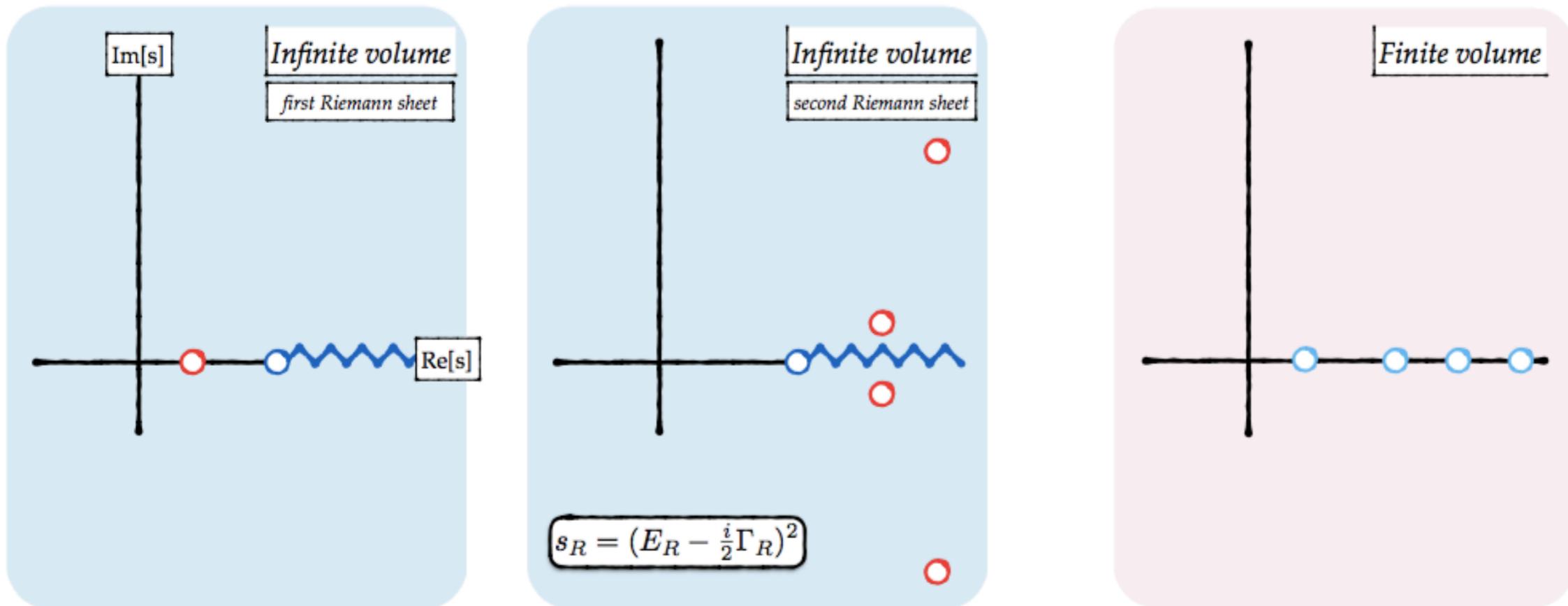
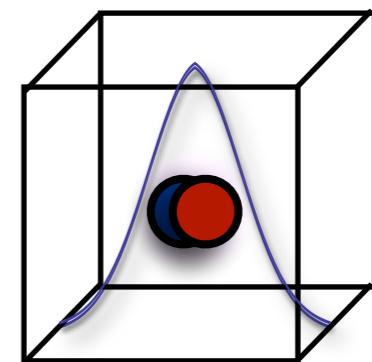
Figures courtesy
R. Briceno

Spectroscopy

- Finite volume energies simple to calculate from correlation functions at large Euclidean time:
- Bound states: infinite volume extrapolation gives binding energies
- Can't directly resolve resonances or scattering states

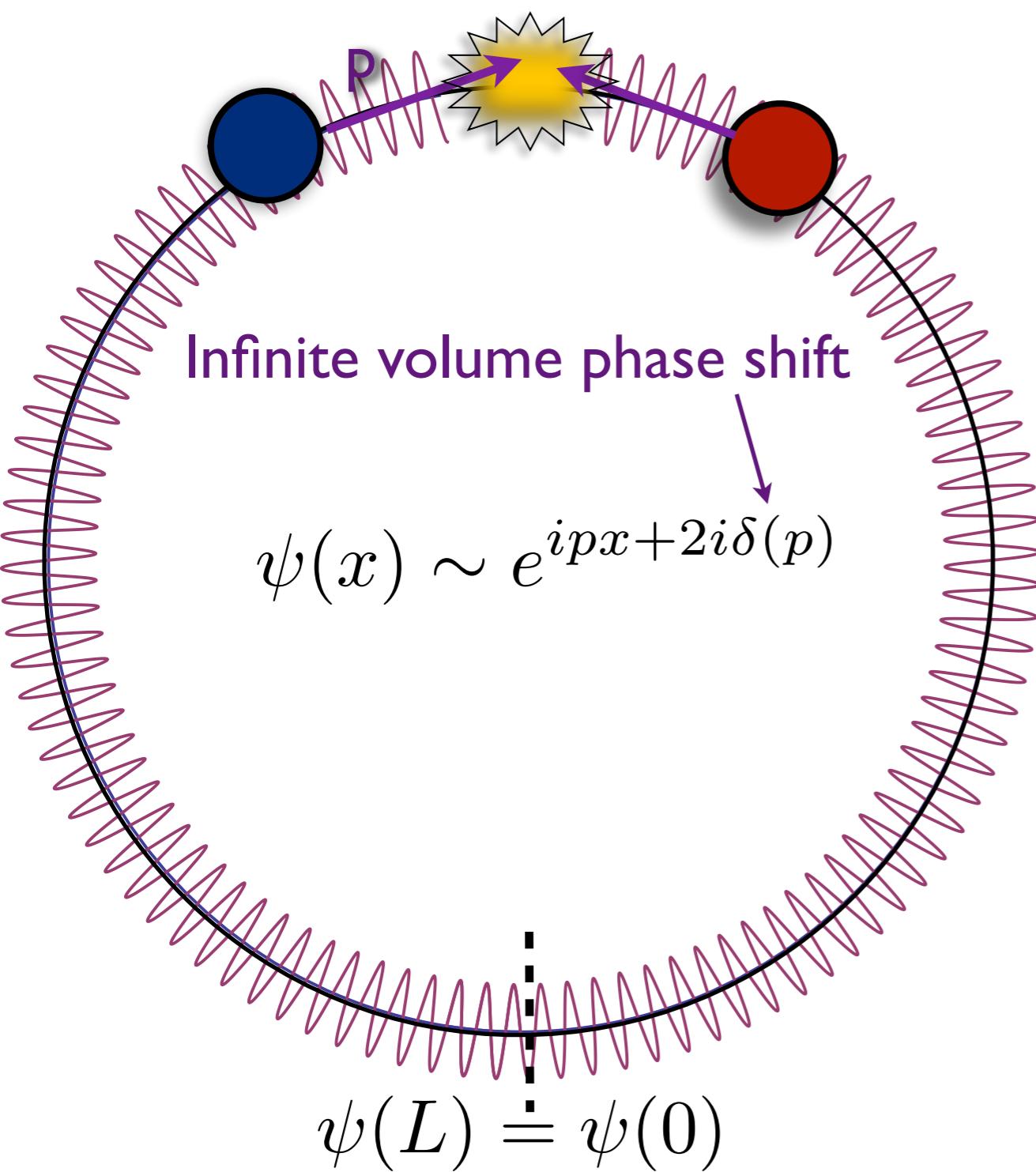
$$\langle \mathcal{O}(t)\mathcal{O}^\dagger(0) \rangle = \sum_n |\langle 0|\mathcal{O}|n\rangle|^2 e^{-E_n t}$$

$$\xrightarrow[t\rightarrow\infty]{} \langle 0|\mathcal{O}|E_0\rangle\langle E_0|\mathcal{O}|0\rangle e^{-E_0 t}$$

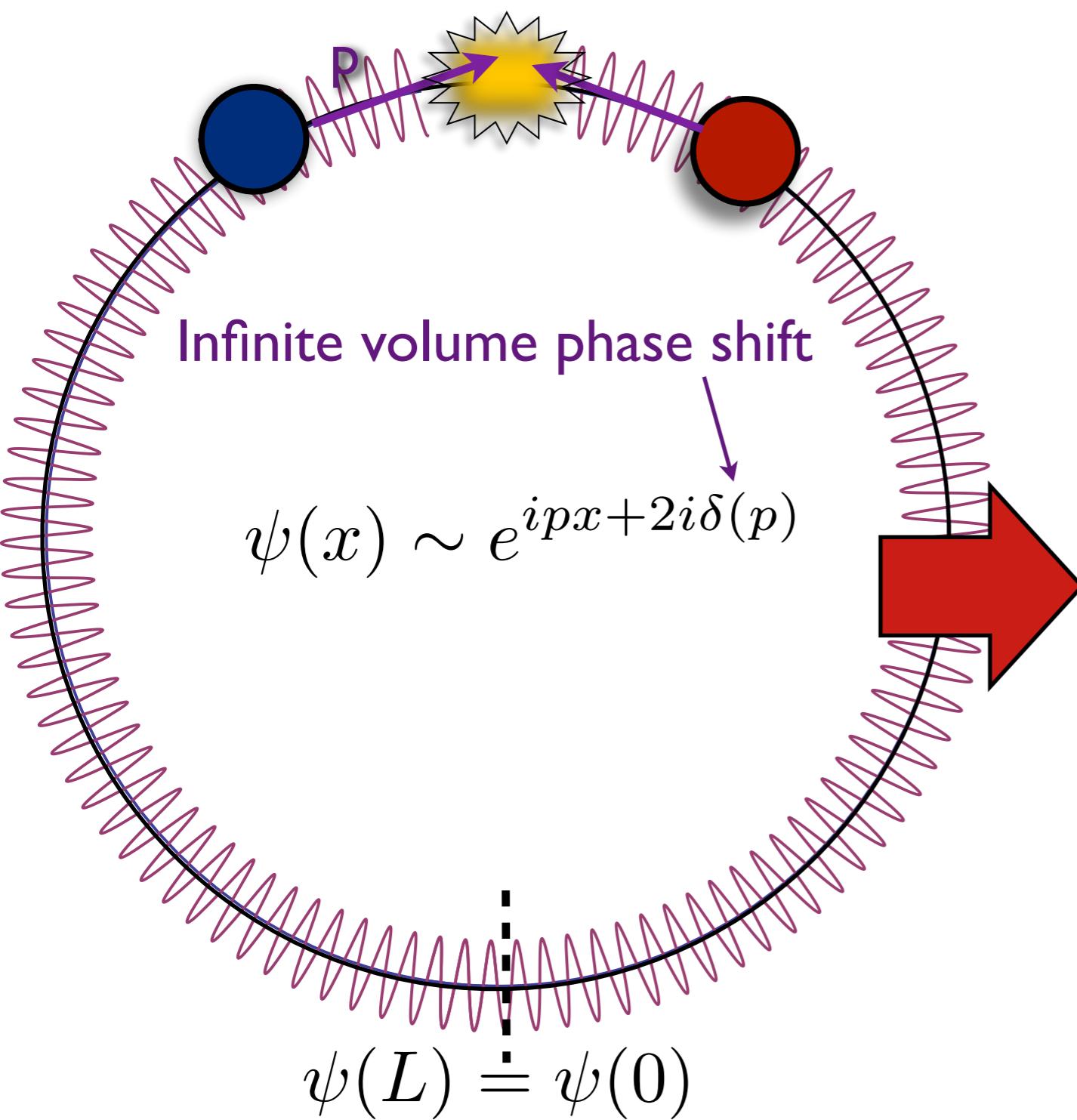


Figures courtesy
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“Lüscher” in 1-d

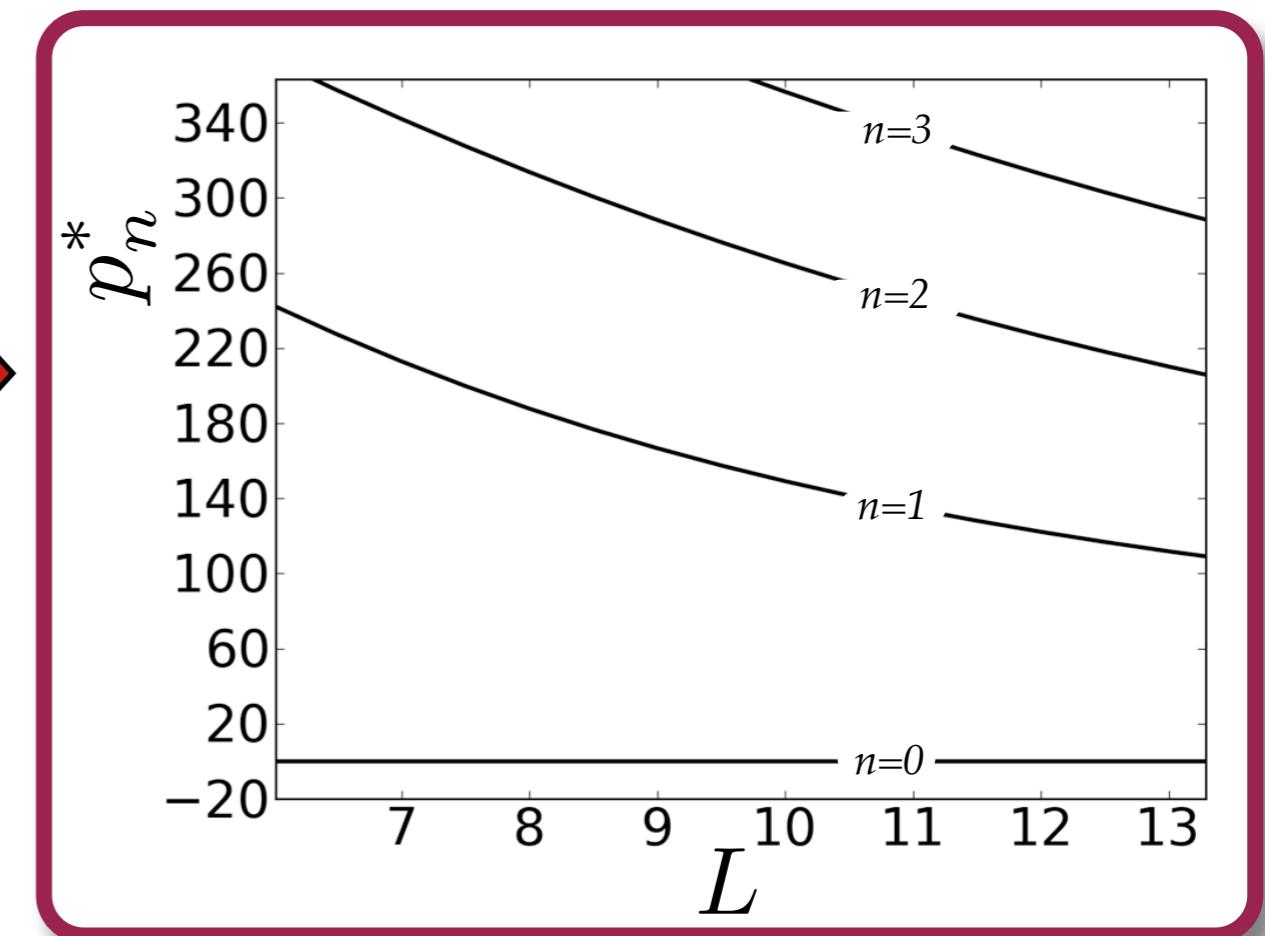


“Lüscher” in 1-d



Quantization condition:

$$Lp_n^* + 2\delta(p_n^*) = 2\pi n$$



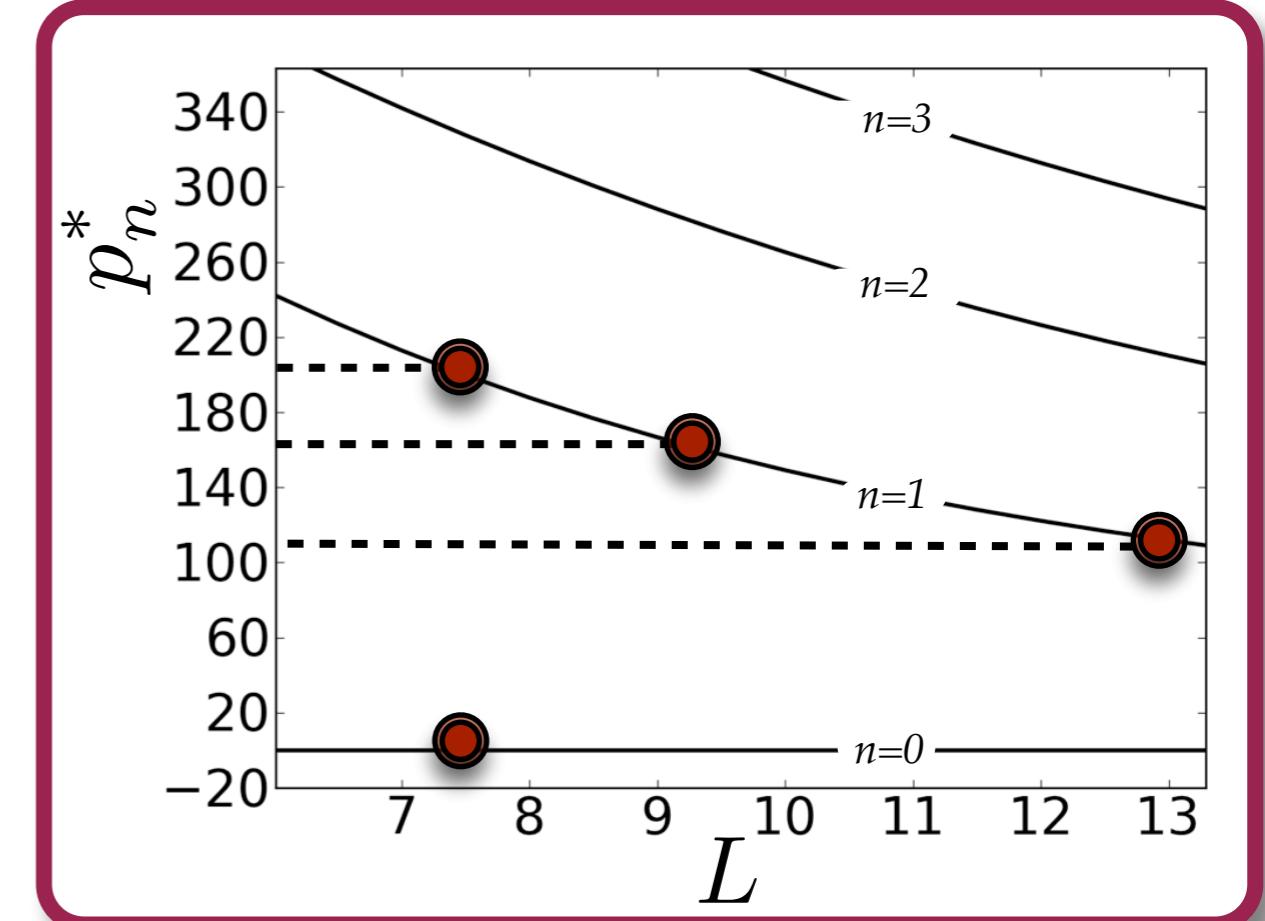
“Lüscher” in 1-d

Slide stolen unabashedly from R. Briceno

Quantization condition:

$$L p_n^* + 2\delta(p_n^*) = 2\pi n$$

Lattice: measure
energies at a given L

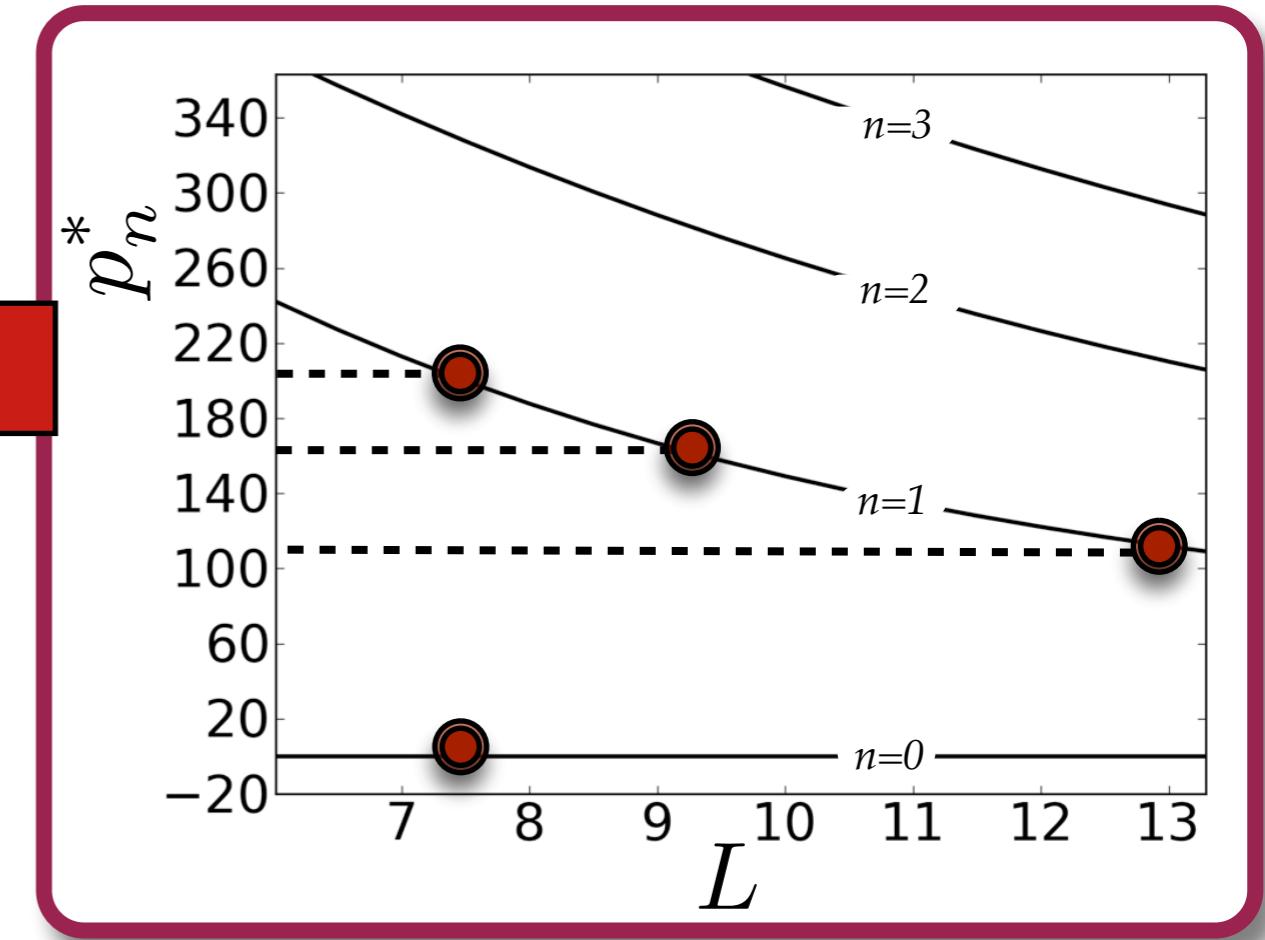
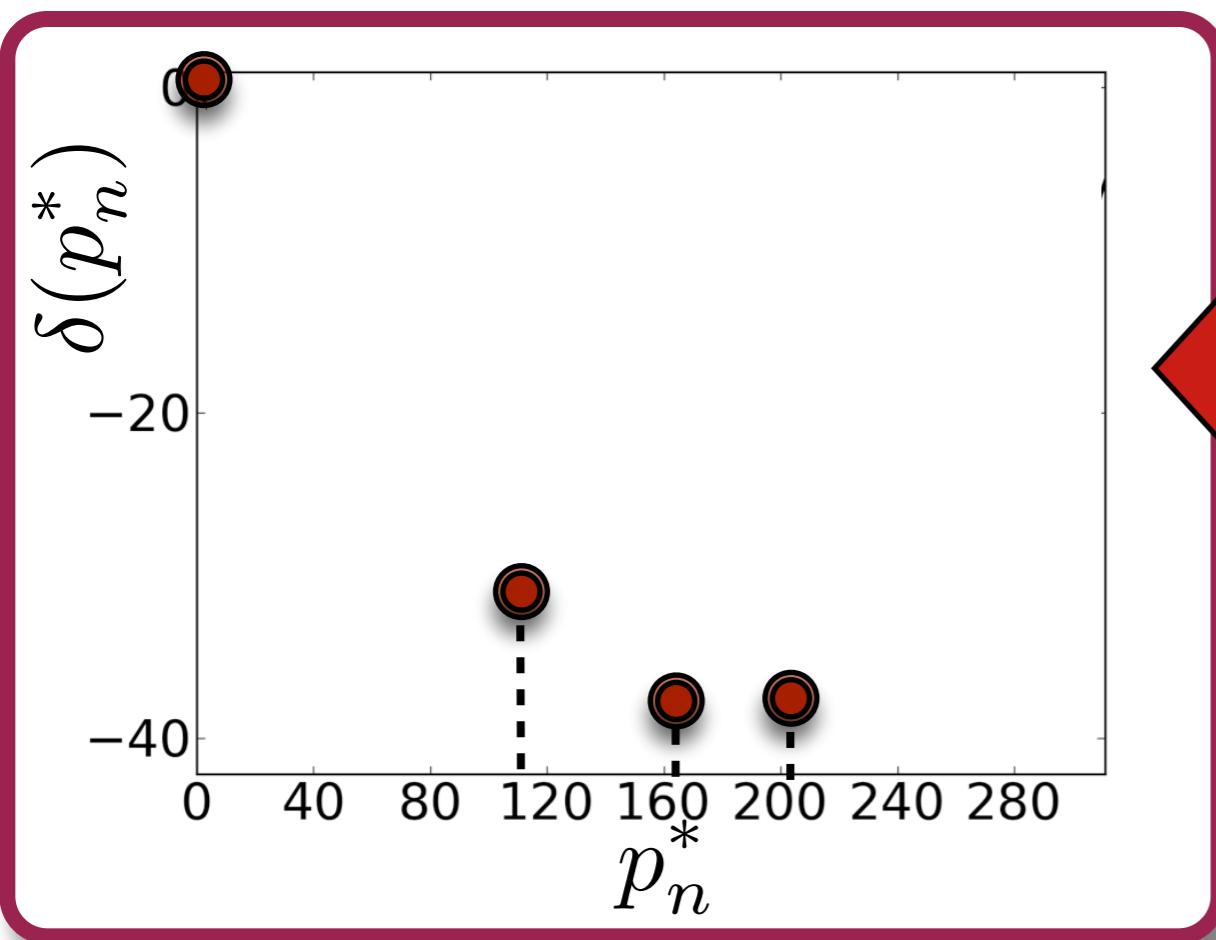


“Lüscher” in 1-d

Slide stolen unabashedly from R. Briceno

Quantization condition:

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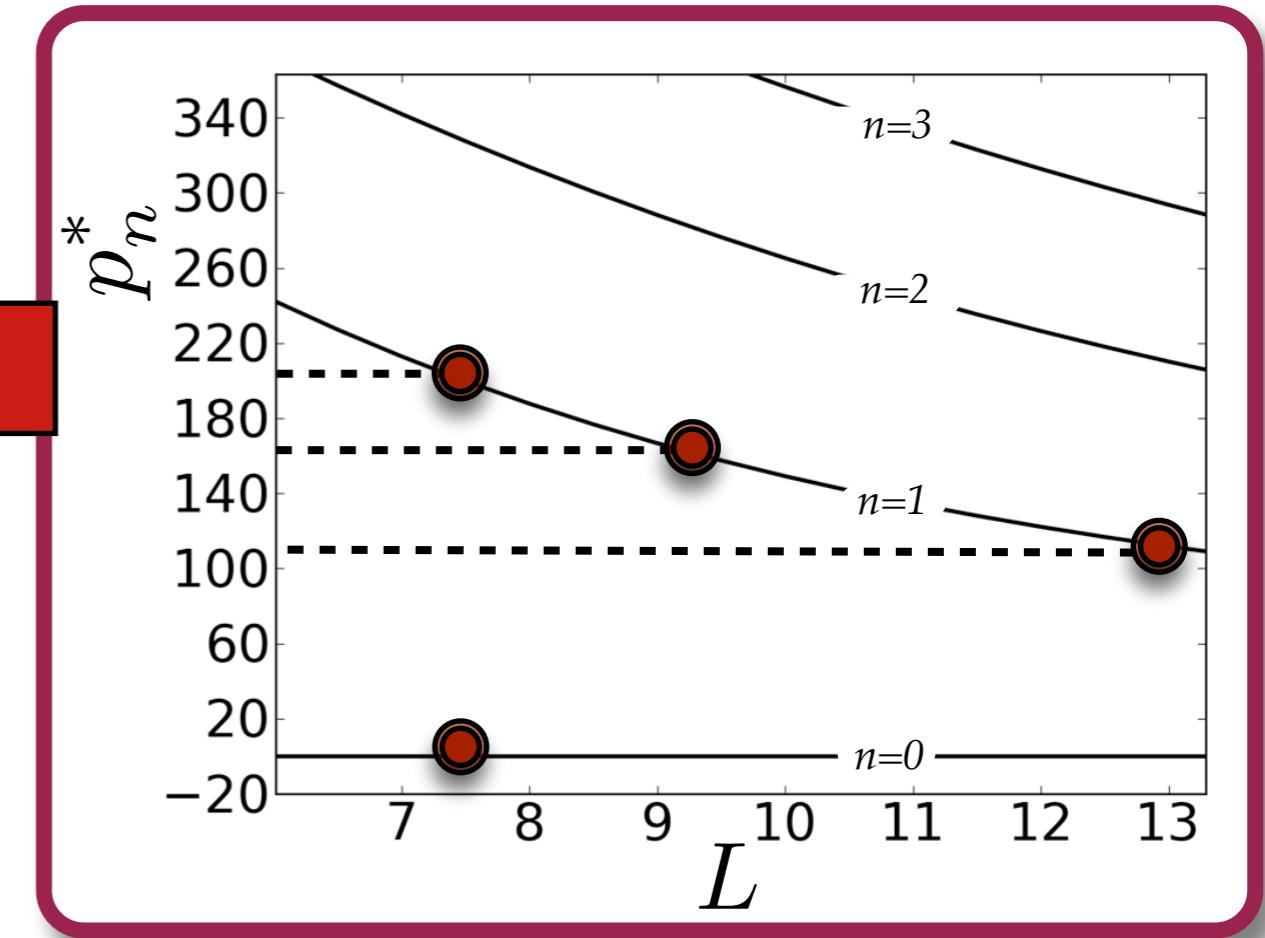
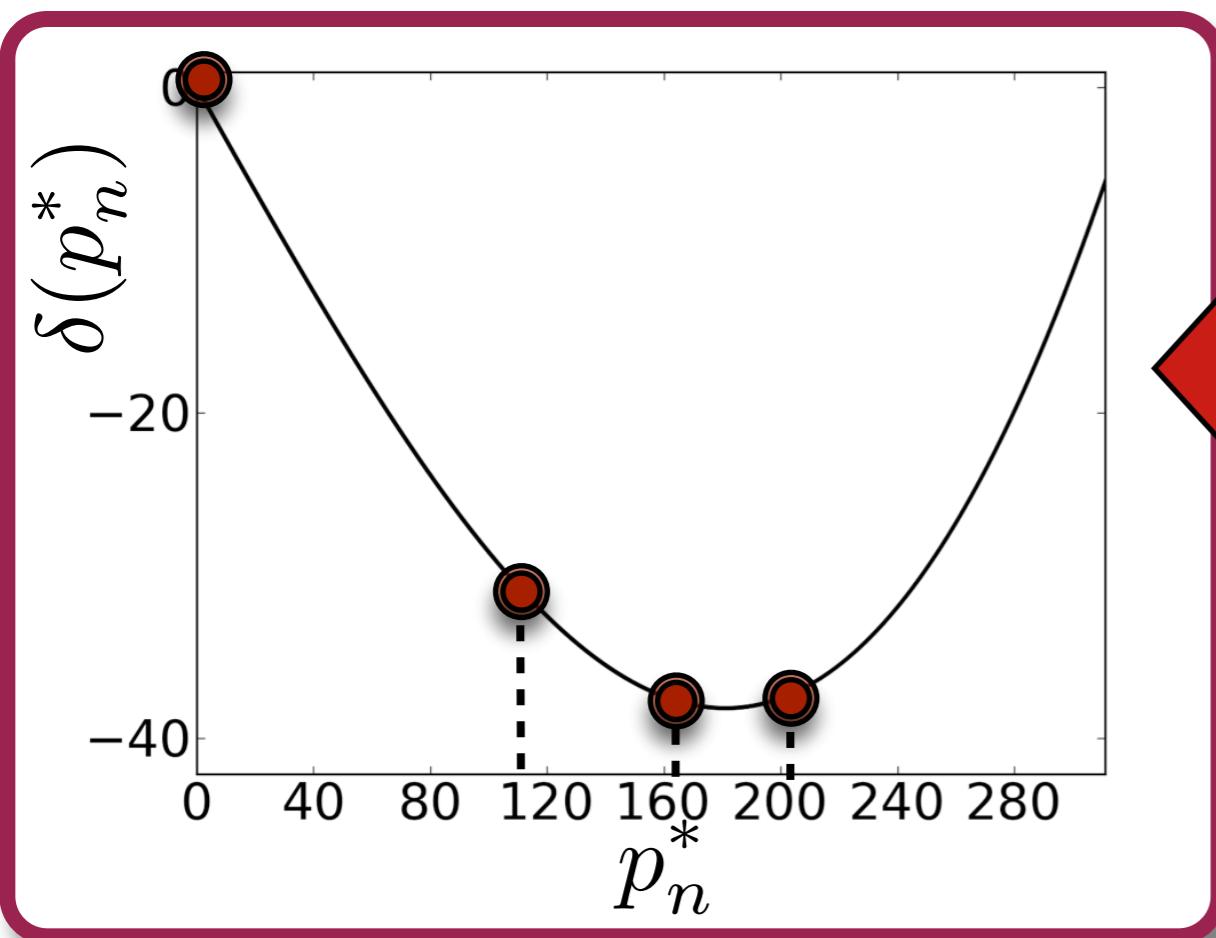


“Lüscher” in 1-d

Slide stolen unabashedly from R. Briceno

Quantization condition:

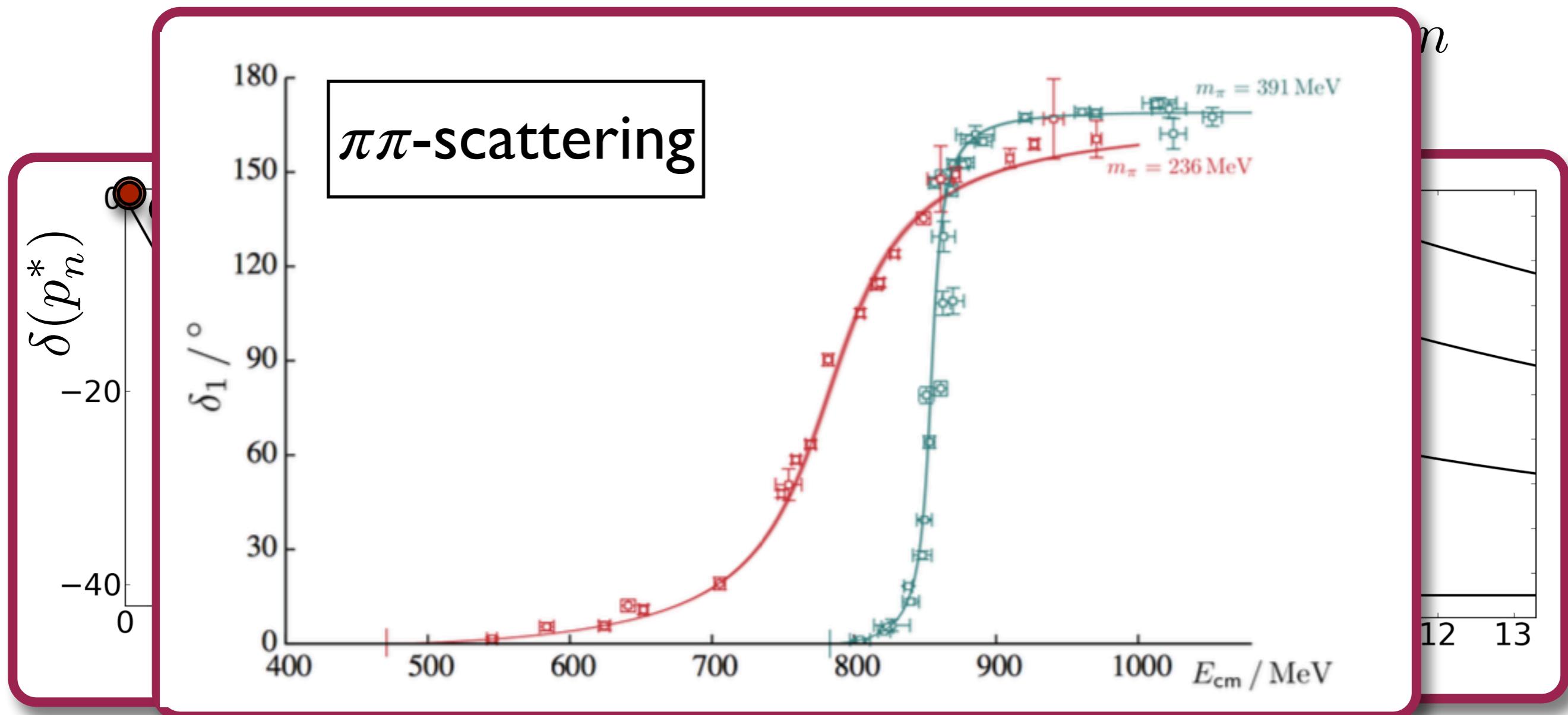
$$L p_n^* + 2\delta(p_n^*) = 2\pi n$$



“Lüscher” in 1-d

D. J. Wilson, R. A. Briceño, J. J. Dudek, R. G. Edwards and C. E. Thomas, Phys. Rev. D 92, 094502 (2015)

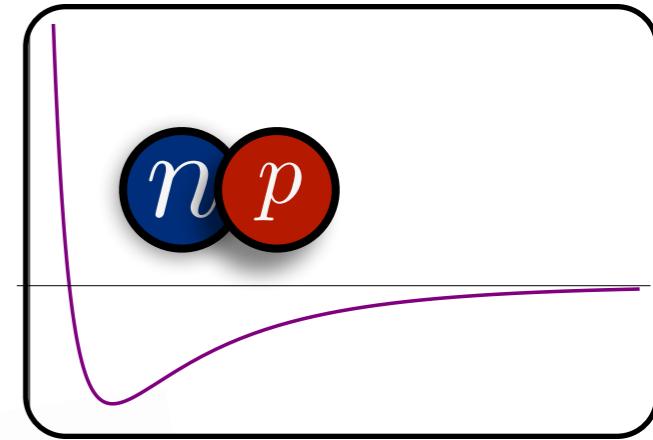
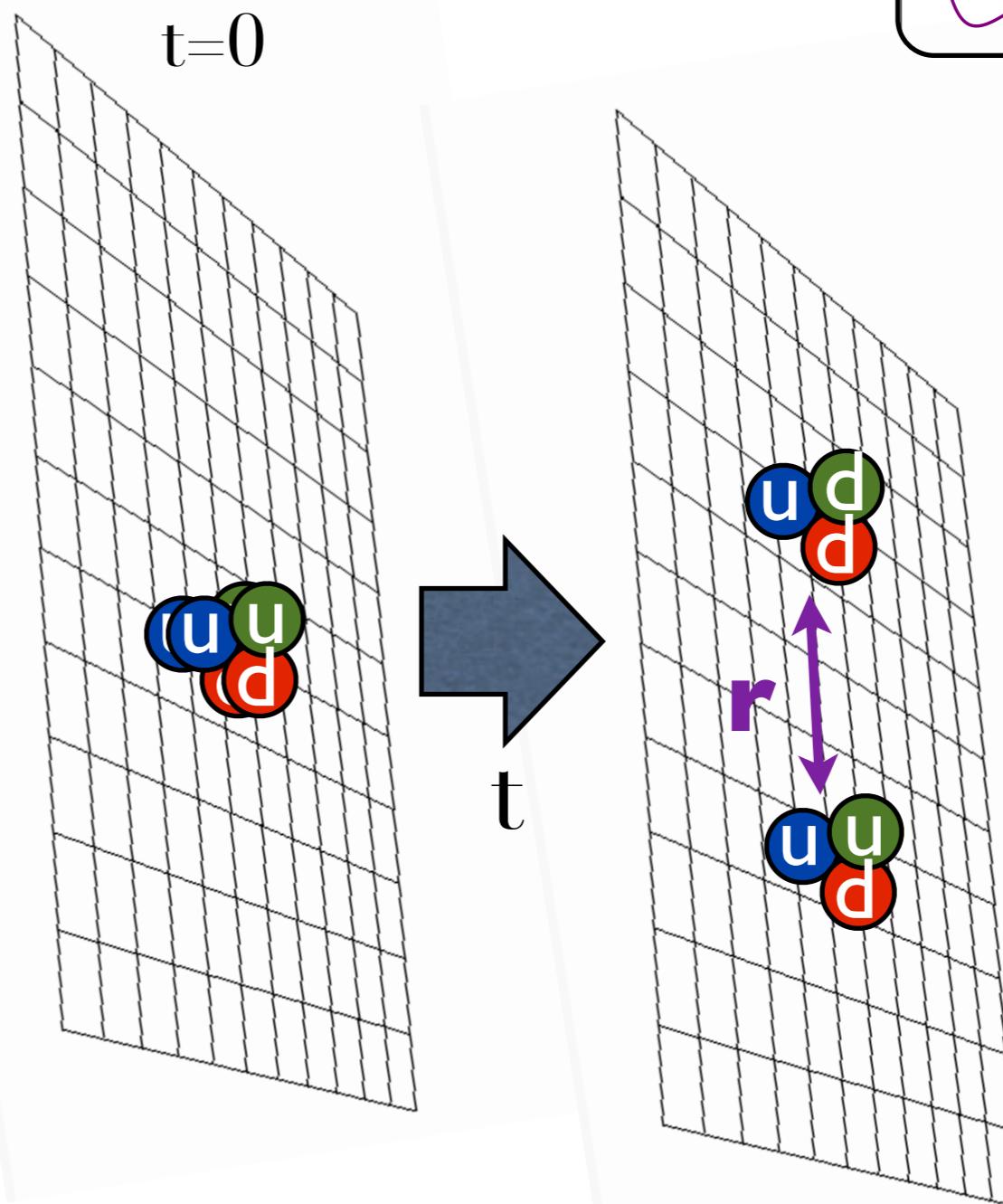
Quantization condition:



Potential method

1. Create the following correlation function:

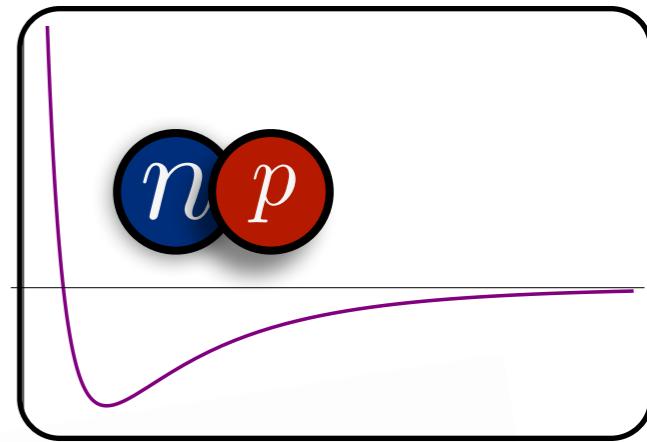
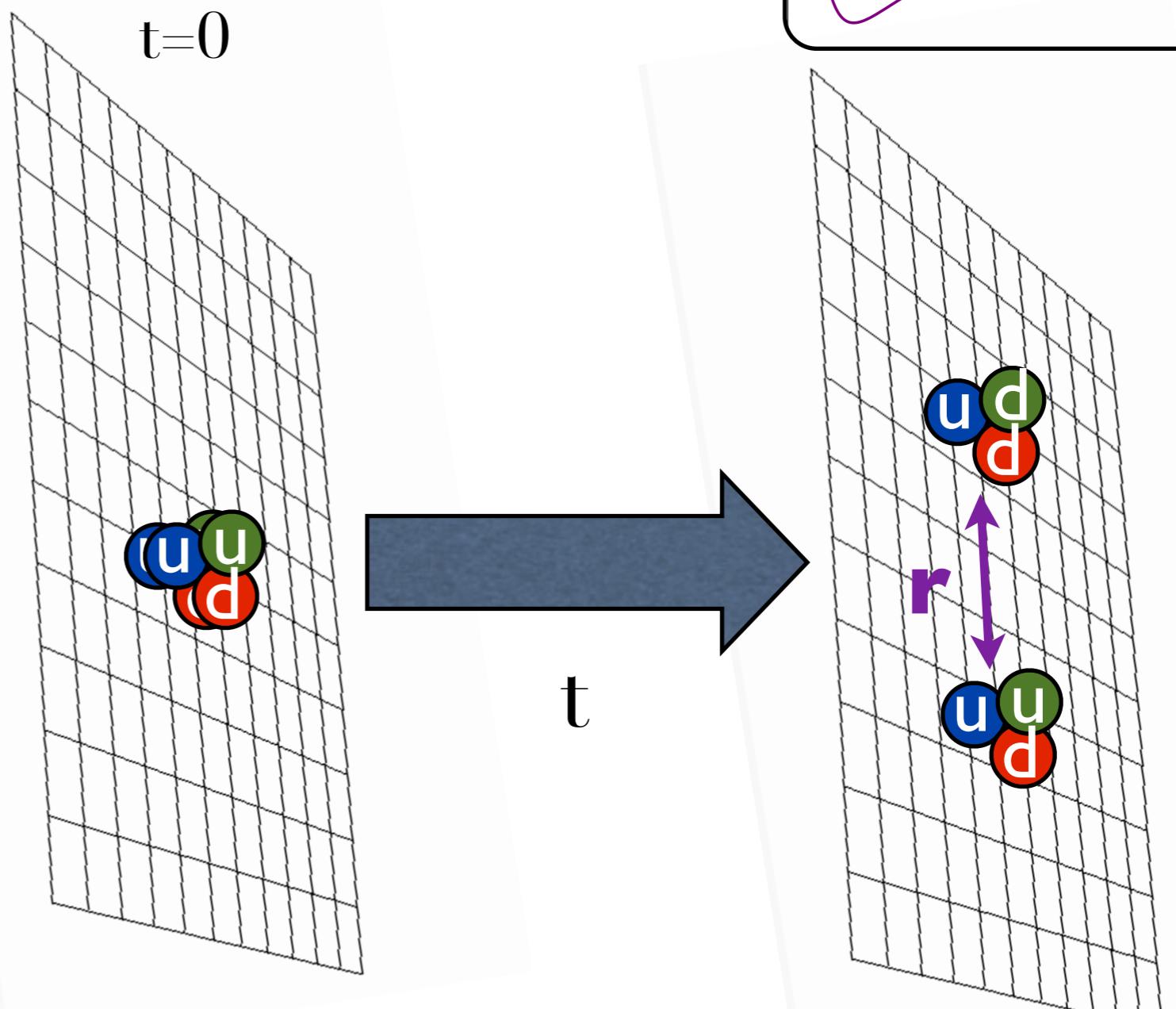
$$C_{NN}(\mathbf{r}, t)$$



Potential method

1. Create the following correlation function:

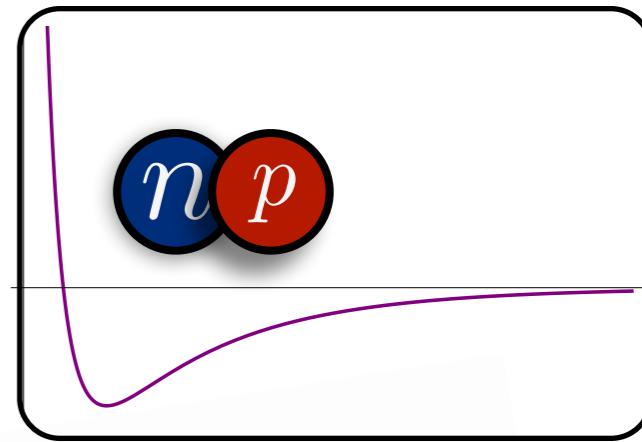
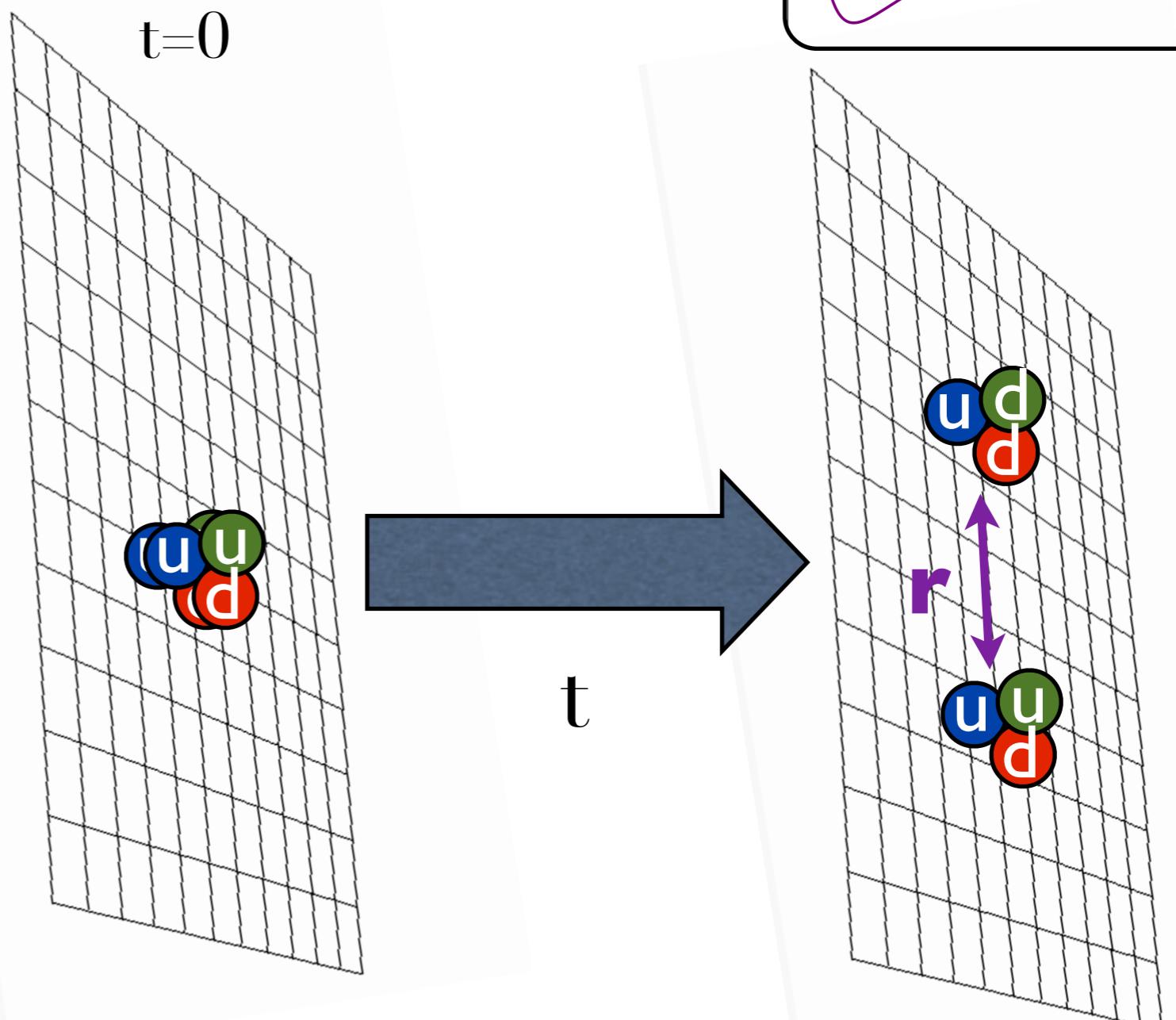
$$\lim_{t \rightarrow \infty} C_{NN}(\mathbf{r}, t) =$$



Potential method

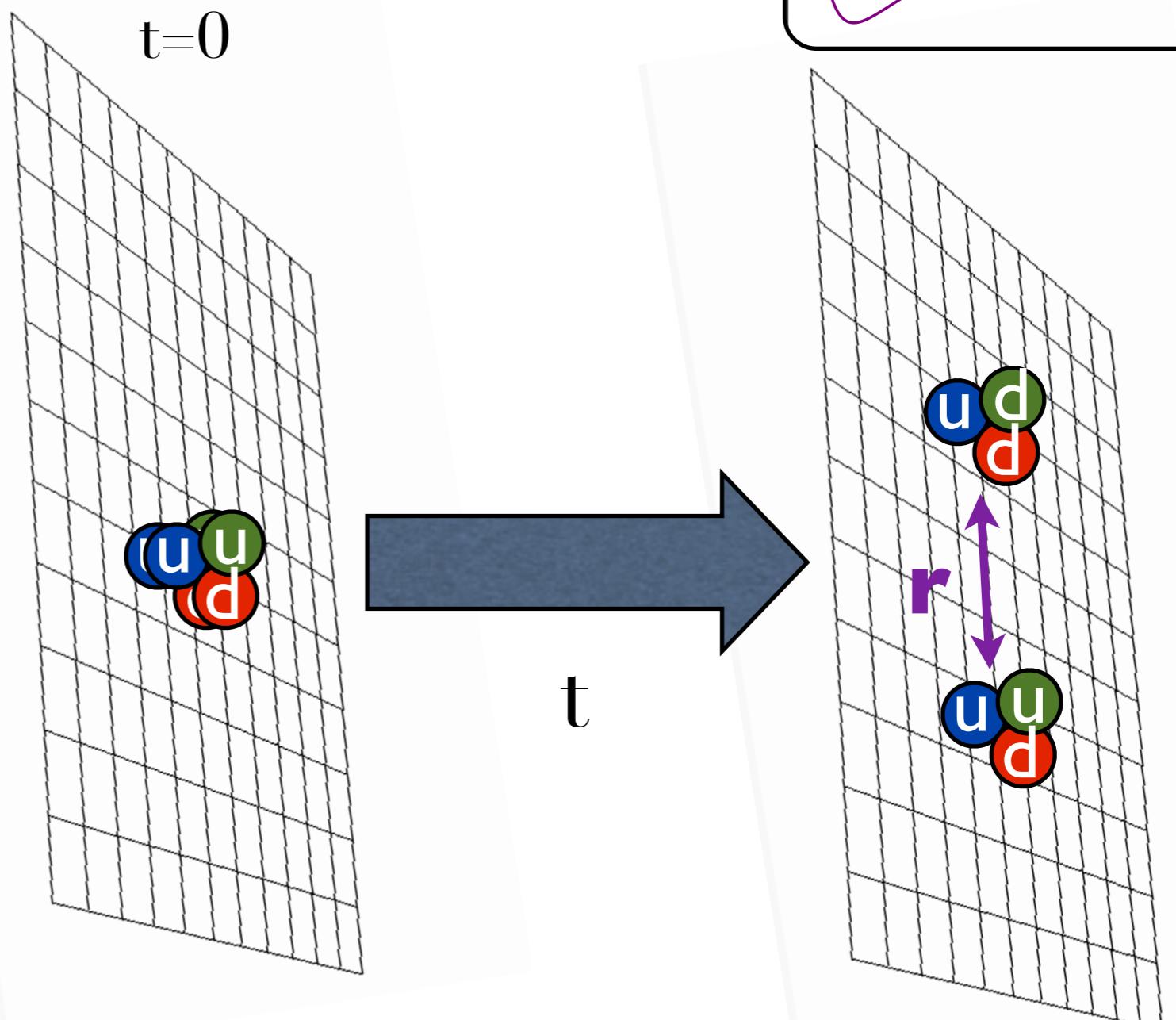
1. Create the following correlation function:

$$\lim_{t \rightarrow \infty} C_{NN}(\mathbf{r}, t) = \psi_0^\dagger$$



Potential method

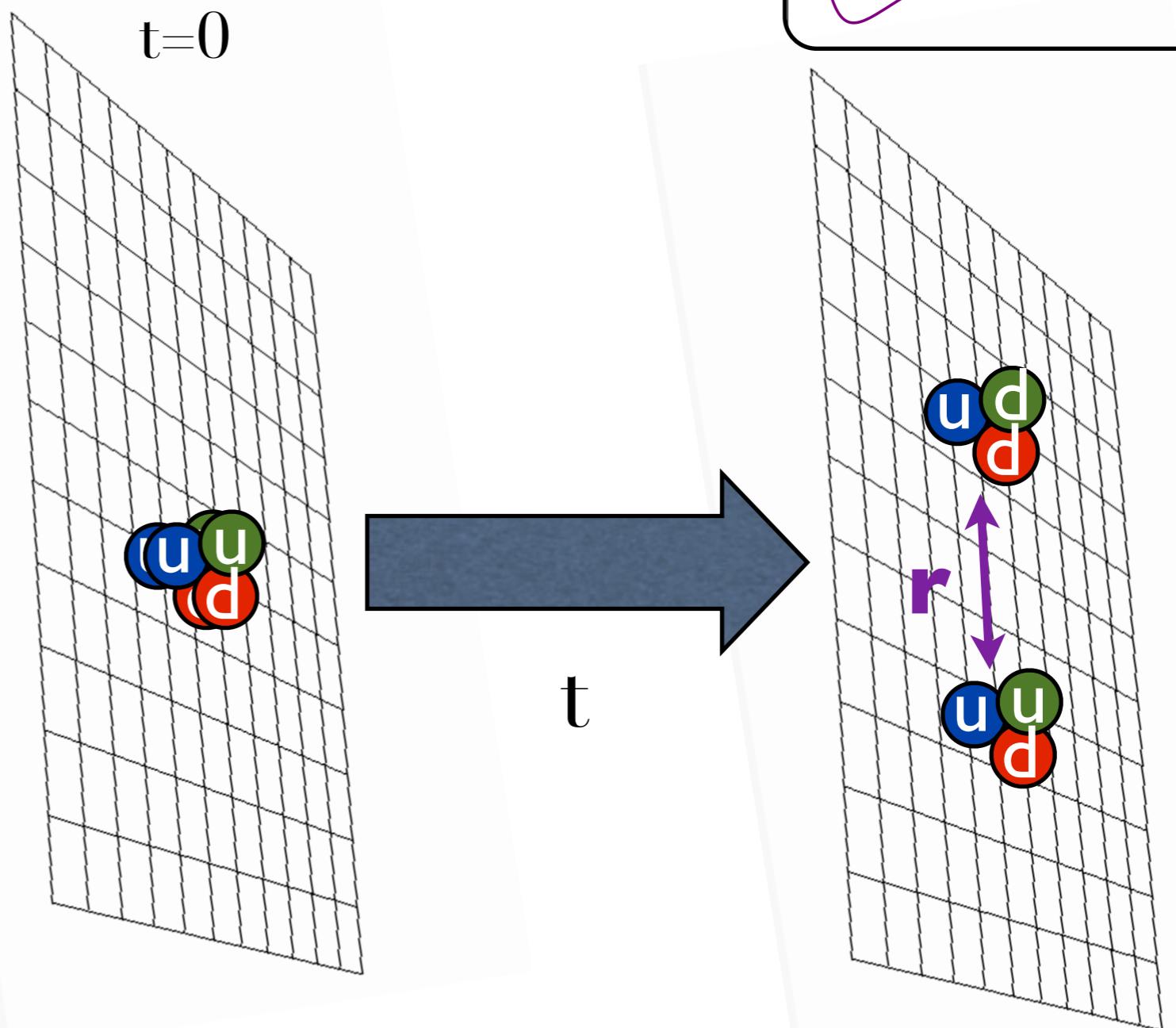
1. Create the following correlation function:



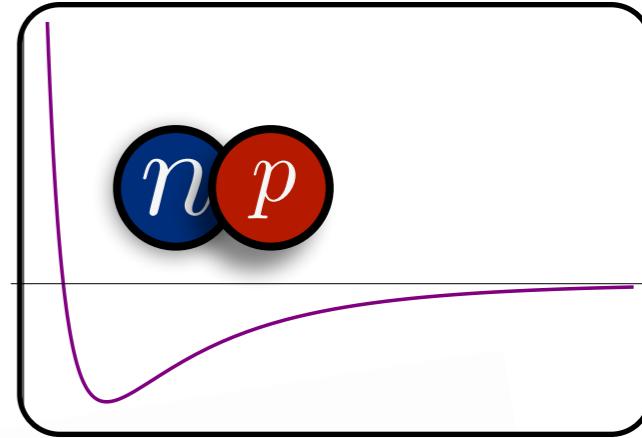
$$\lim_{t \rightarrow \infty} C_{NN}(\mathbf{r}, t) = \psi_0^\dagger \times e^{-E_0 t}$$

Potential method

1. Create the following correlation function:

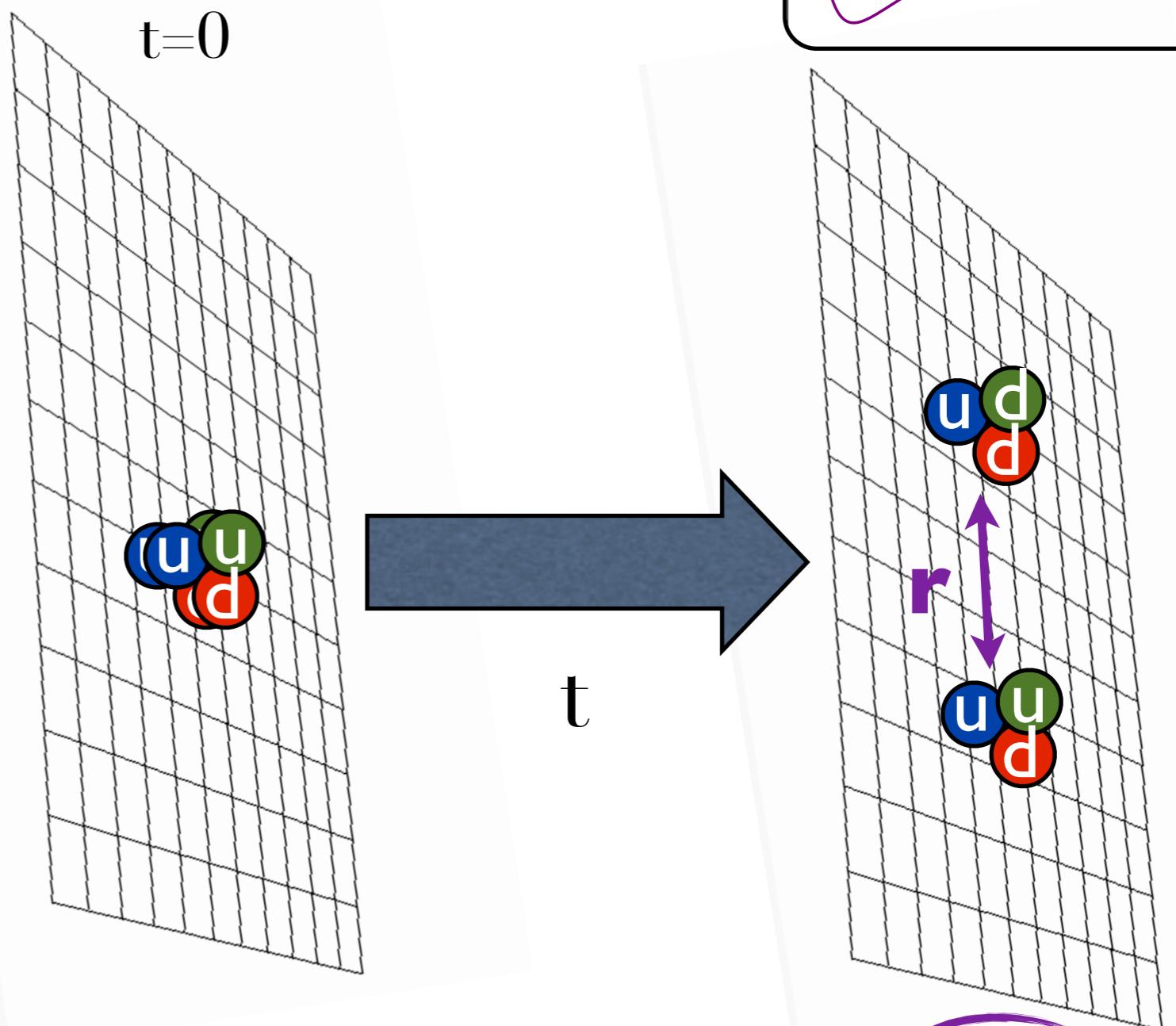


$$\lim_{t \rightarrow \infty} C_{NN}(\mathbf{r}, t) = \psi_0^\dagger \times e^{-E_0 t} \times \psi_0(\mathbf{r})$$



Potential method

1. Create the following correlation function:

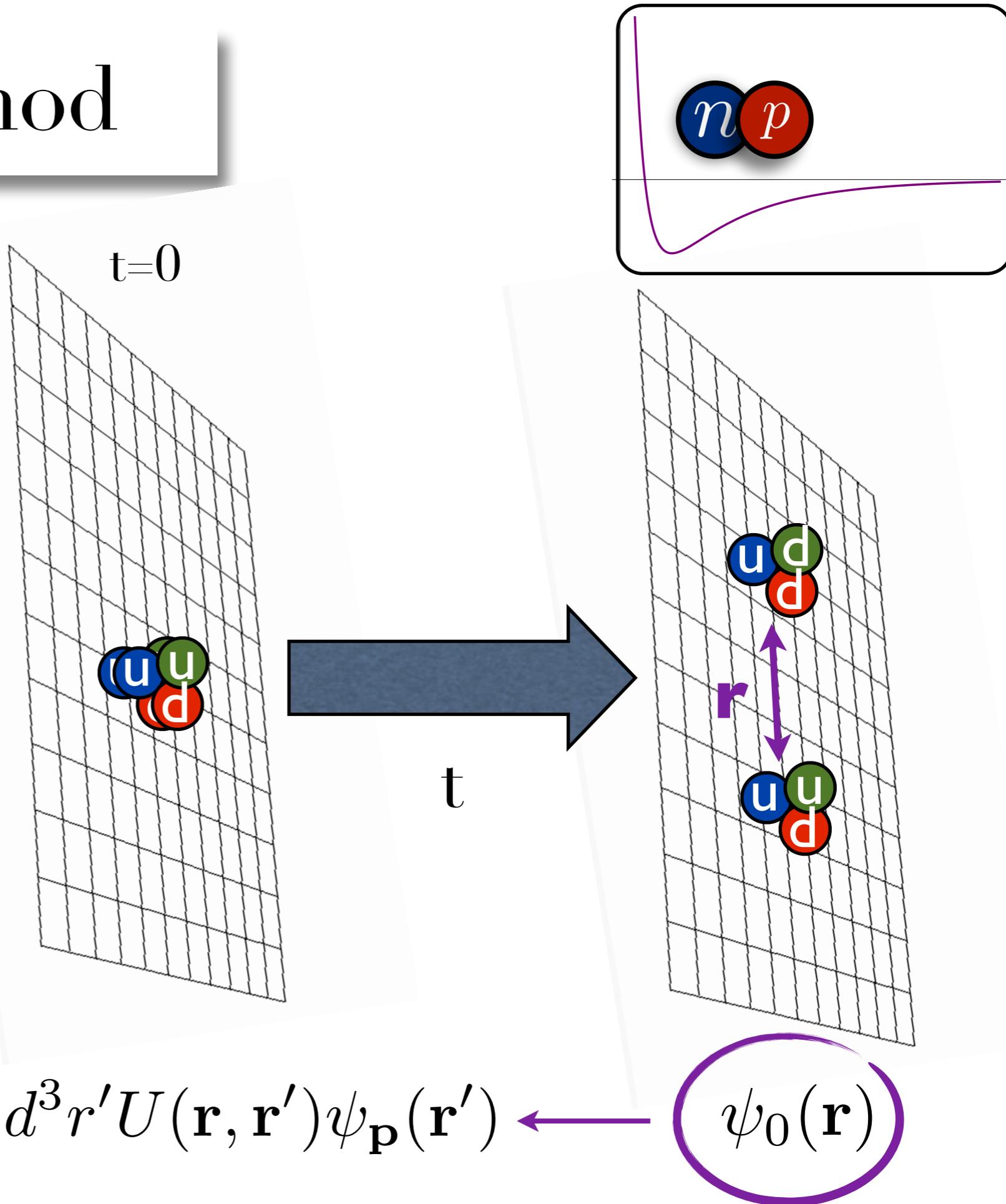


$$\lim_{t \rightarrow \infty} C_{NN}(\mathbf{r}, t) = \psi_0^\dagger \times e^{-E_0 t} \times \psi_0(\mathbf{r})$$

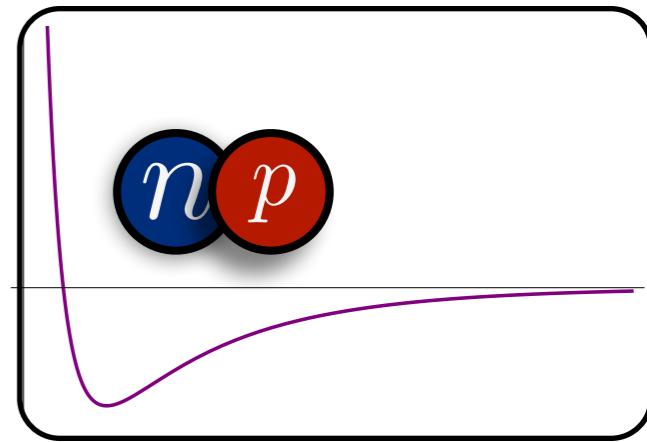
Potential method

2. Plug NBS wave-function into Schrödinger Eq. to determine the potential:

$$\left[\frac{\mathbf{p}^2}{2\mu} - H_0 \right] \psi_{\mathbf{p}}(\mathbf{r}) = \int d^3 r' U(\mathbf{r}, \mathbf{r}') \psi_{\mathbf{p}}(\mathbf{r}') \quad \psi_0(\mathbf{r})$$



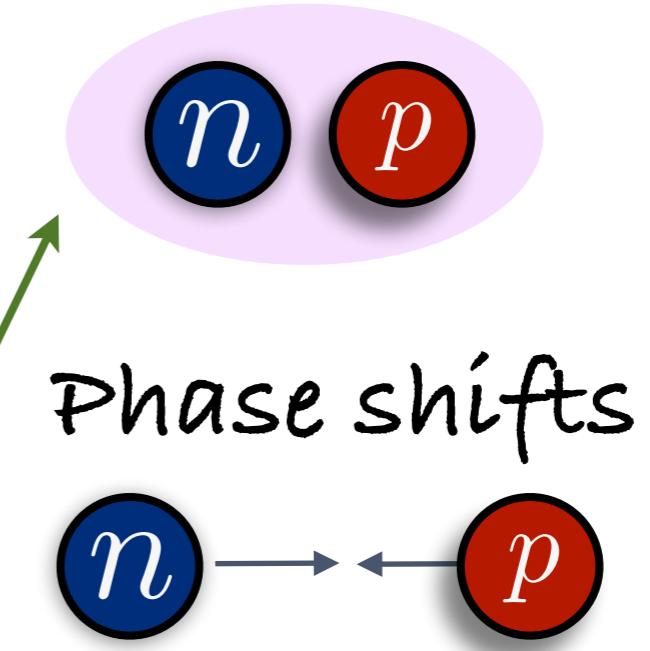
Potential method



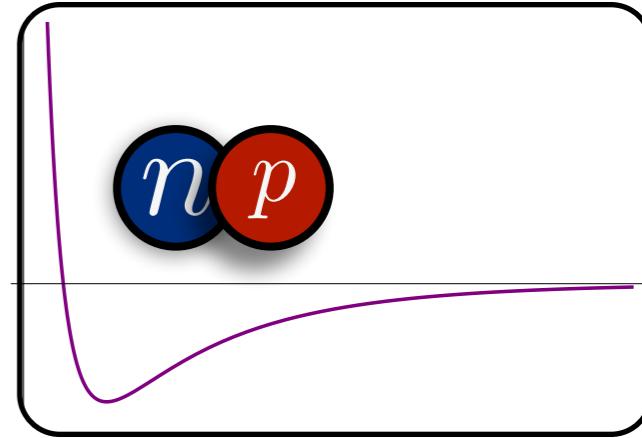
Binding energies

2. Plug NBS
wave-function
into Schrödinger
Eq. to determine
the potential:

$$\left[\frac{\mathbf{p}^2}{2\mu} - H_0 \right] \psi_{\mathbf{p}}(\mathbf{r}) = \int d^3r' U(\mathbf{r}, \mathbf{r}') \psi_{\mathbf{p}}(\mathbf{r}') \quad \psi_0(\mathbf{r})$$



Potential method



3. Use derivative expansion to determine the leading order potential:

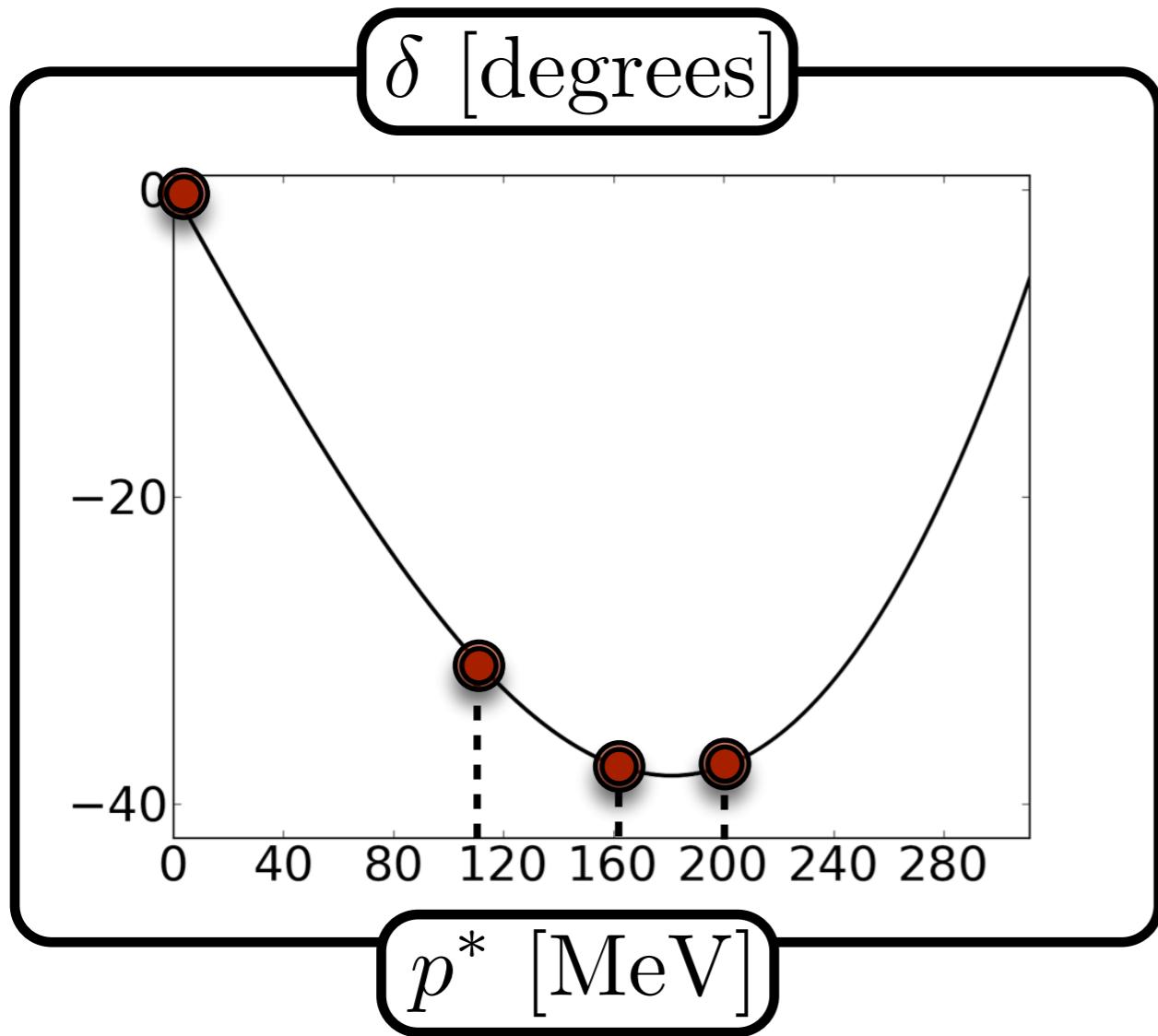
$$U(\mathbf{r}, \mathbf{r}') = V_C(\mathbf{r})\delta(\mathbf{r} - \mathbf{r}') + \mathcal{O}(\nabla_{\mathbf{r}}^2/\Lambda^2)$$

$$V_C(\mathbf{r}) \simeq \frac{\mathbf{p}^2}{2\mu} + \lim_{t \rightarrow \infty} \frac{1}{2\mu} \frac{\nabla_{\mathbf{r}}^2 C_{NN}(\mathbf{r}, t)}{C_{NN}(\mathbf{r}, t)}$$

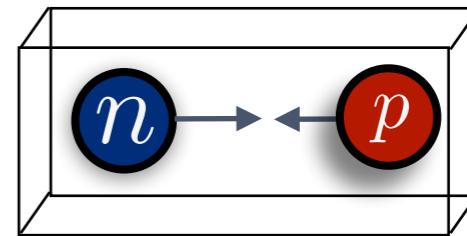
$$\left[\frac{\mathbf{p}^2}{2\mu} - H_0 \right] \psi_{\mathbf{p}}(\mathbf{r}) = \int d^3 r' U(\mathbf{r}, \mathbf{r}') \psi_{\mathbf{p}}(\mathbf{r}') \longleftarrow \psi_0(\mathbf{r})$$

Some comparisons

see Drischler, et al, 1910.07961

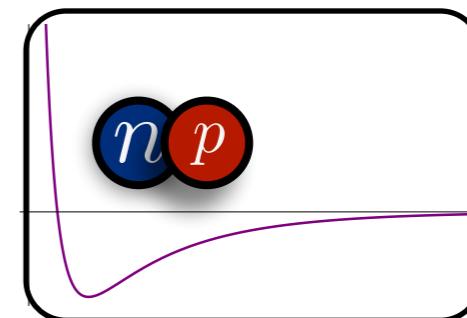


$$U(\mathbf{r}, \mathbf{r}') = V_C(\mathbf{r})\delta(\mathbf{r} - \mathbf{r}') + \mathcal{O}(\nabla_{\mathbf{r}}^2/\Lambda^2)$$



Luscher

- discrete phase shifts
- need ground state saturation
- no volume extrapolation
- no uncontrolled approximations

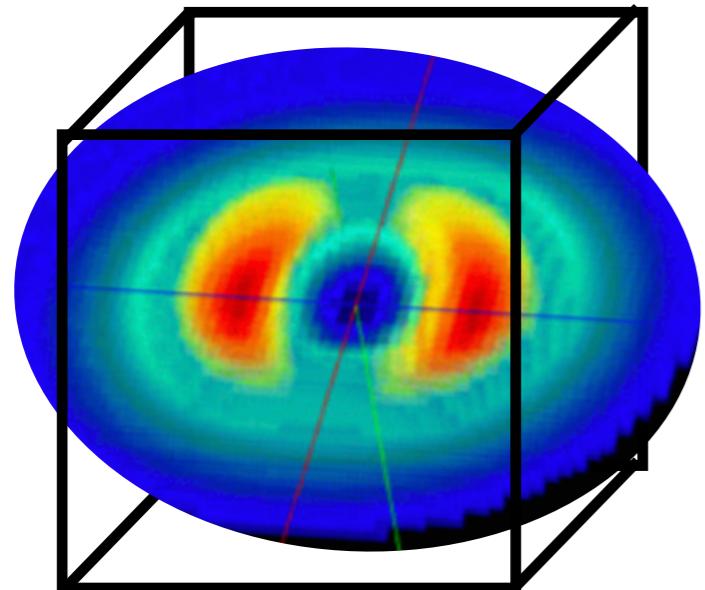


Potential

- nearly continuous phase shifts
- only need elastic state saturation
- need volume extrapolation
- cutoff in gradient expansion

LQCD connection to HOBET

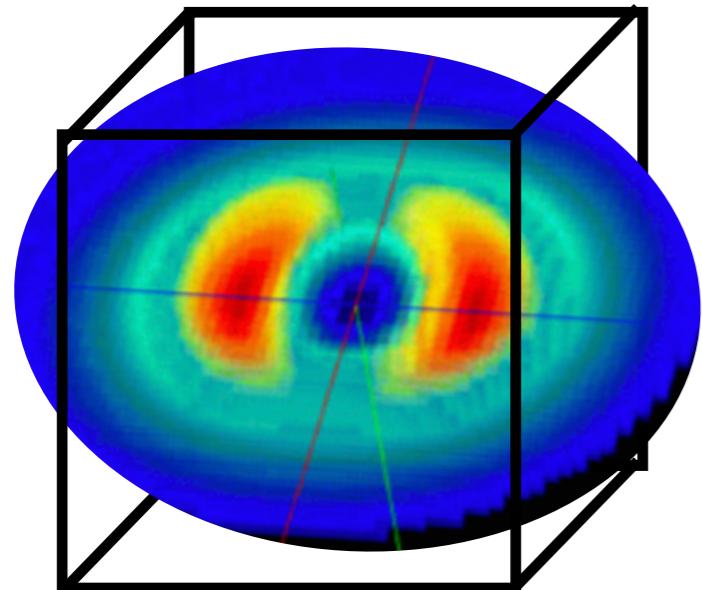
(K. McElvain and W. Haxton)



Formulate HOBET with the same (IR) BC as LQCD, fix (UV) couplings to reproduce LQCD energy levels, then remove the BC and make predictions

LQCD connection to HOBET

(K. McElvain and W. Haxton)

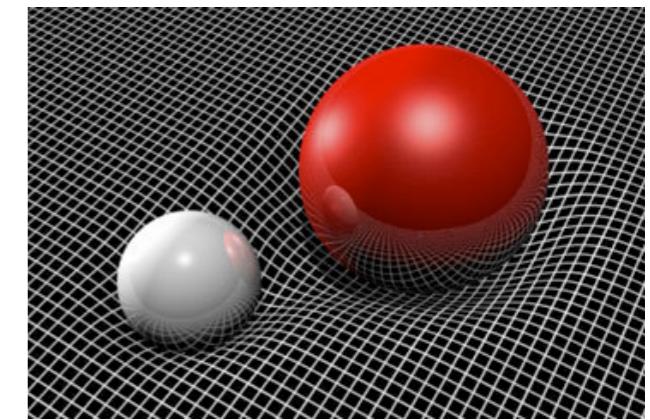


Formulate HOBET with the same (IR) BC as LQCD, fix (UV) couplings to reproduce LQCD energy levels, then remove the BC and make predictions

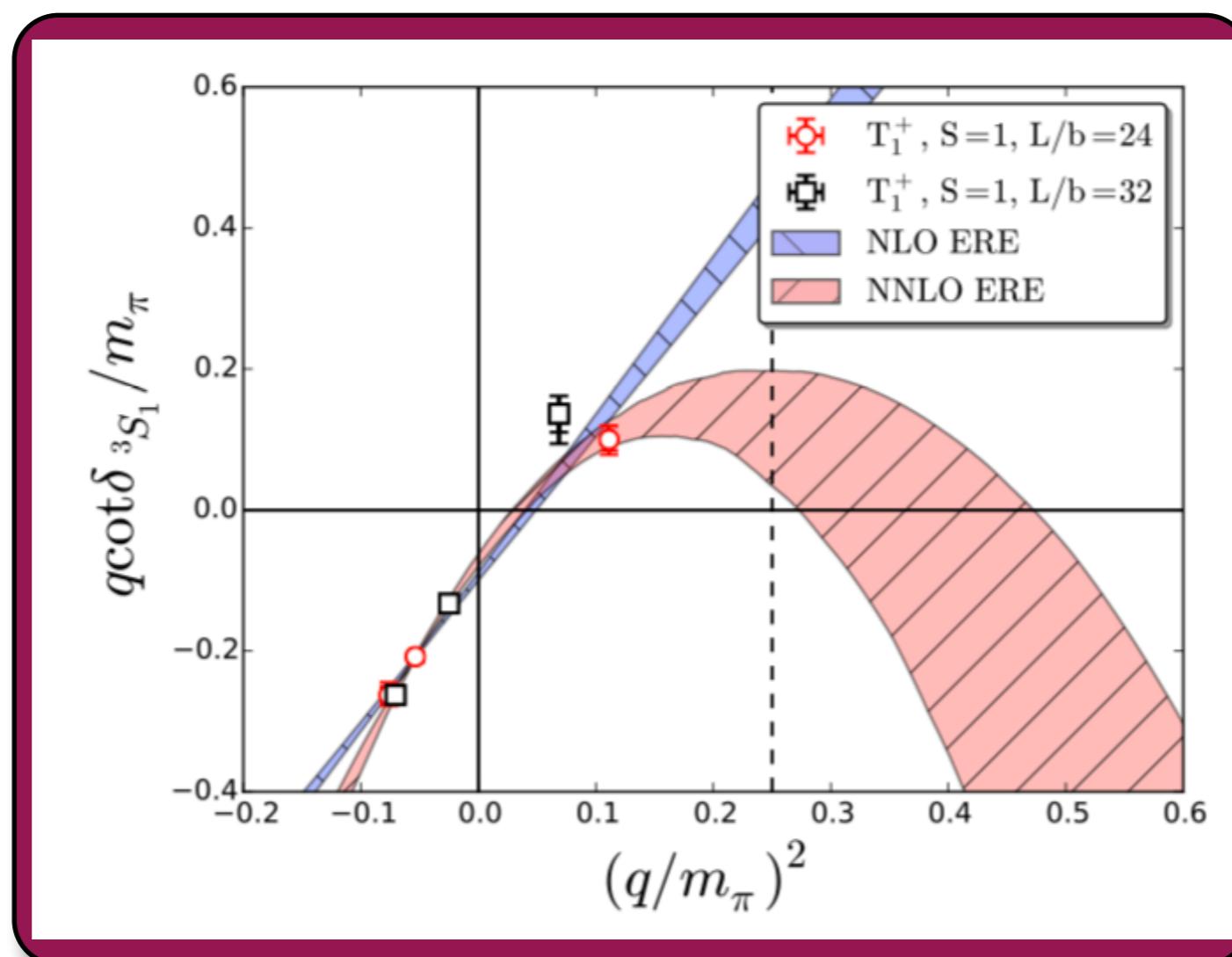
- No need to truncate partial wave expansion
- Can deal with volumes smaller than Compton wavelength of the pion
- Luscher formalism for $N>2$ is messy
- Alternate method for determining binding energies

Composite states at $m_\pi \sim 800$ MeV

- L=32
- L=24

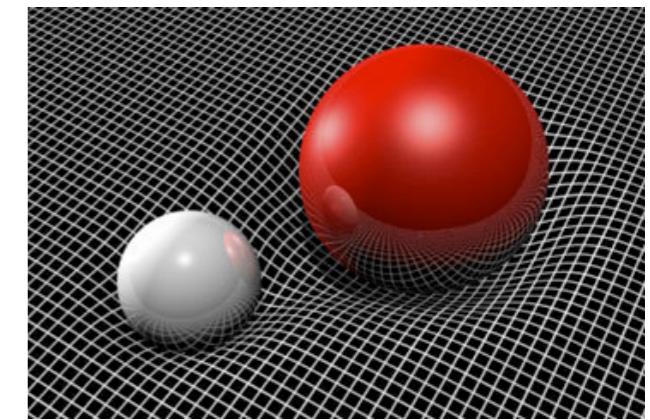


$$pcot\delta = ip$$

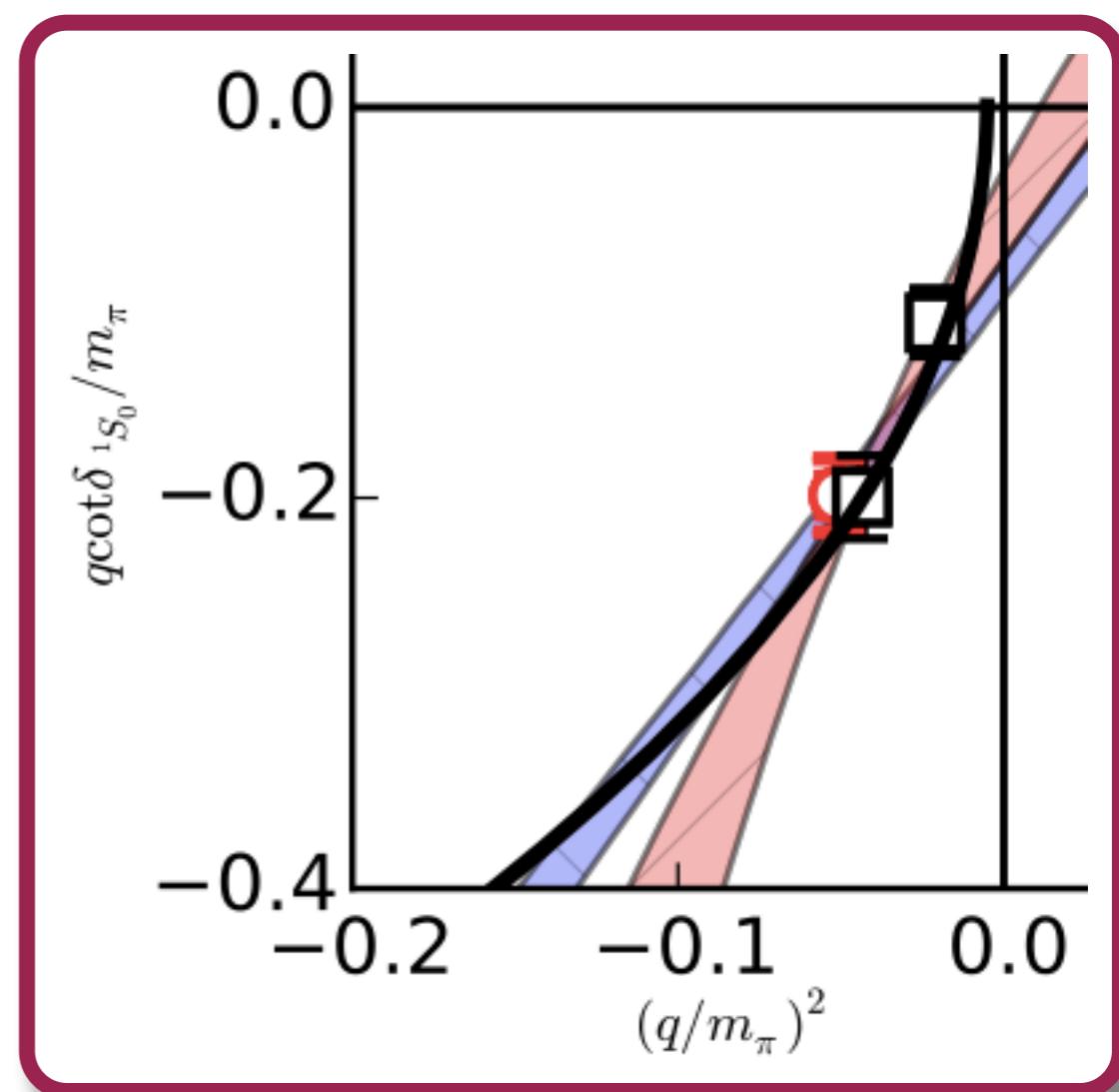
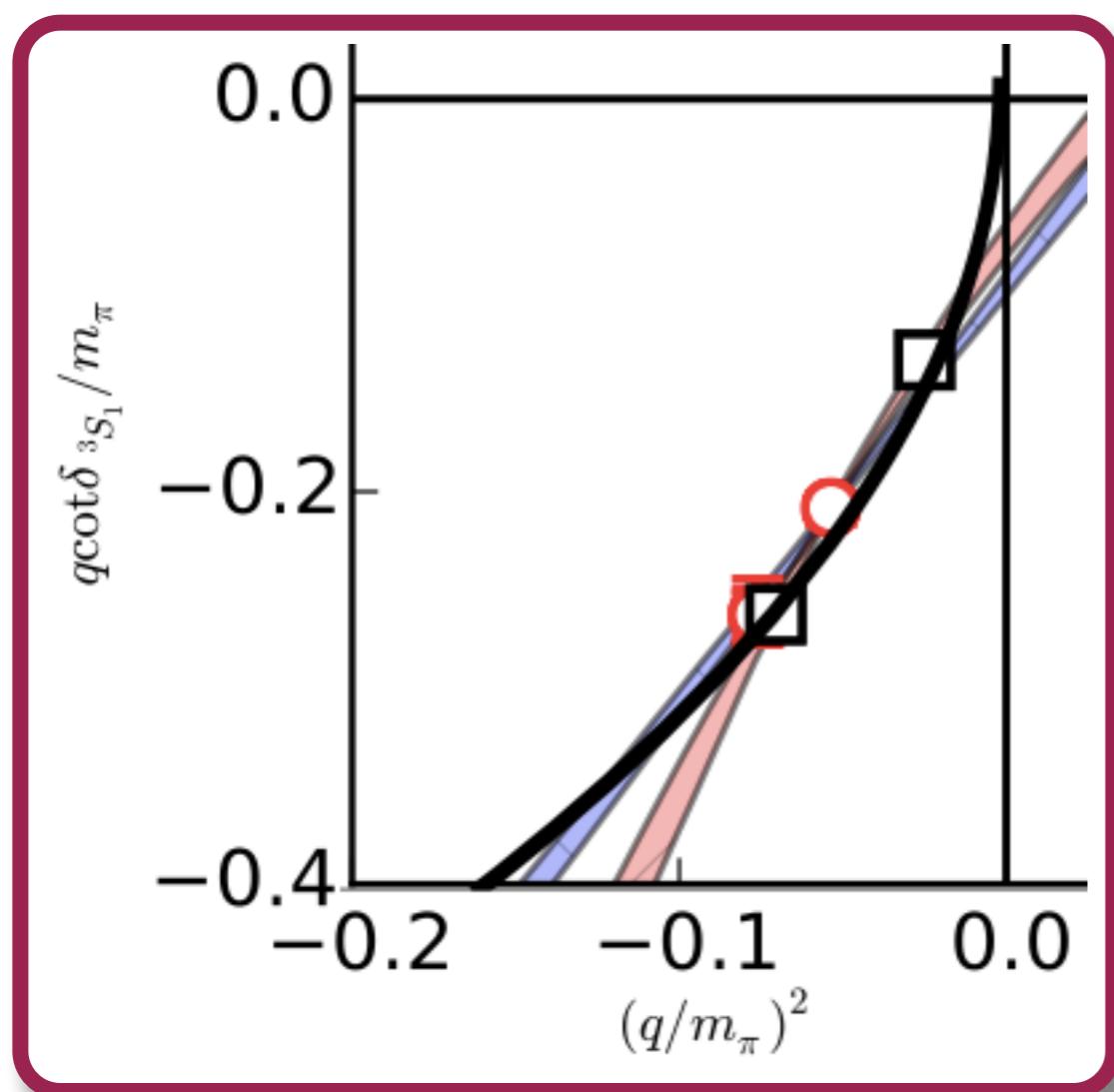


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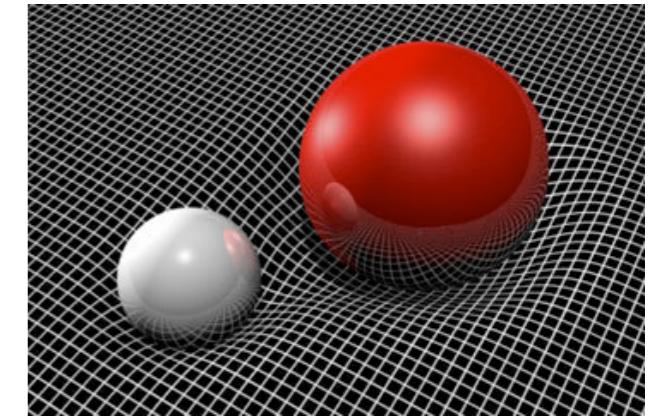


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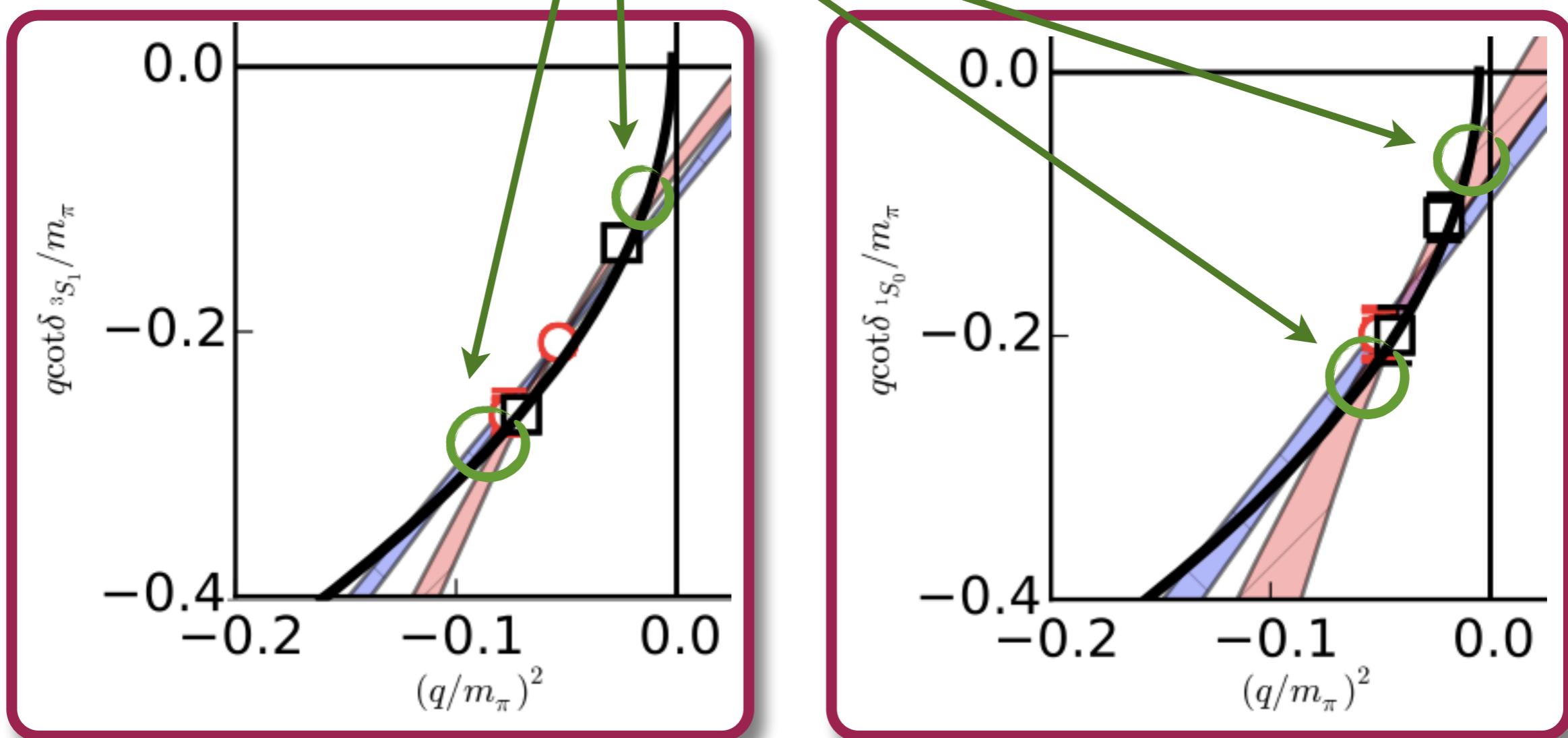
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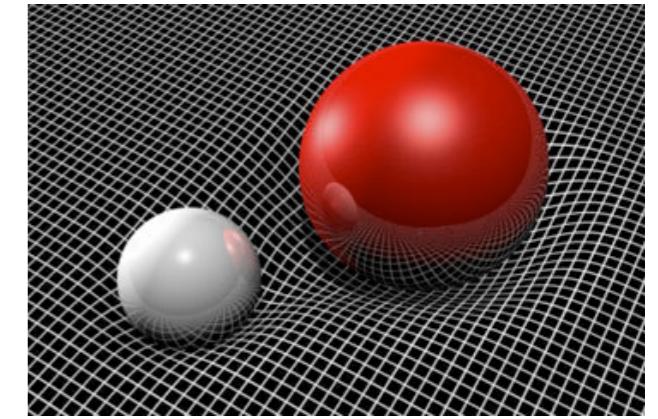
NNLO crossings

$$pcot\delta = ip$$



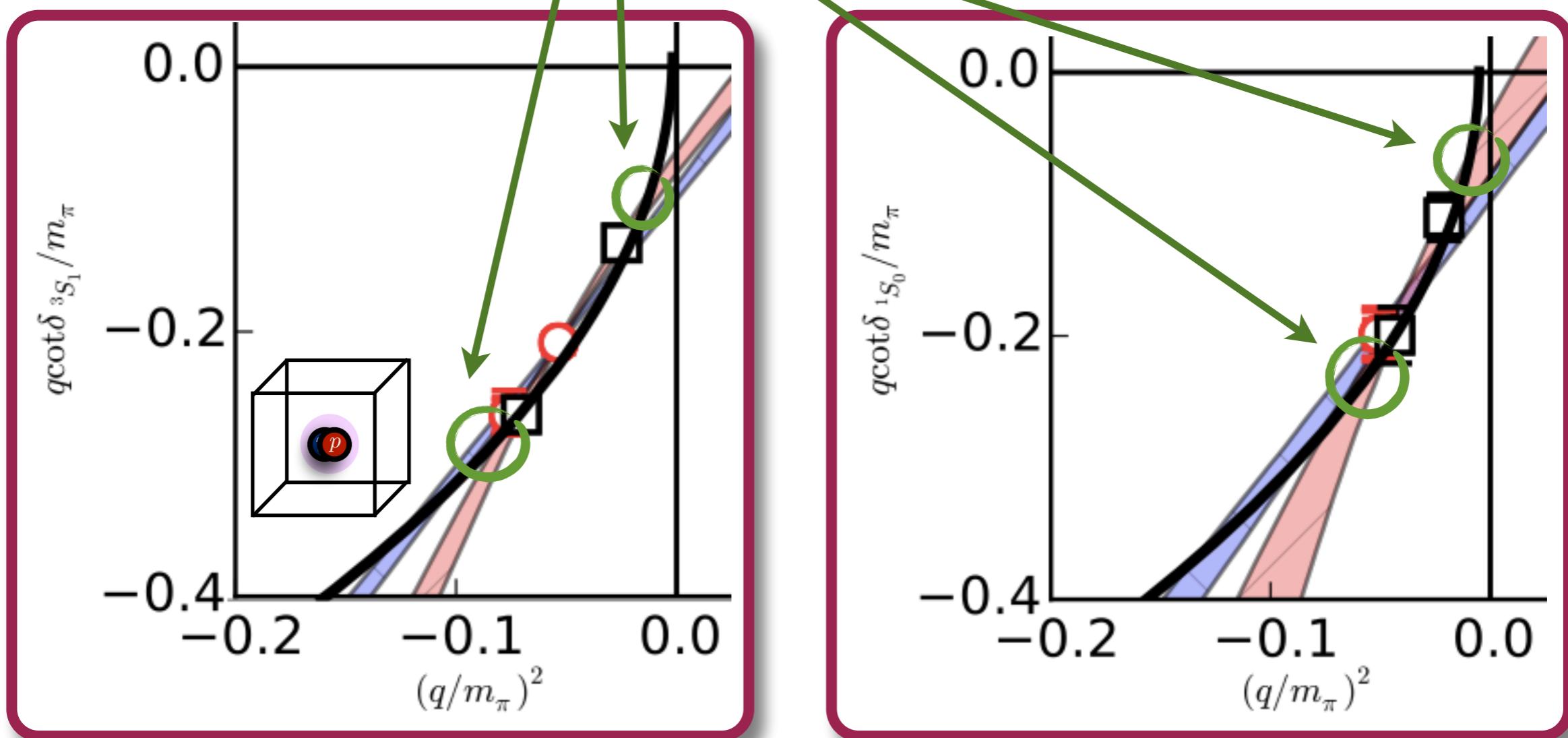
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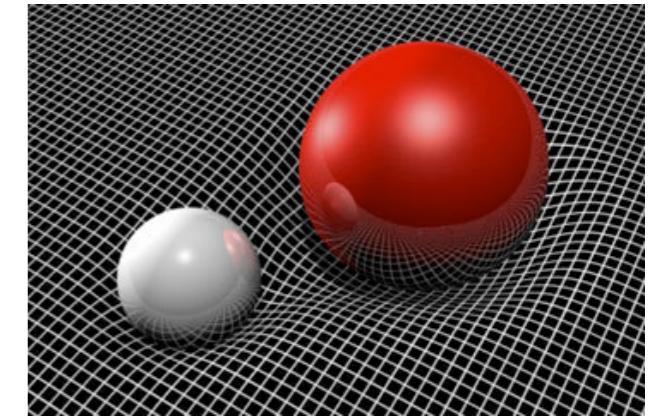
NNLO crossings

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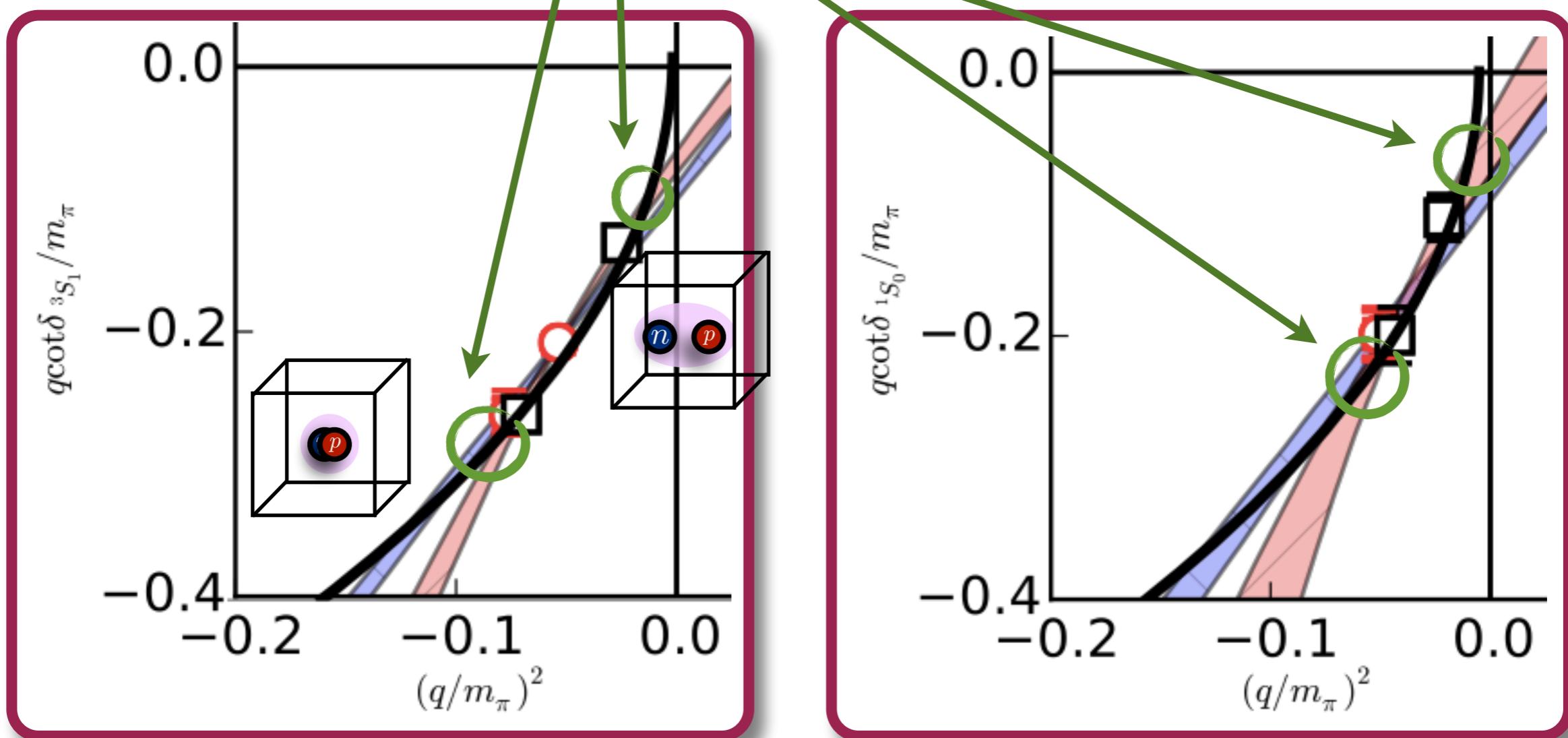
Composite states at $m_\pi \sim 800$ MeV

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NNLO crossings

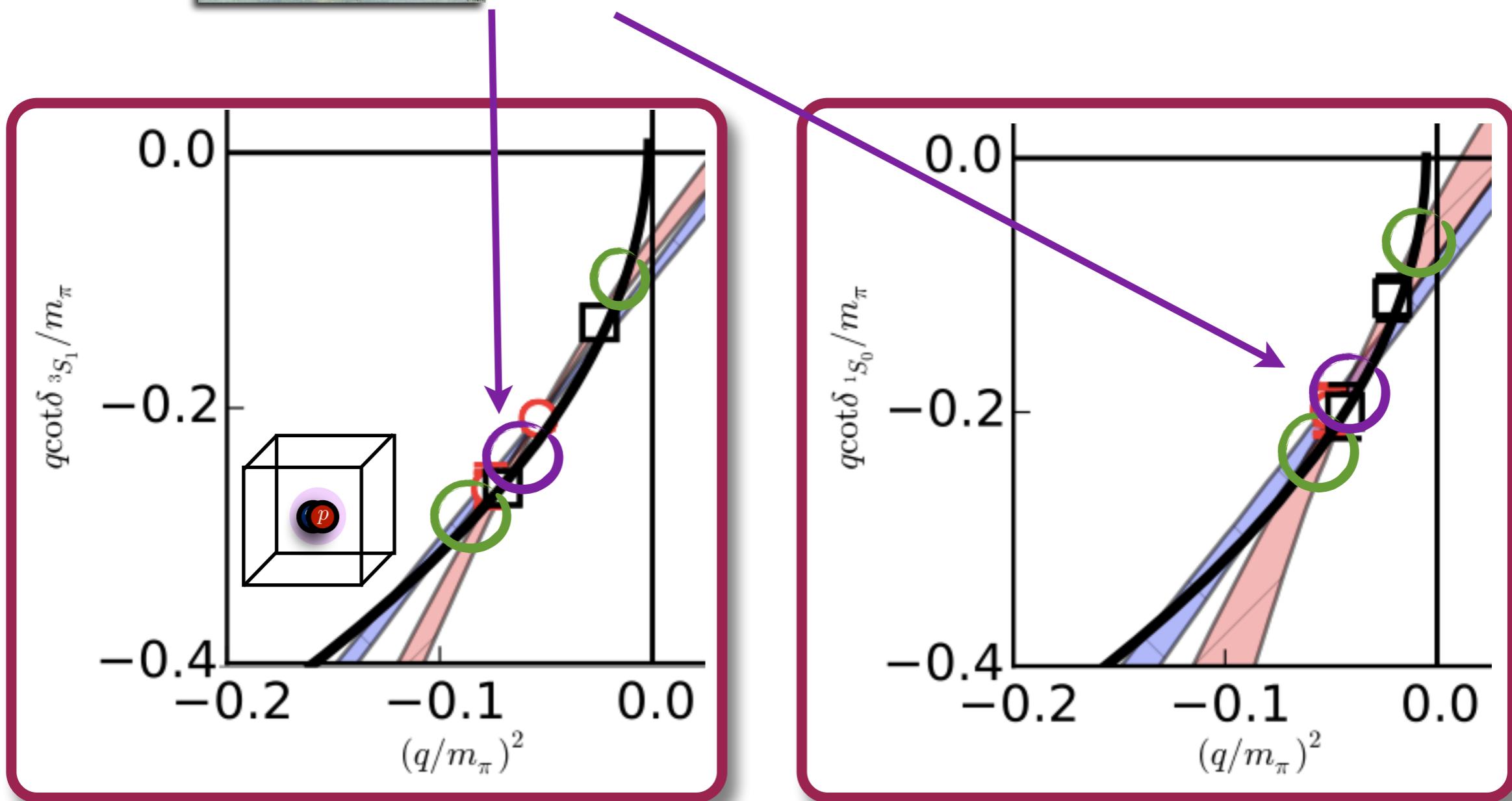
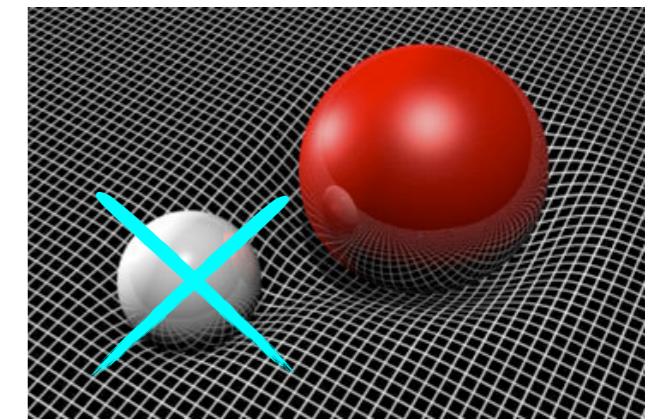
$$pcot\delta = ip$$

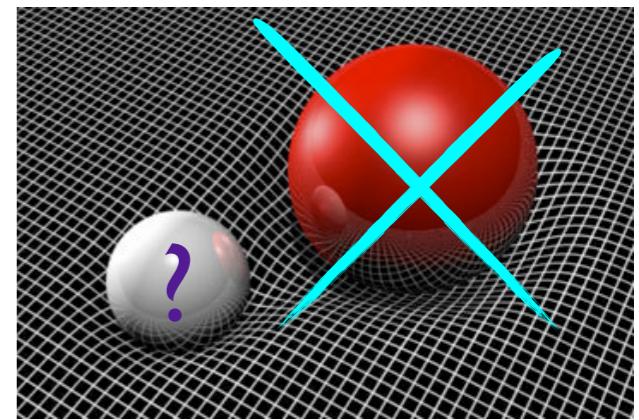
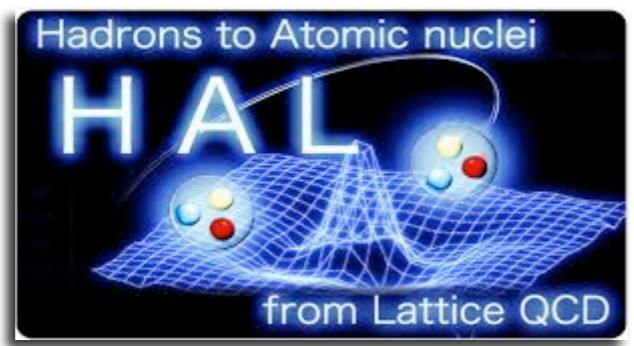




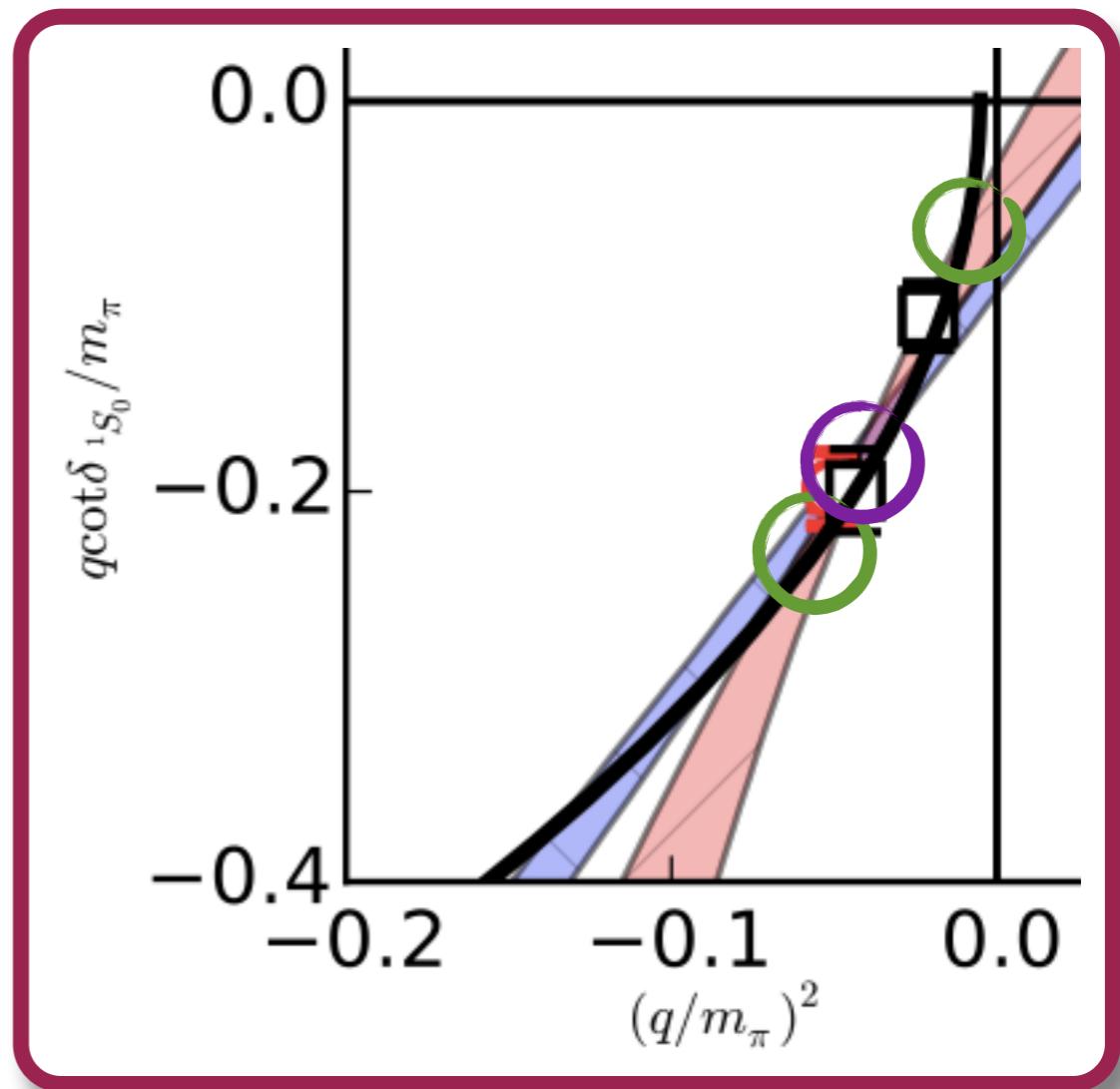
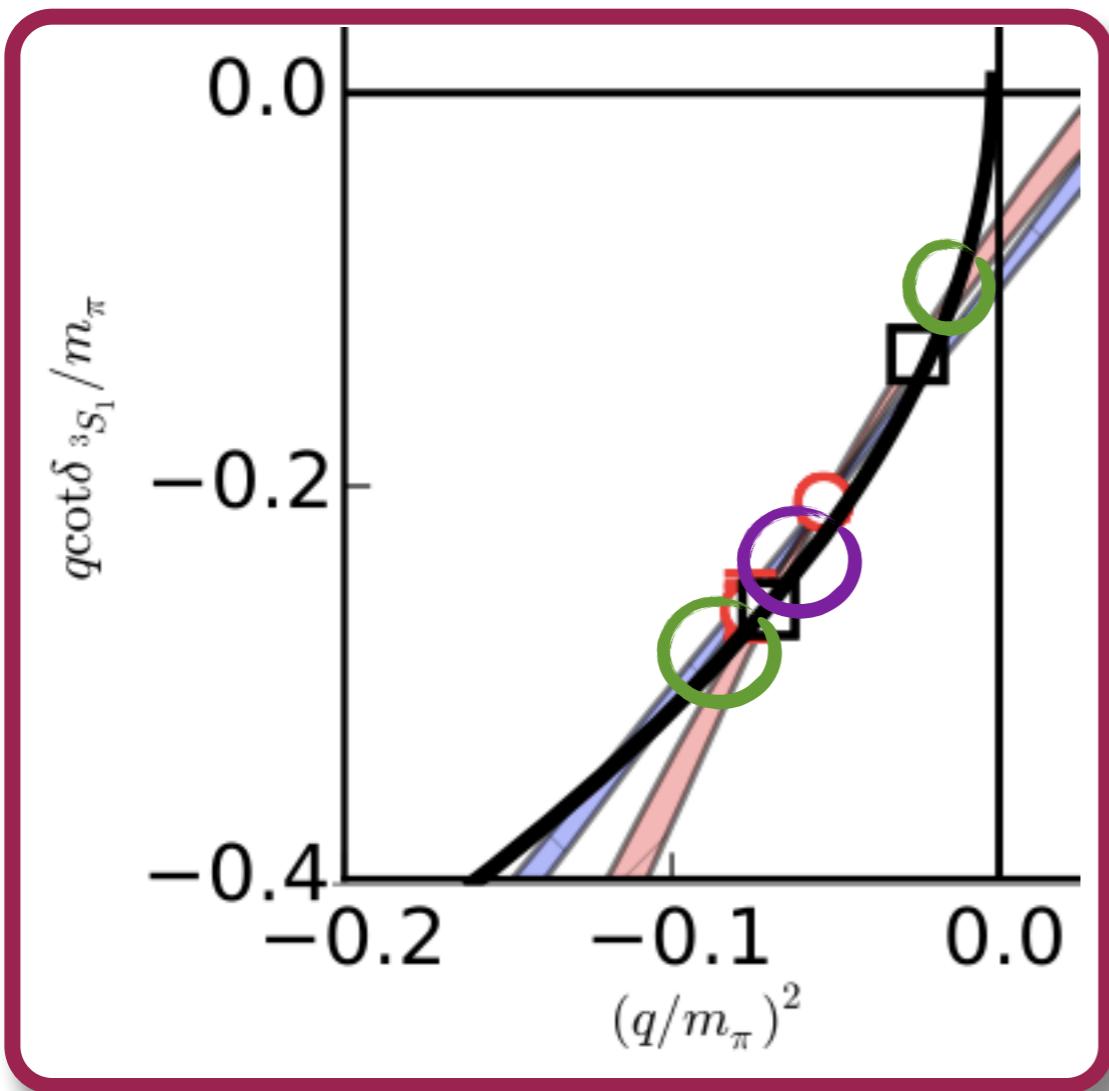
Yamazaki, et. al.

$\text{pcot}\delta = \text{ip}$



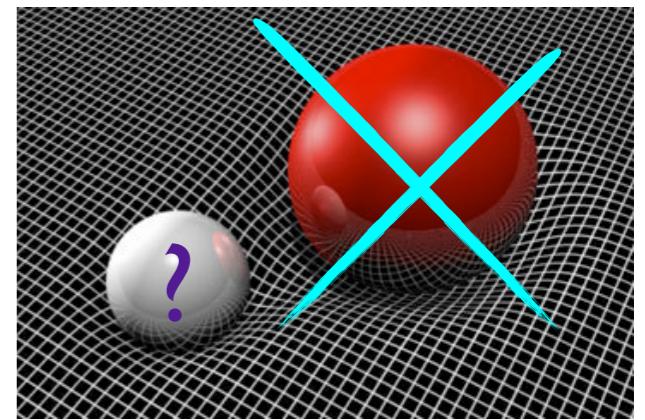


$$pcot\delta = ip$$

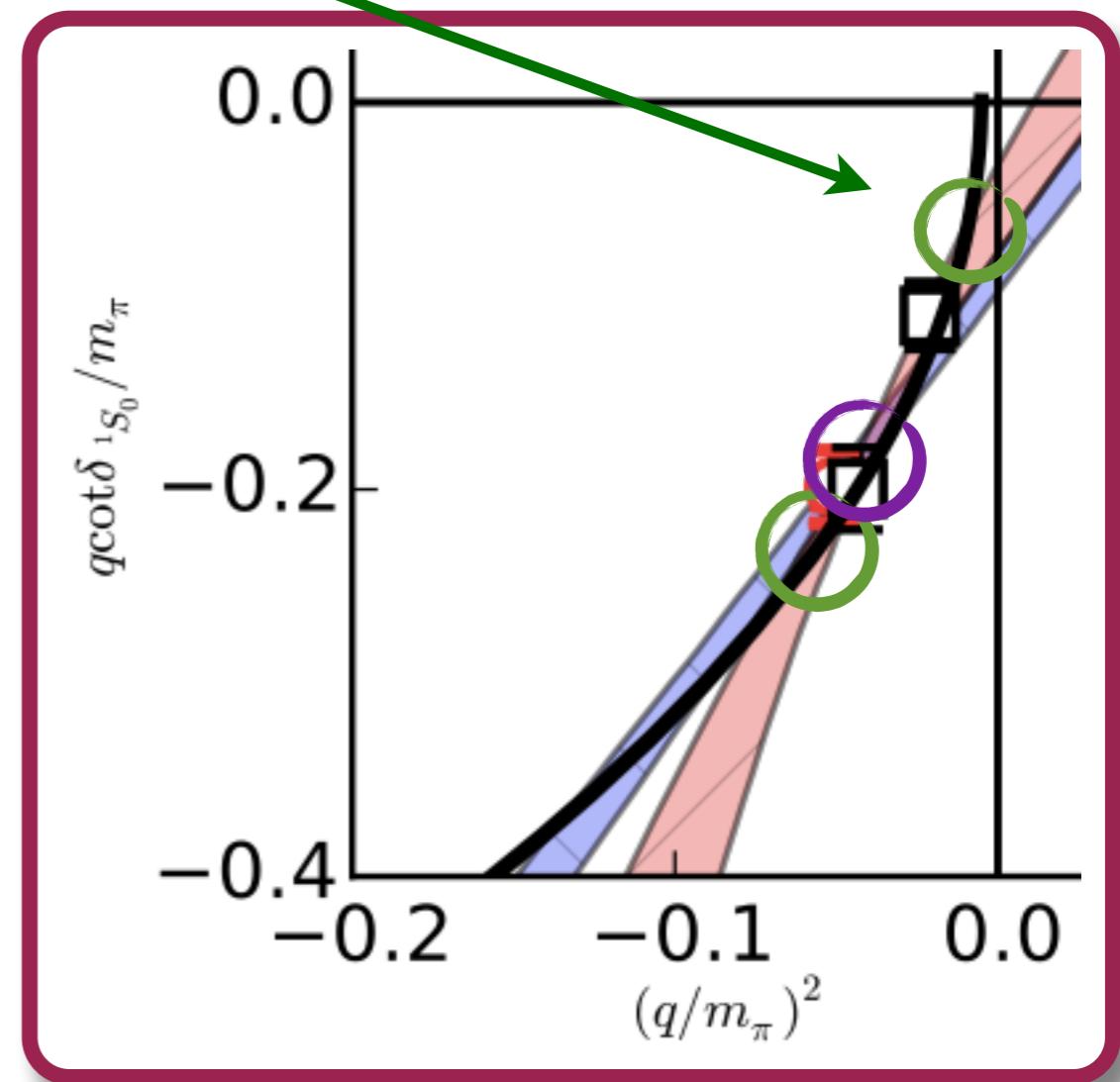
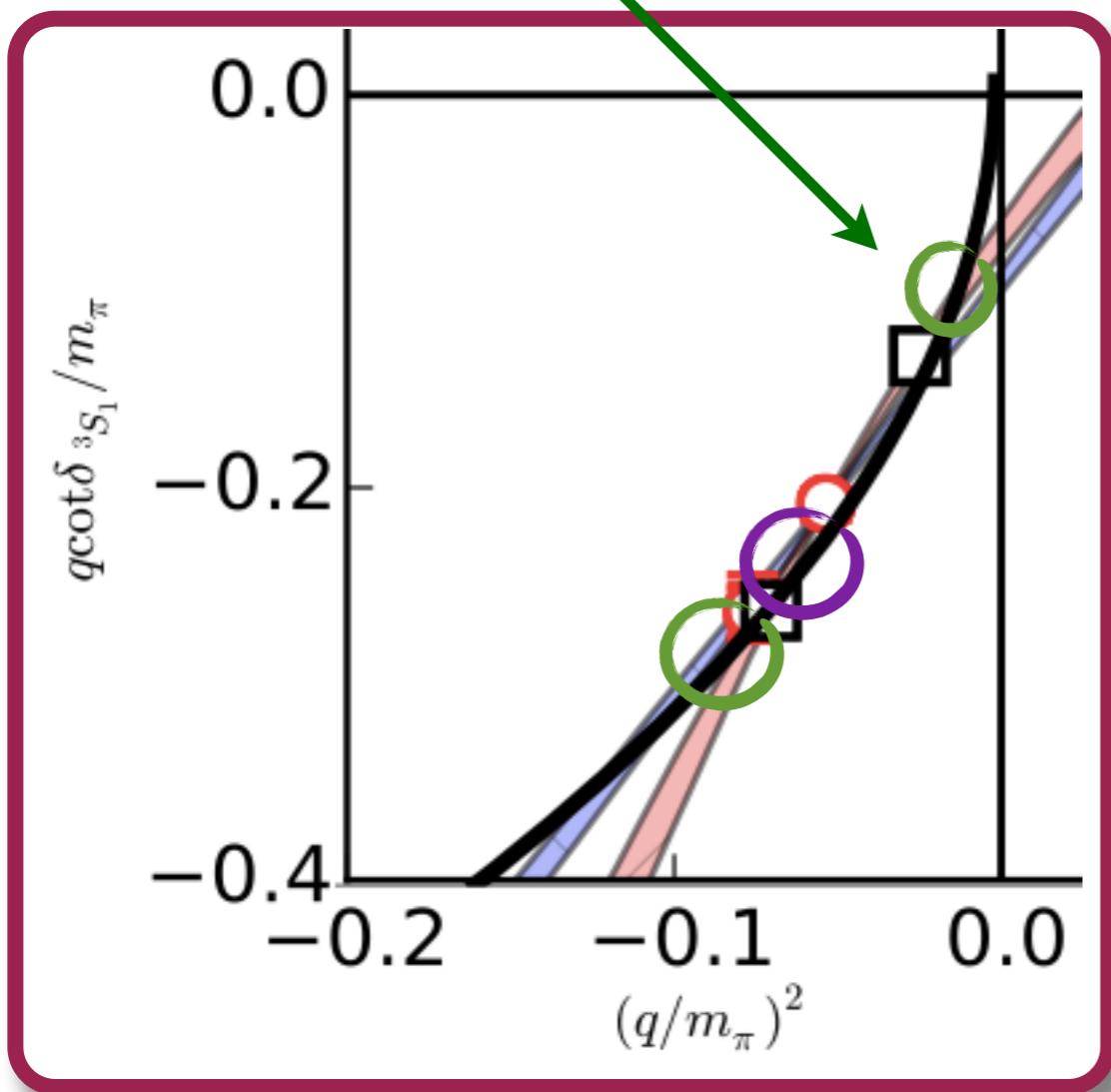


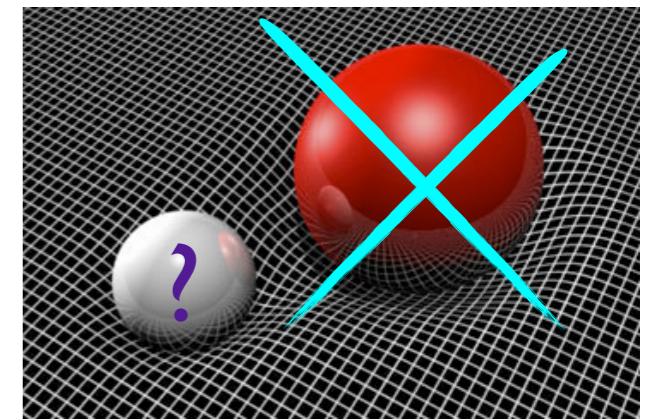


?



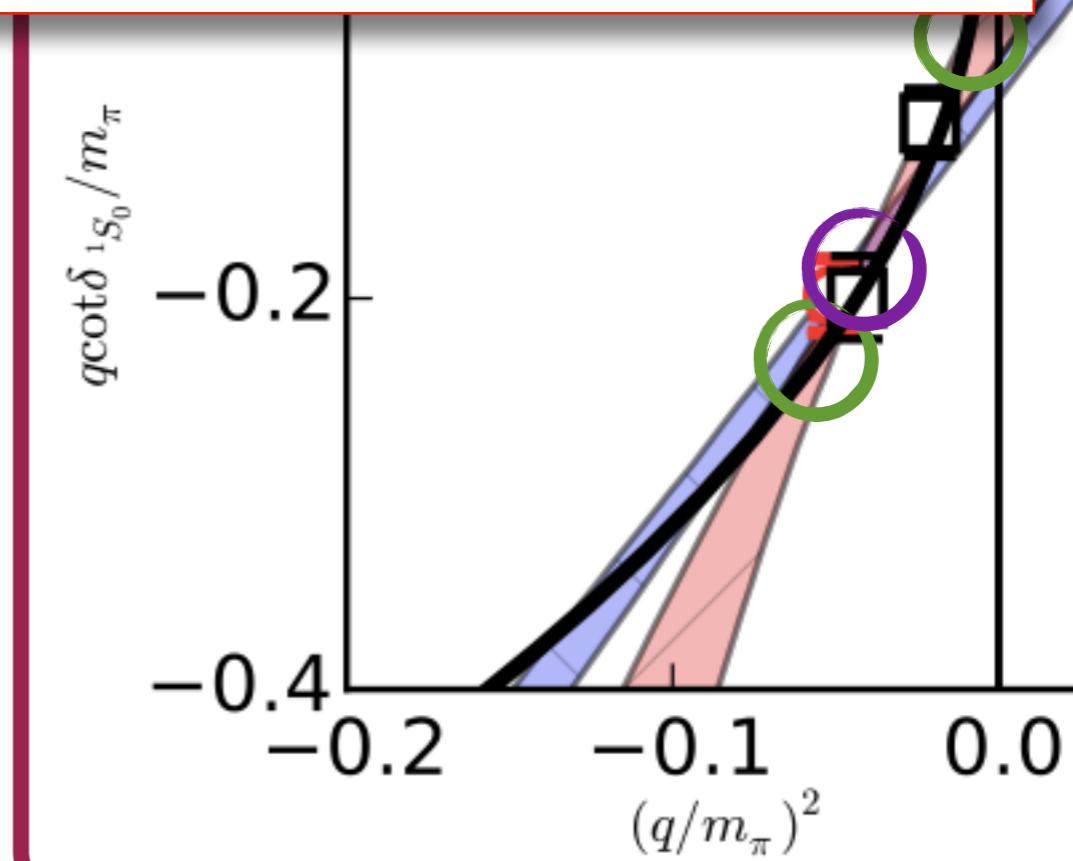
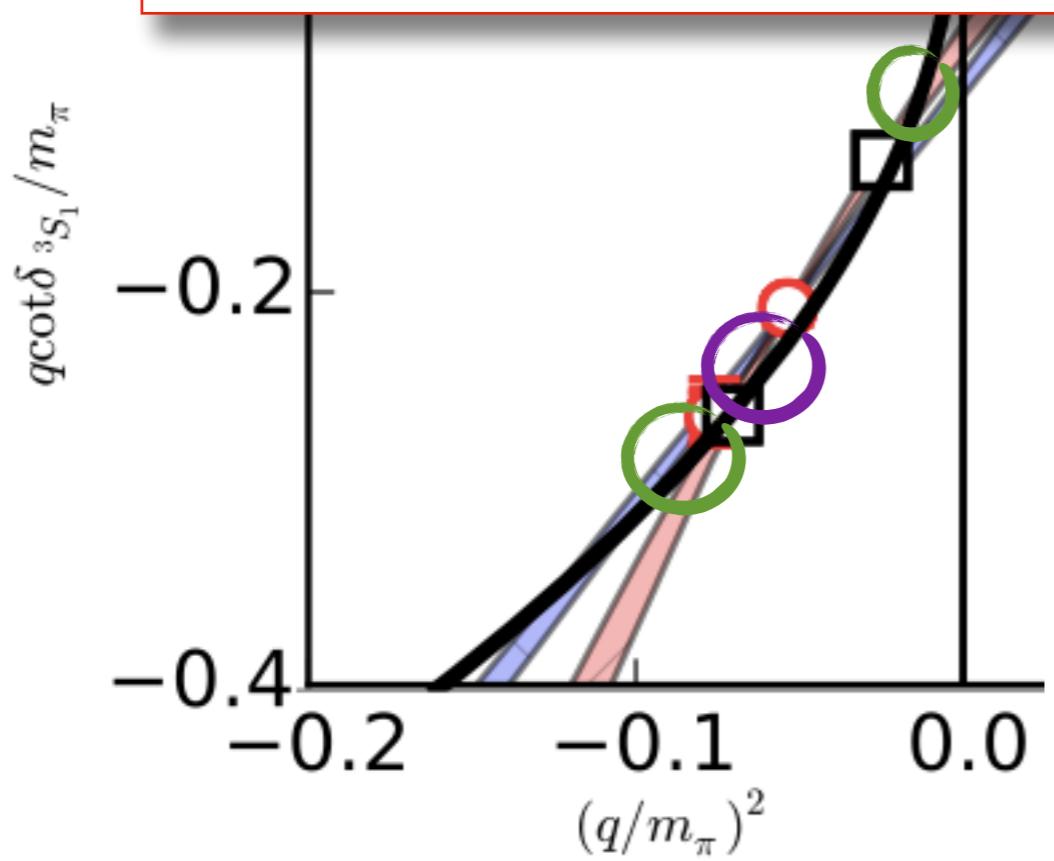
$$pcot\delta = ip$$



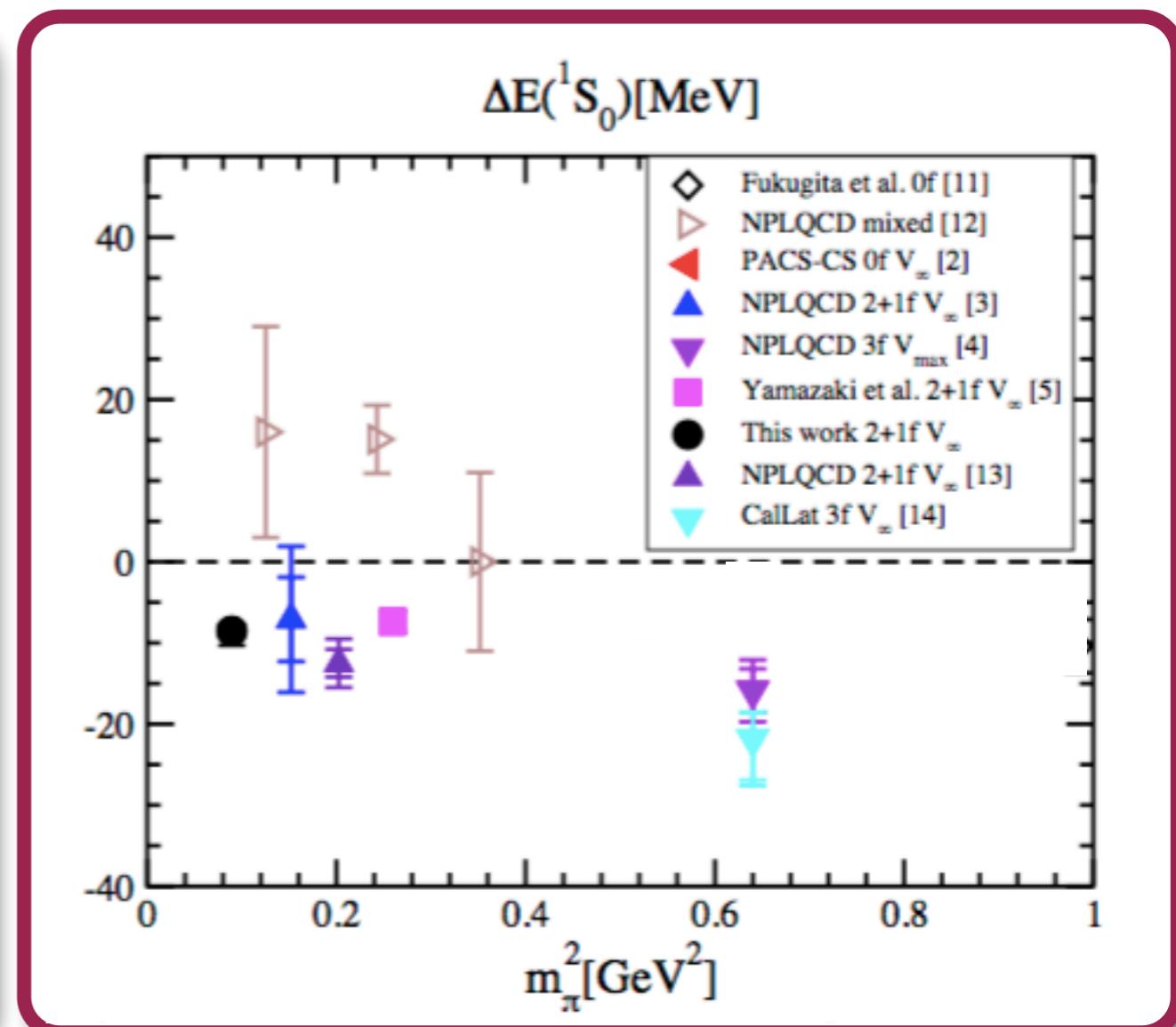
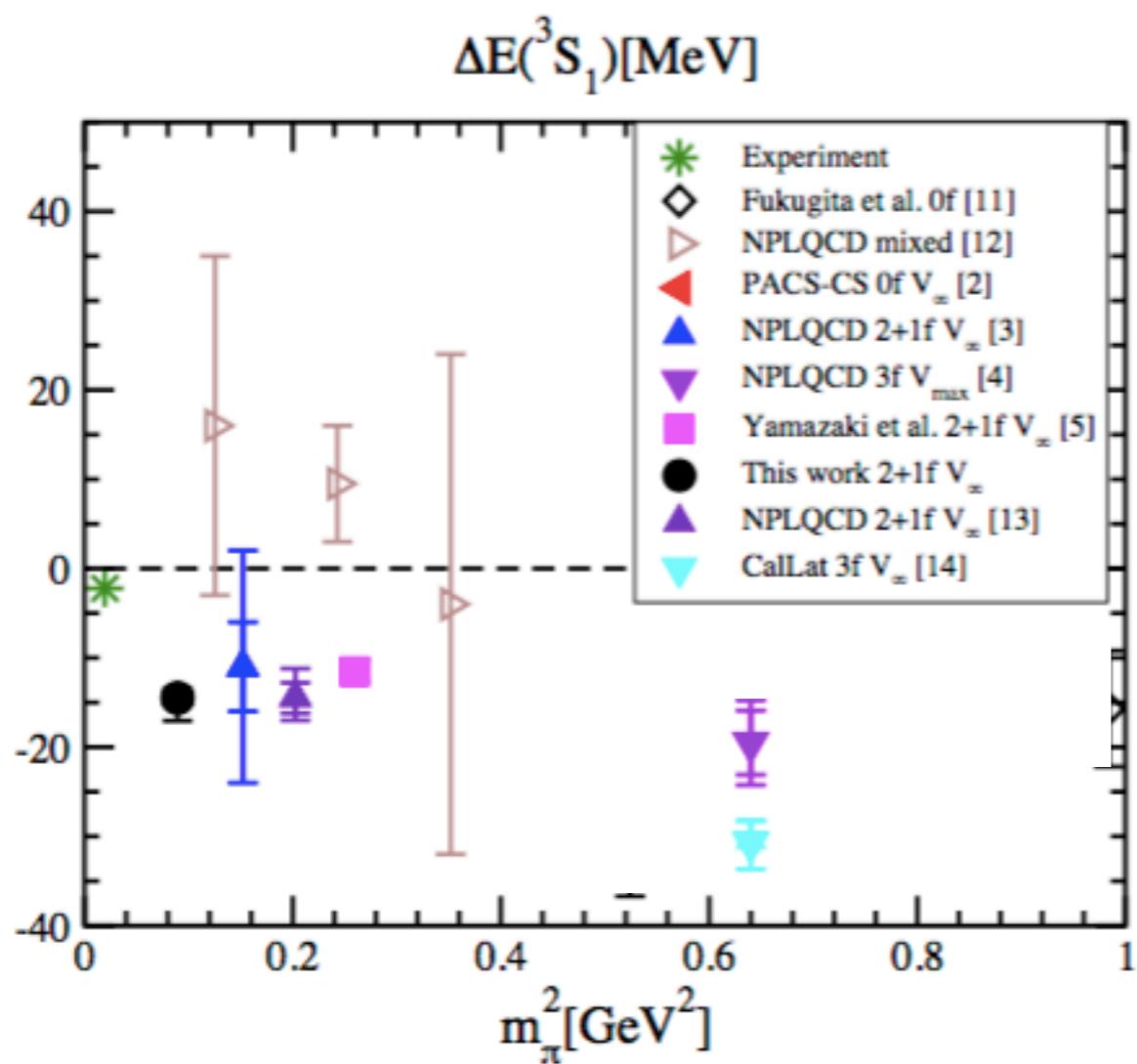
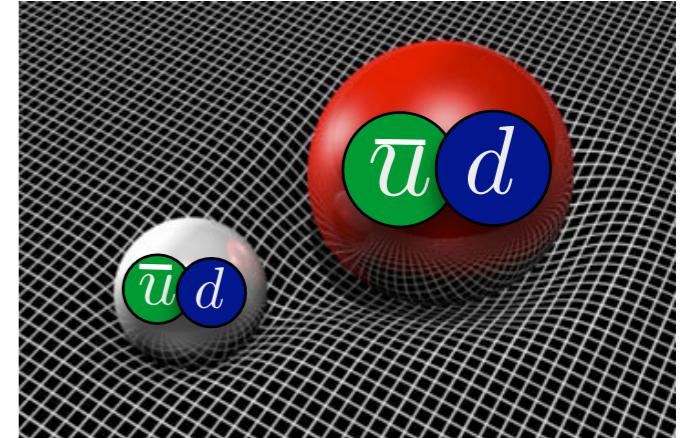


$$pcot\delta = ip$$

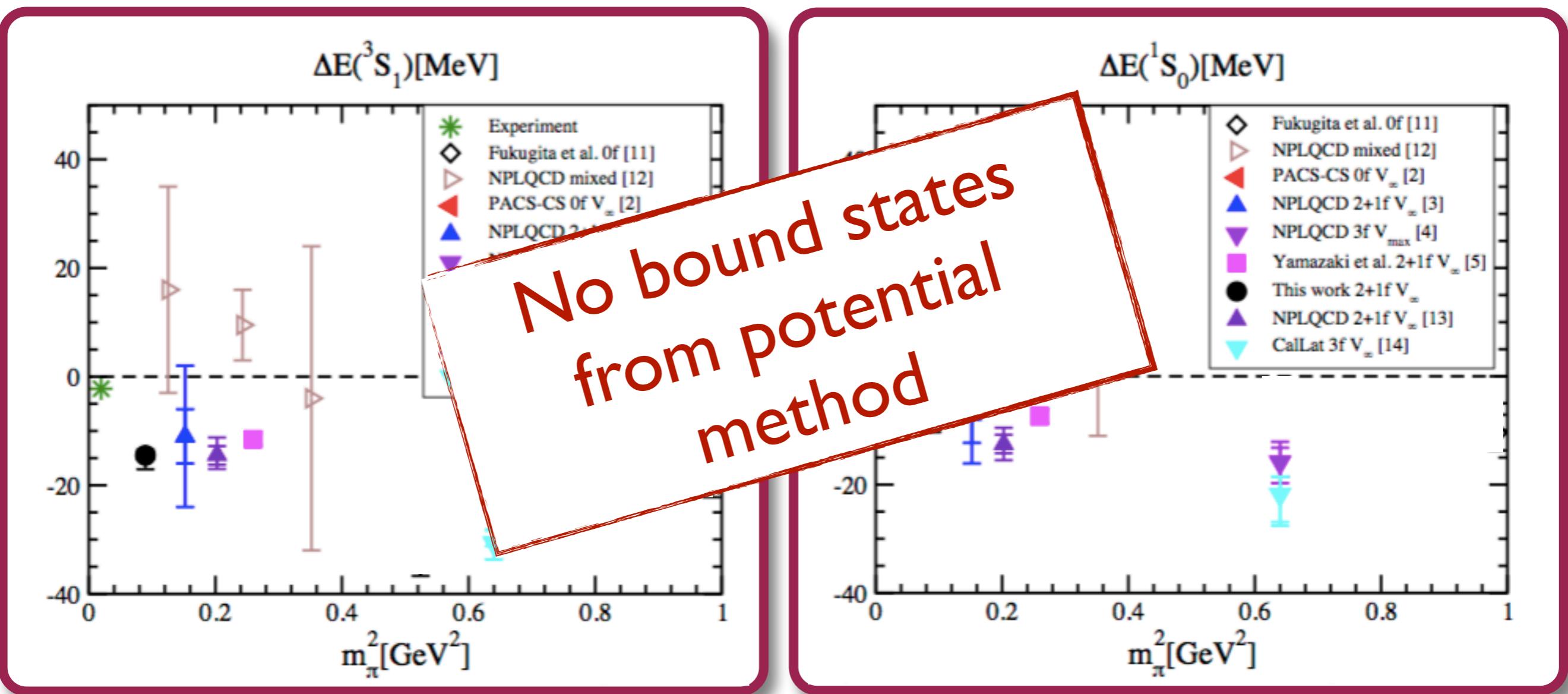
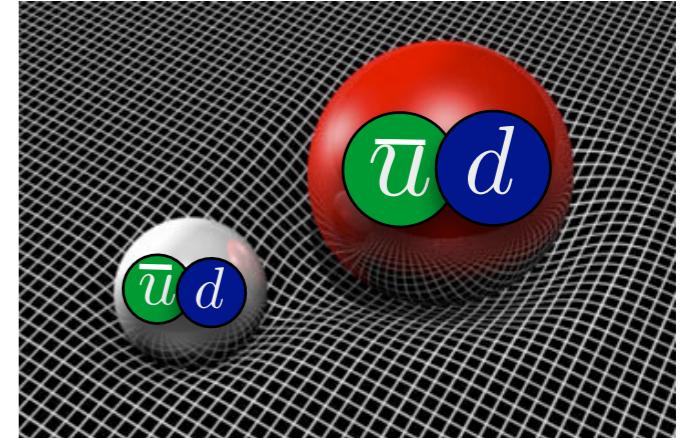
HAL has pointed to issues with an ERE interpretation of some of these results



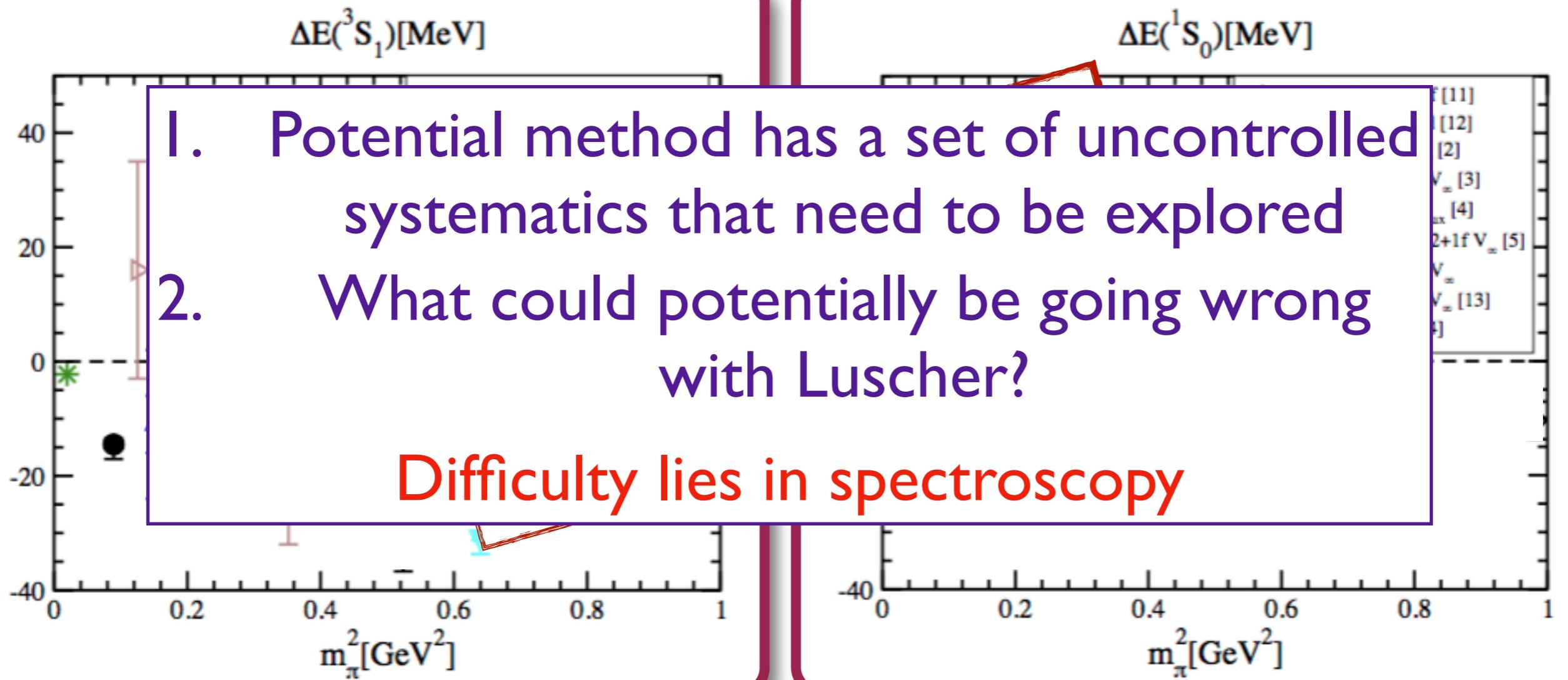
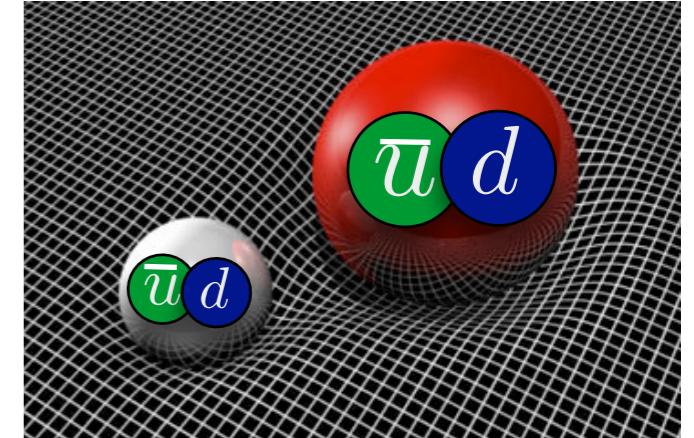
NN Binding energies



NN Binding energies



NN Binding energies



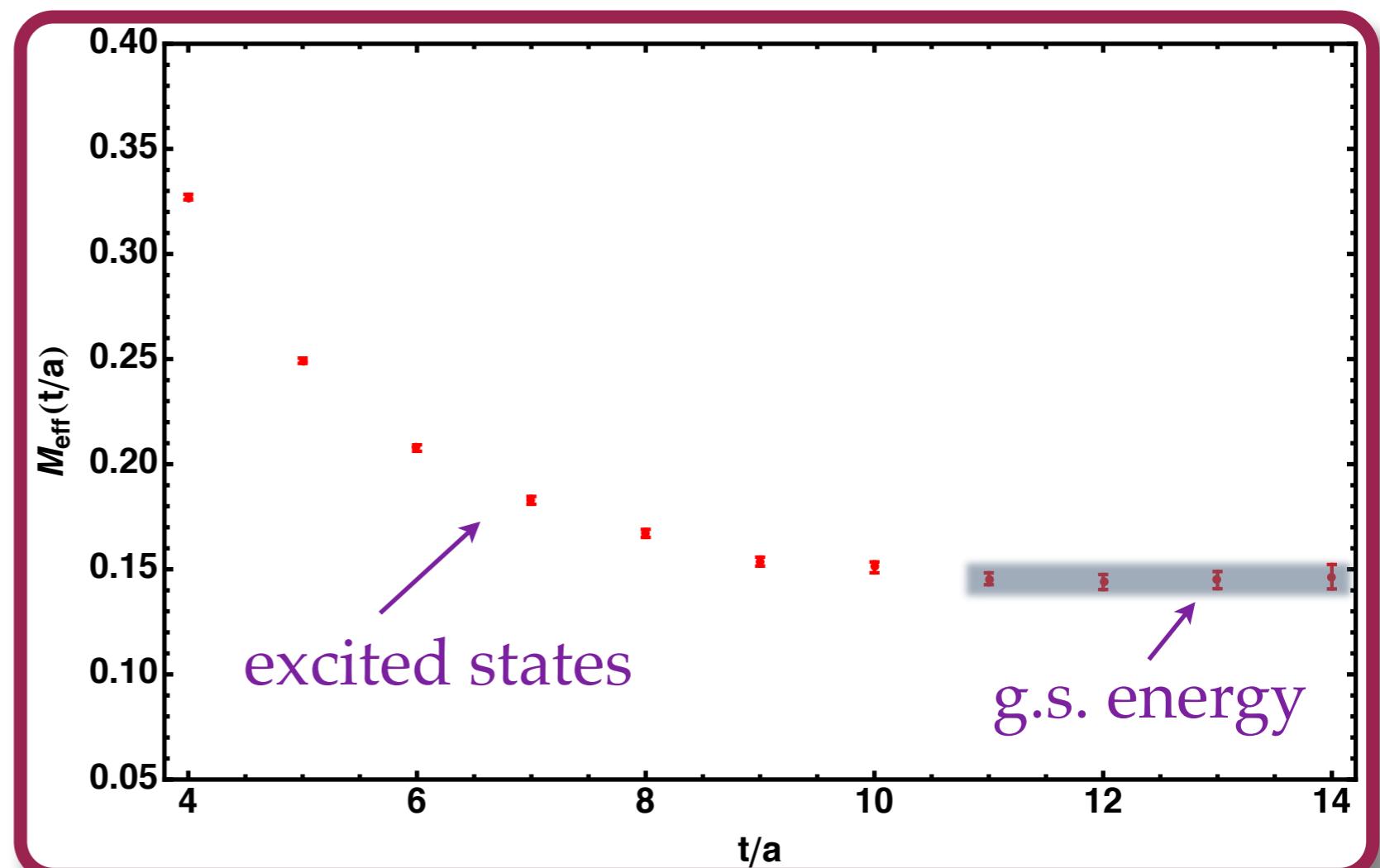
Calculating the energies

Imaginary time projection:

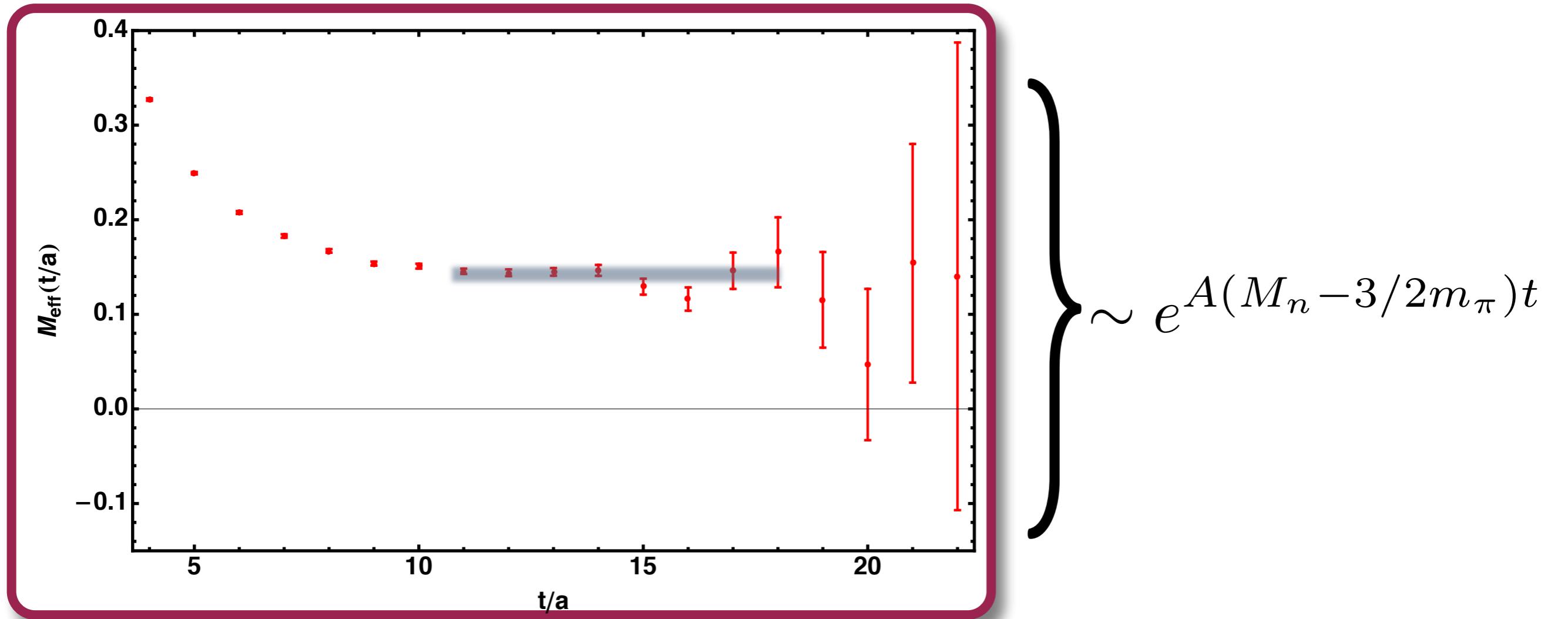
$$C(t) = \langle \mathcal{O}(t)\mathcal{O}^\dagger(0) \rangle = \sum_n |\langle 0|\mathcal{O}|n\rangle|^2 e^{-E_n t}$$
$$\xrightarrow[t \rightarrow \infty]{} Z_0 e^{-E_0 t}$$

Effective mass plot:

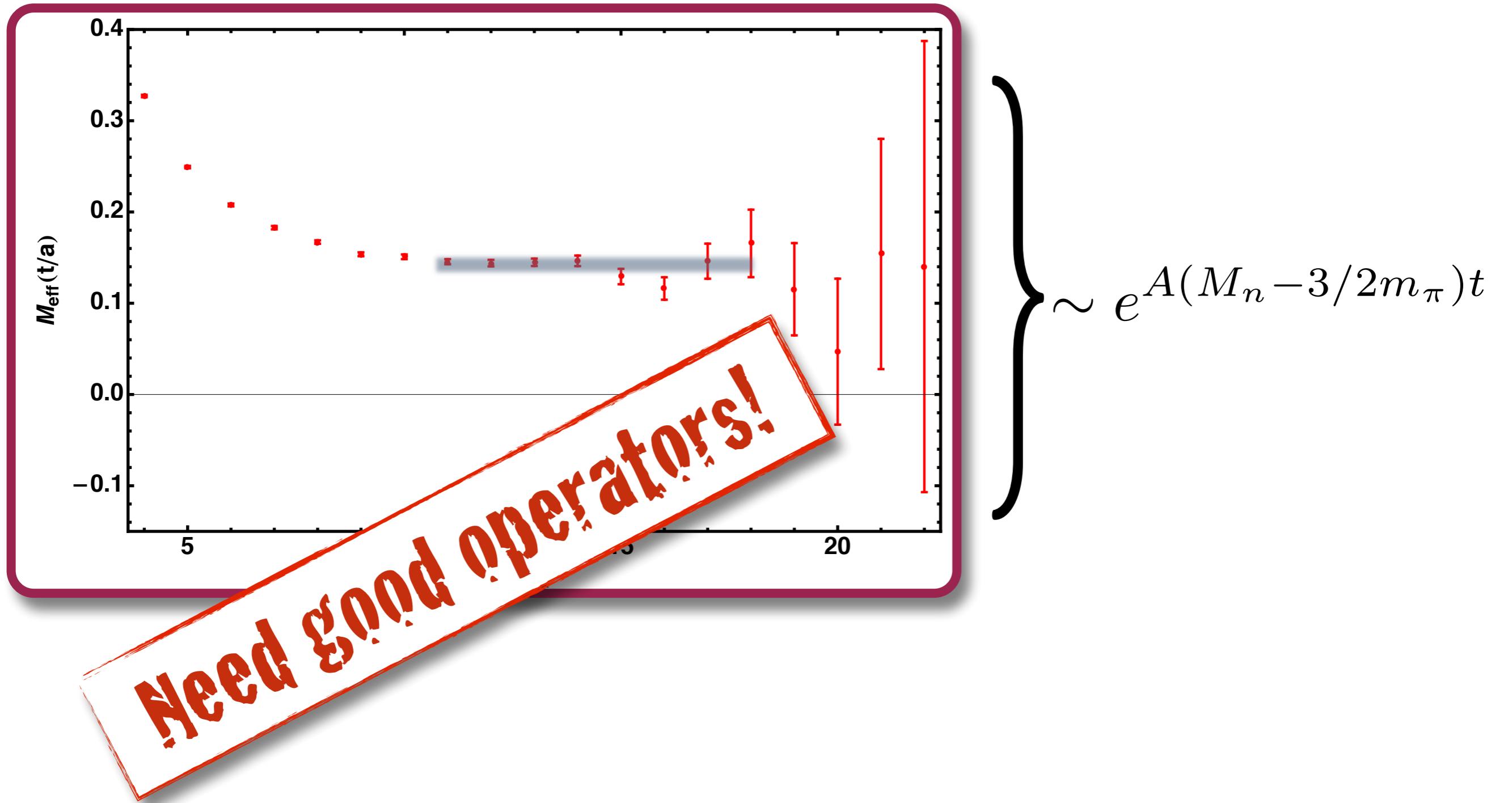
$$M_{\text{eff}} \equiv \ln \frac{C(t)}{C(t+1)}$$
$$\xrightarrow[t \rightarrow \infty]{} E_0$$



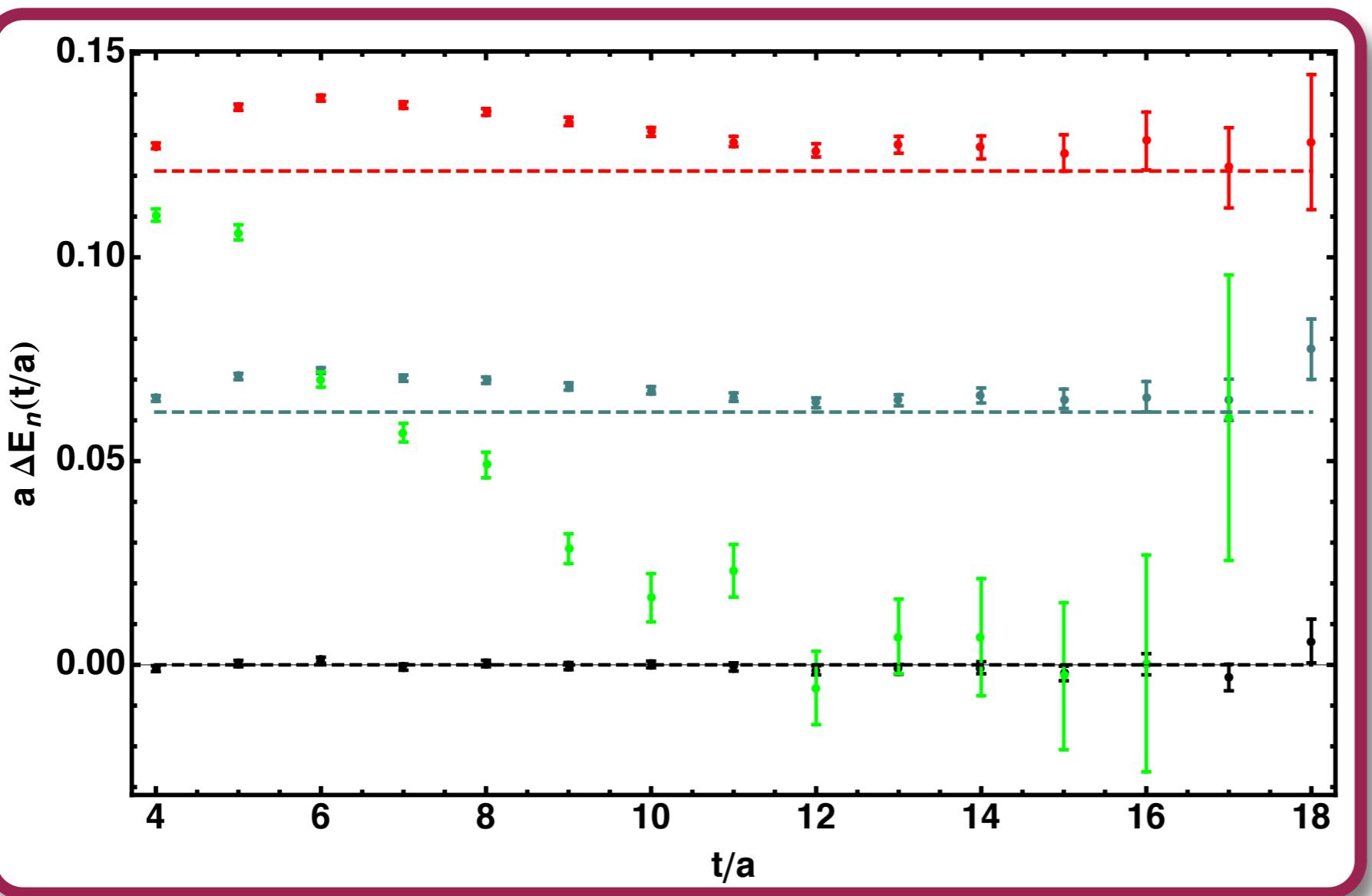
Nucleons: Signal-to-noise



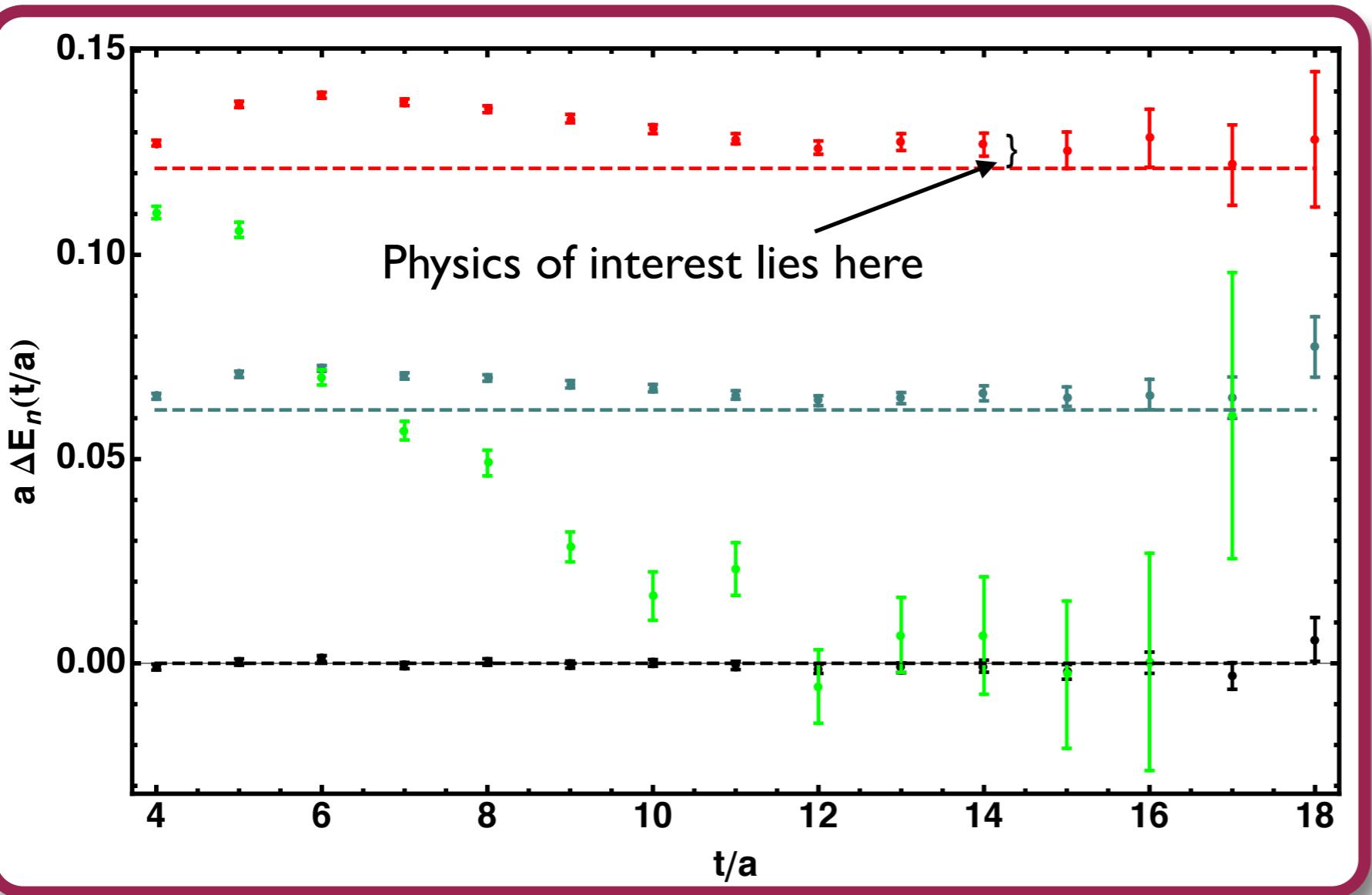
Nucleons: Signal-to-noise



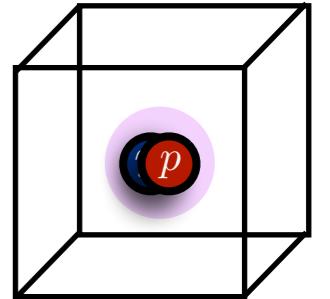
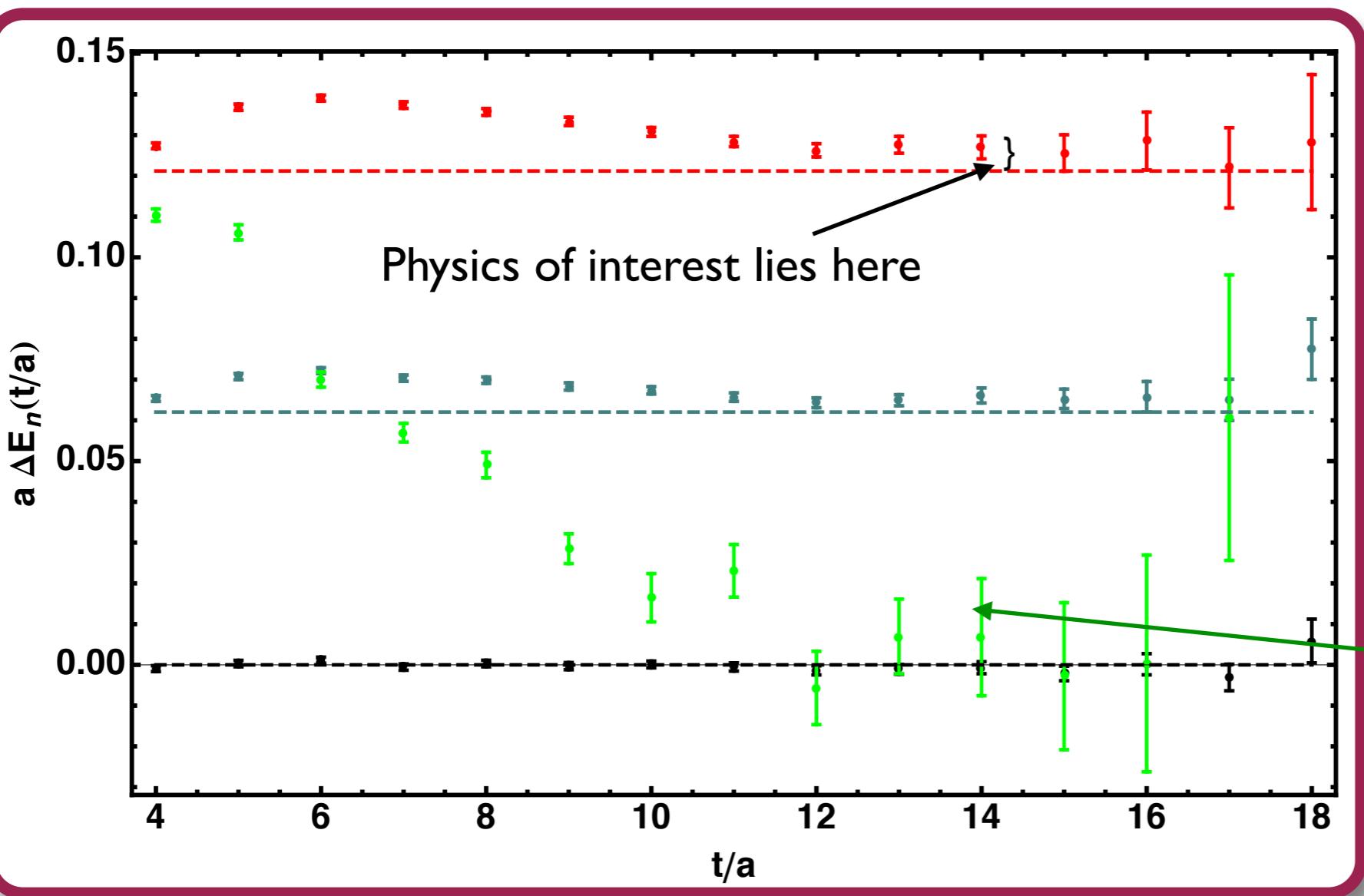
Trying to pull off tiny correction
compared to large nucleon mass:
 $\Delta E = E_{NN} - 2E_N$



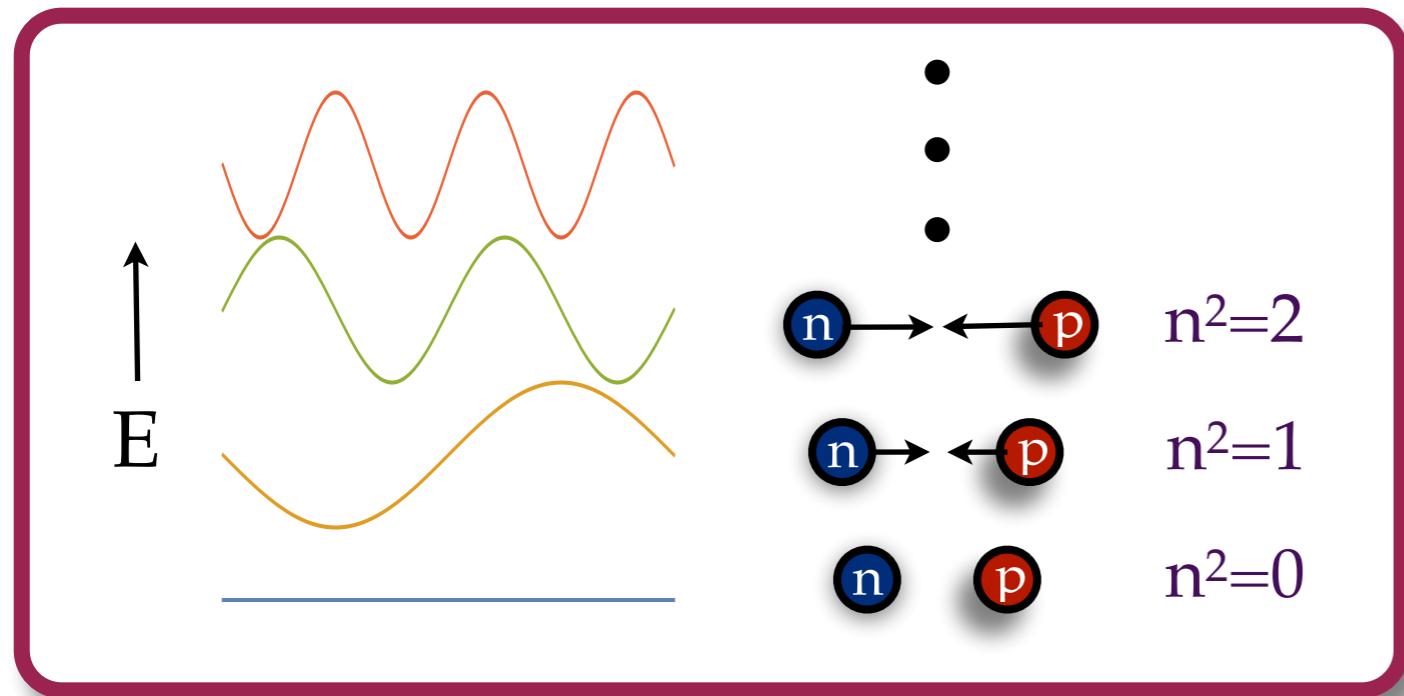
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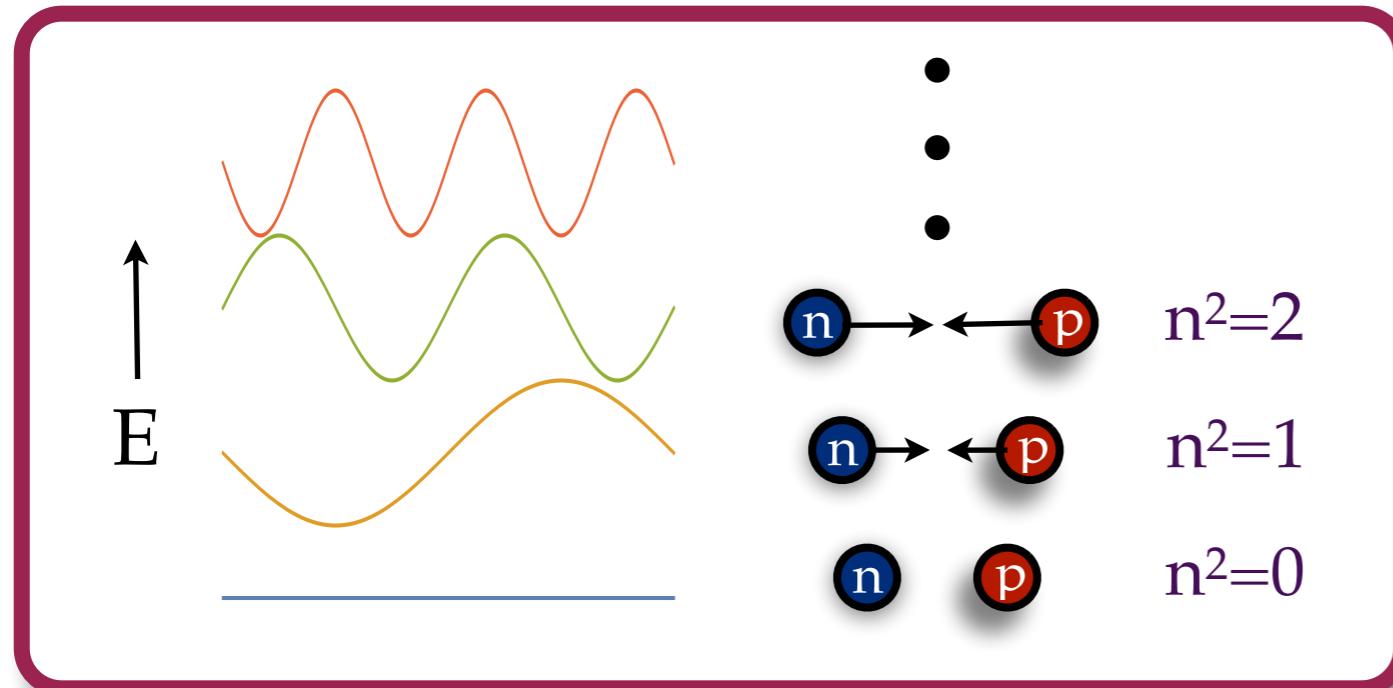


Excited state contamination

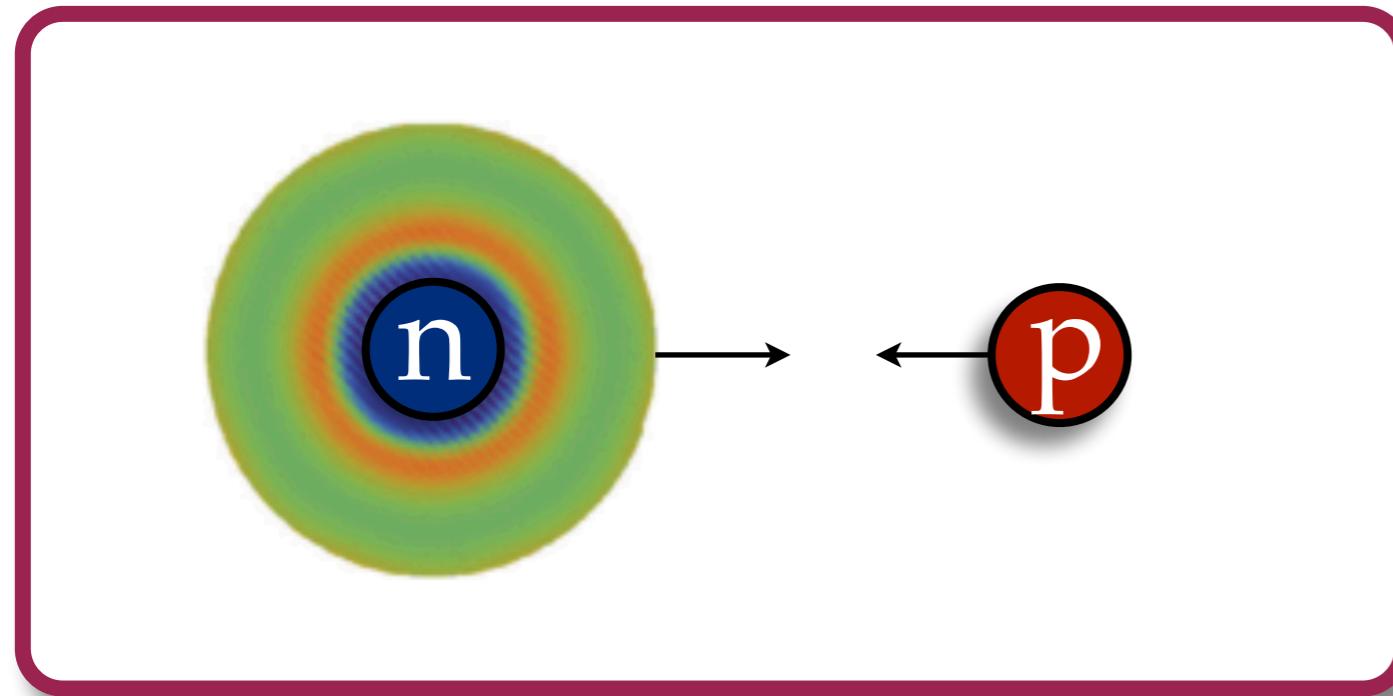


Elastic scattering
(2-body)
 $\Delta E \sim 50$ MeV
(Luscher)

Excited state contamination



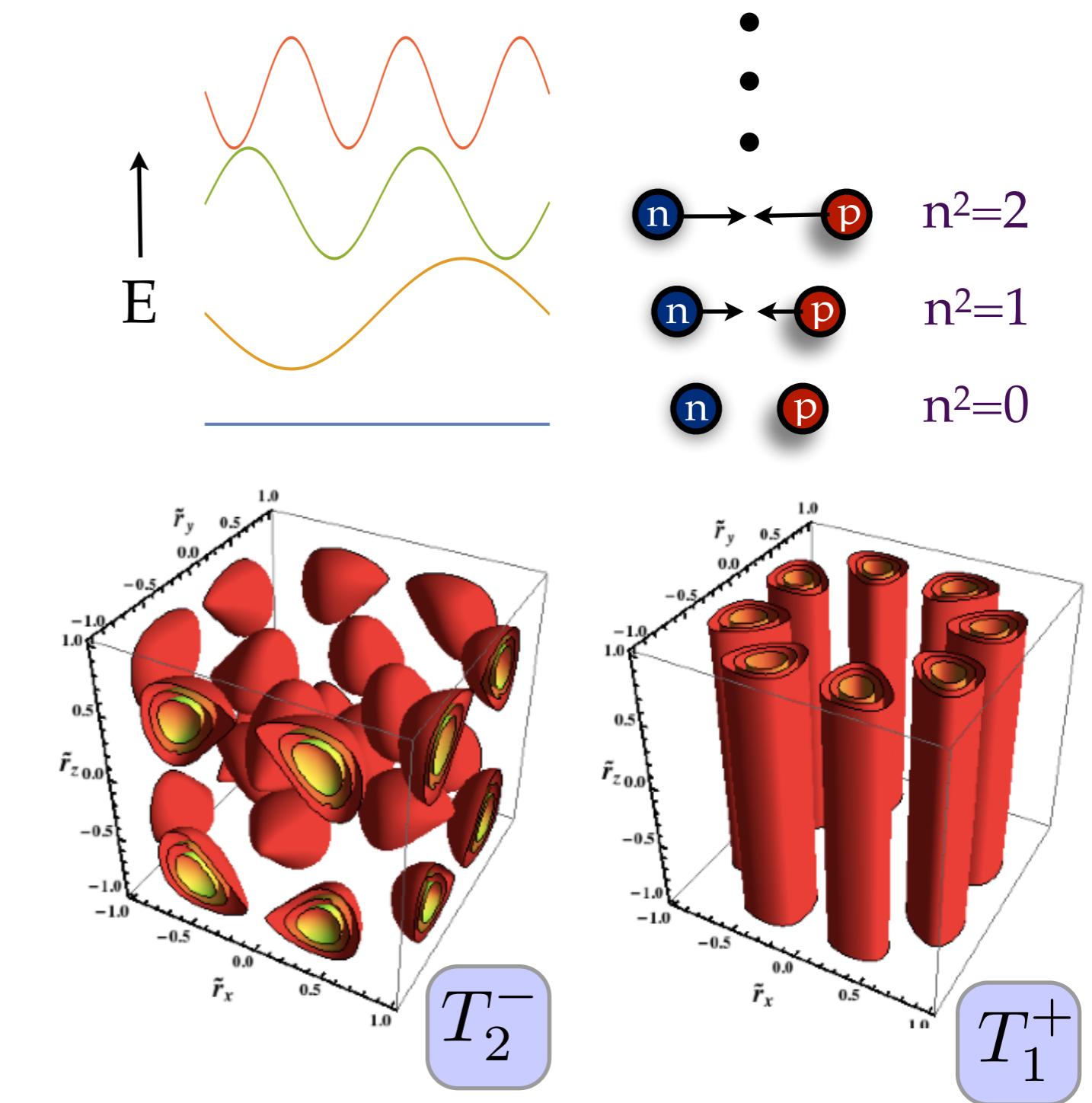
Elastic scattering
(2-body)
 $\Delta E \sim 50$ MeV
(Luscher)



Inelastic single body
 $\Delta E \sim m_\pi$
(HAL, Luscher)

Reducing elastic 2-body excited states

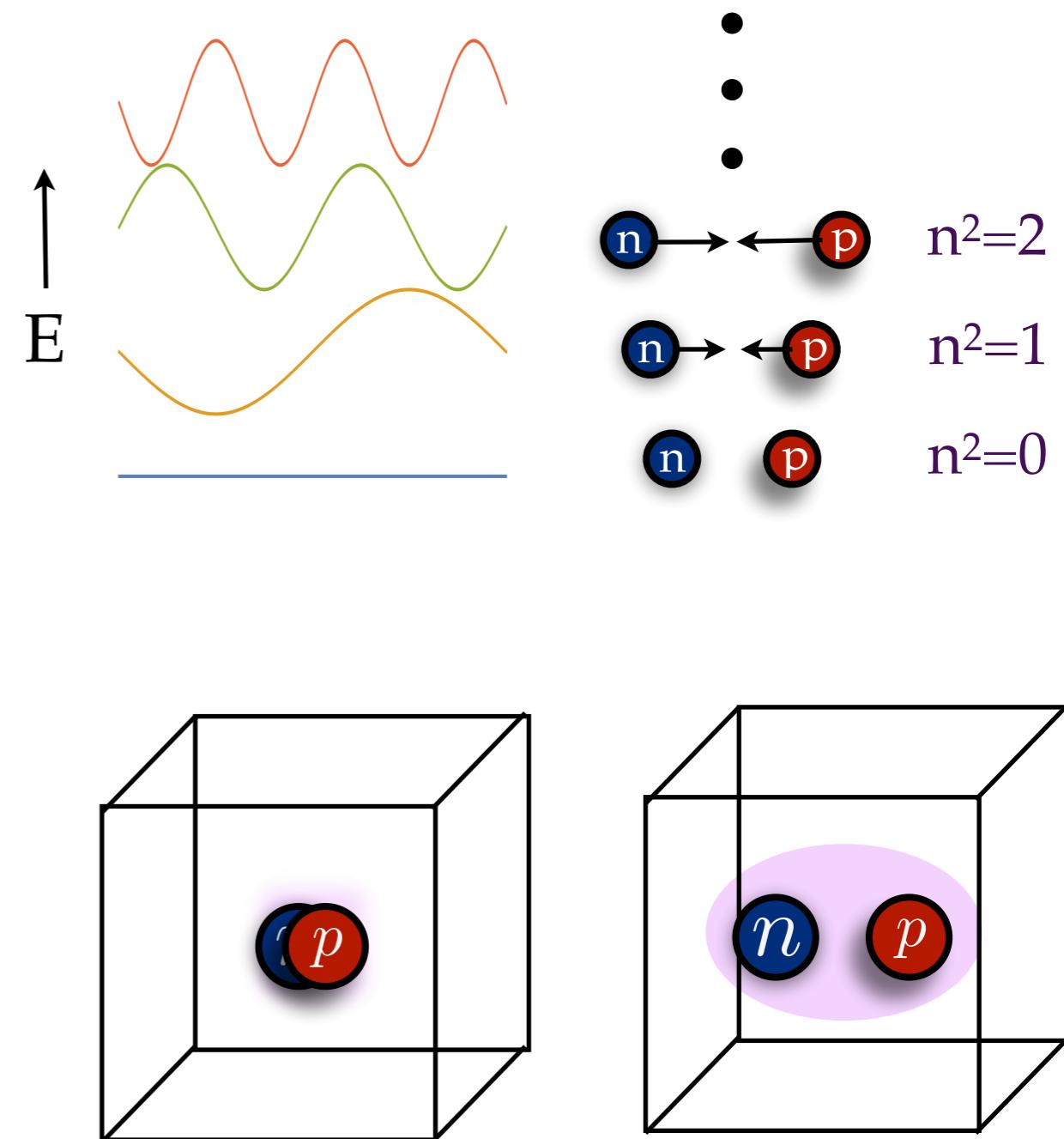
- Project onto non-interacting eigenstates of the box
- Very costly to perform exact momentum/angular momentum projection at both source & sink ($\sim V$)
- Perform exact projection only at the sink



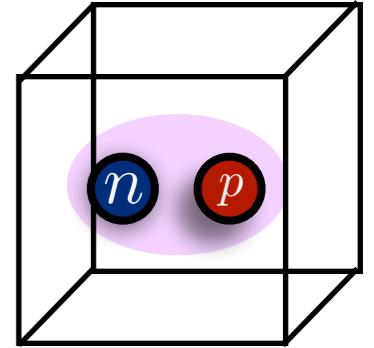
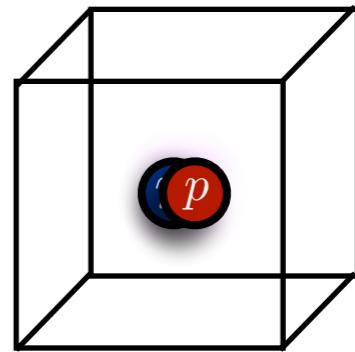
Figures from Luu & Savage (2011)

Reducing elastic 2-body excited states

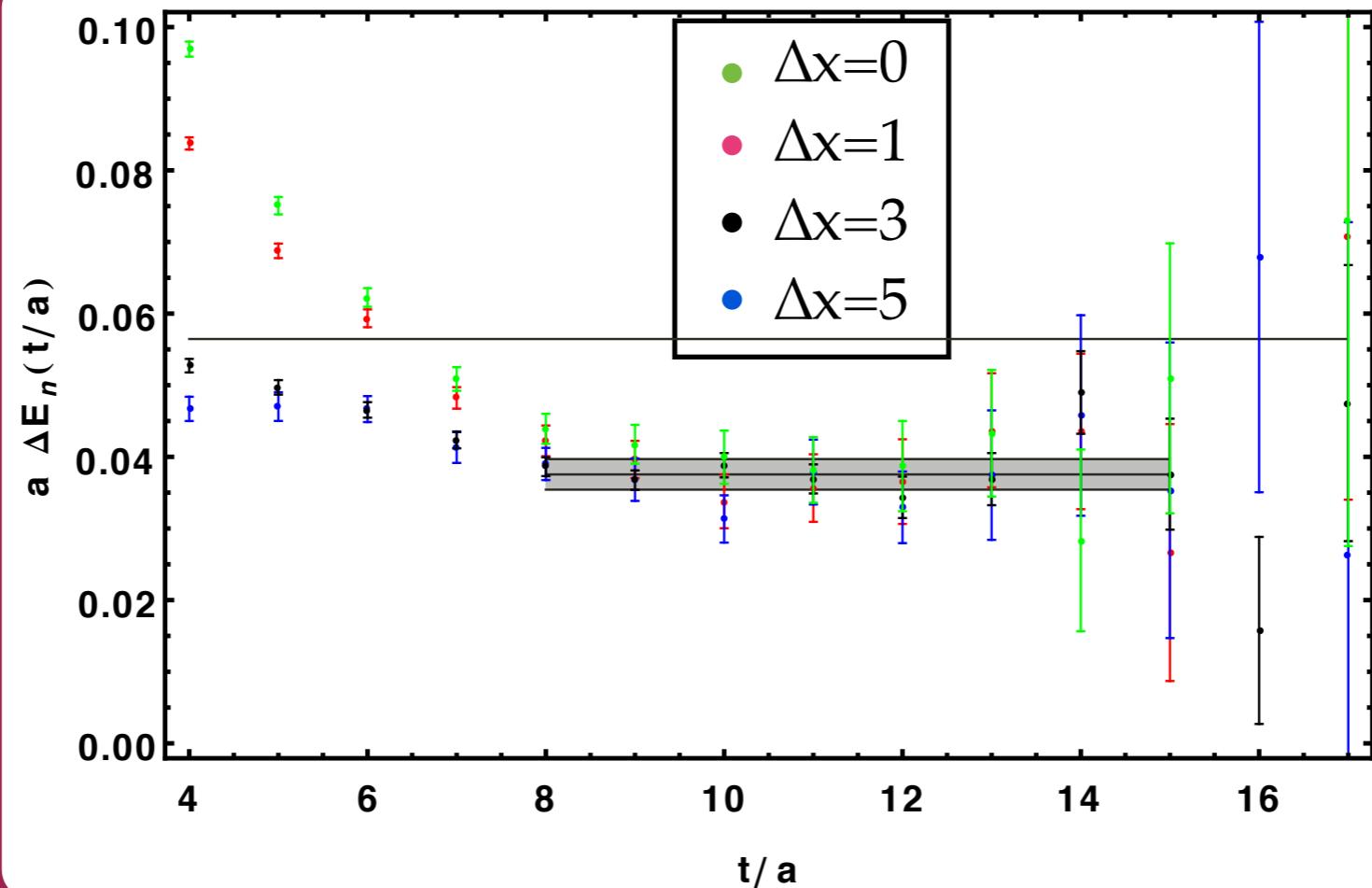
- Project onto non-interacting eigenstates of the box
- Very costly to perform exact momentum/angular momentum projection at both source & sink ($\sim V$)
- Source: need spatially displaced source operators to have overlap with $\ell > 0$
- Even for s-wave, displaced sources are cleaner



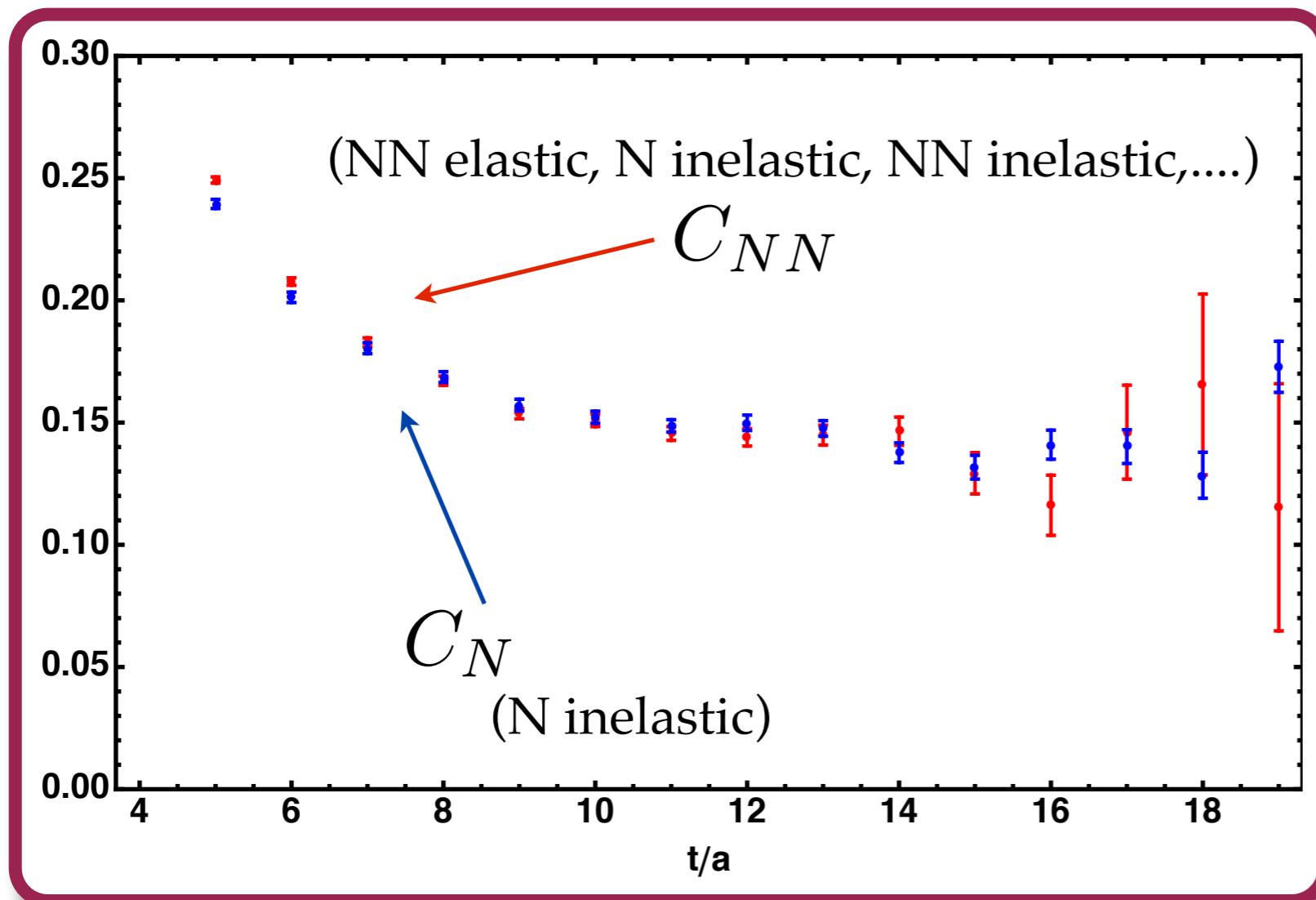
Source: position space



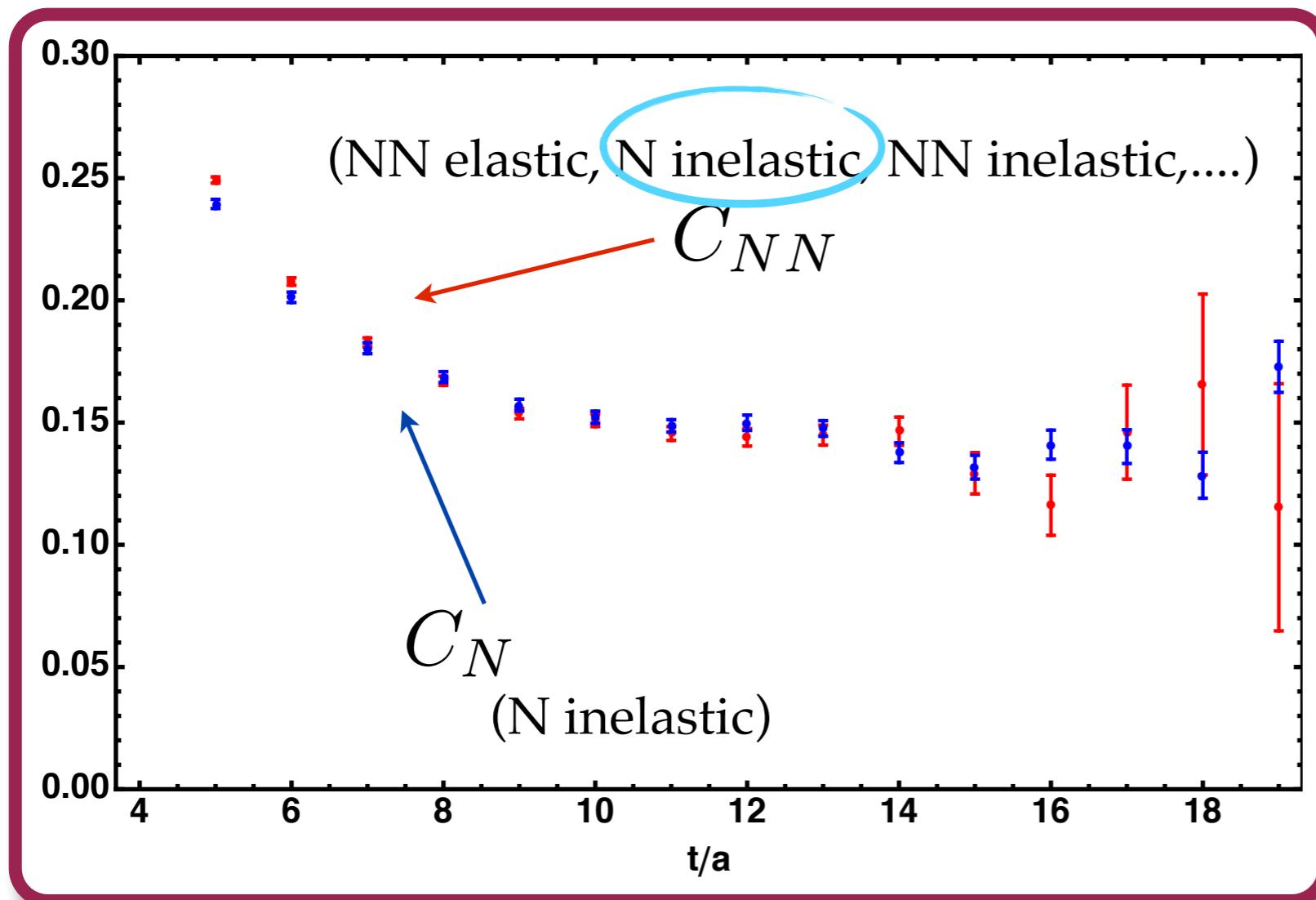
Large displacements are necessary for maximal overlap with low-energy states



Excited state contributions to NN

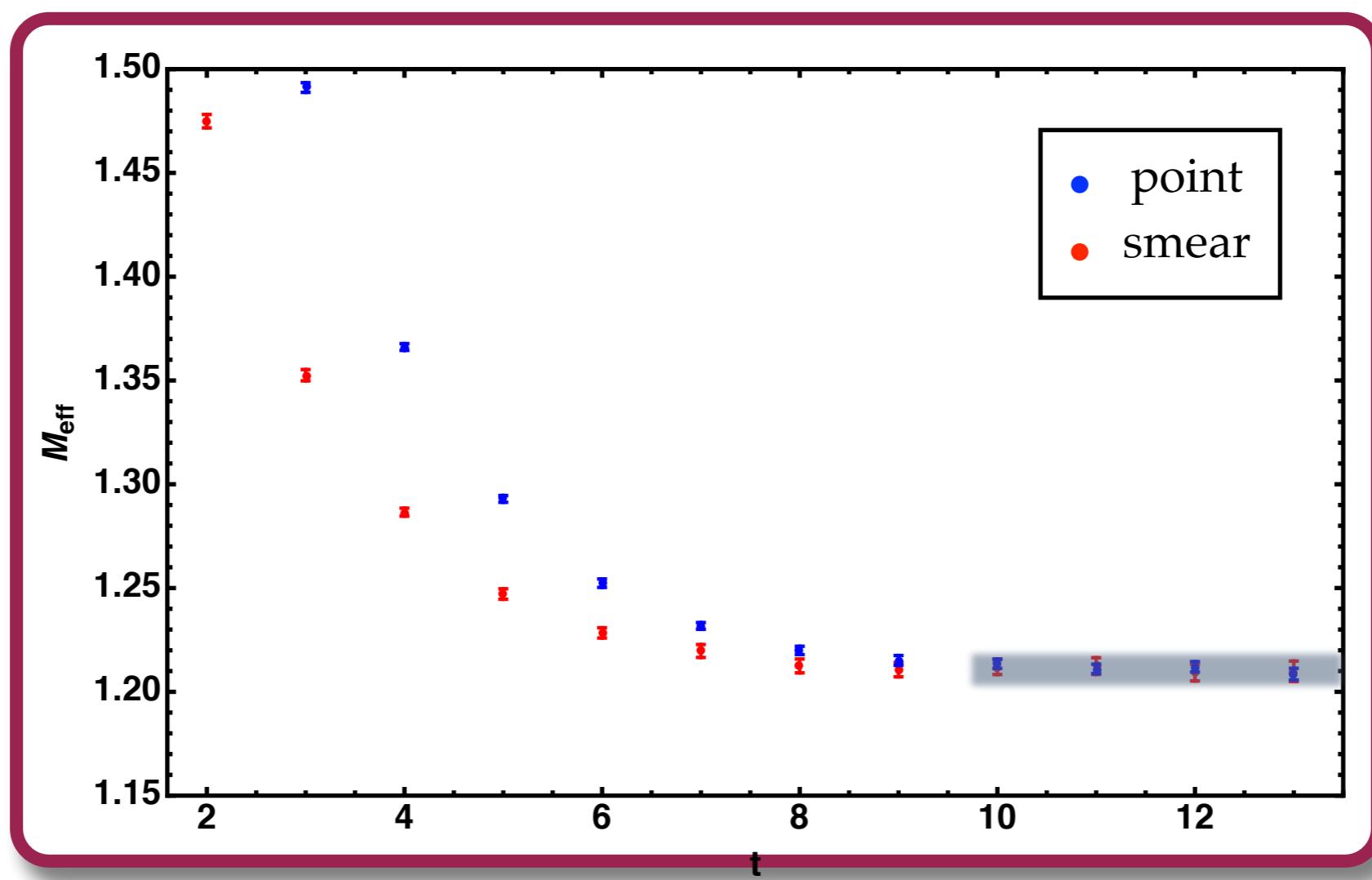
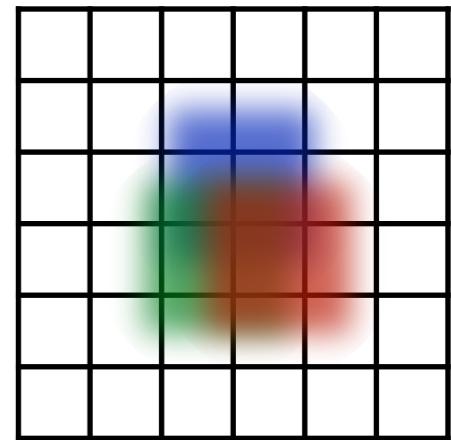
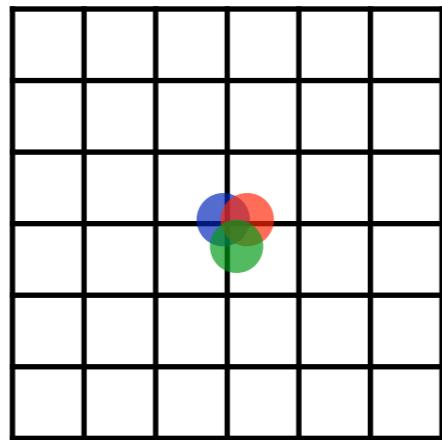


Excited state contributions to NN



Long time behavior
of NN correlator
dominated by
inelastic single
nucleon excited state
(problem for HAL
method!)

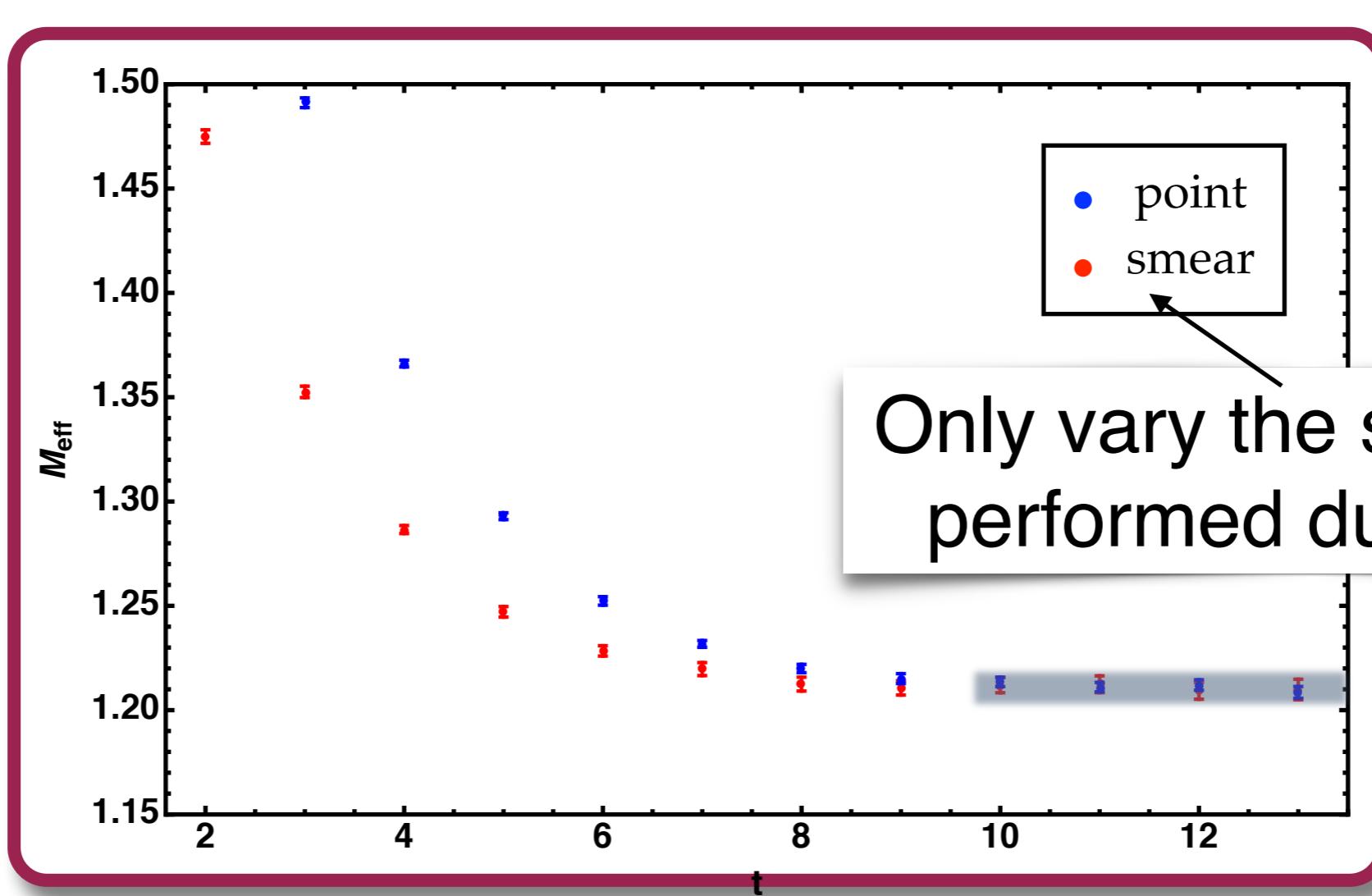
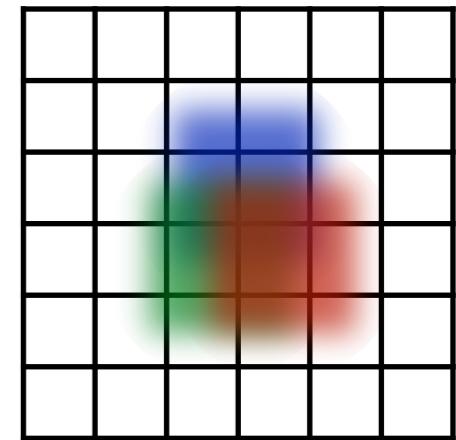
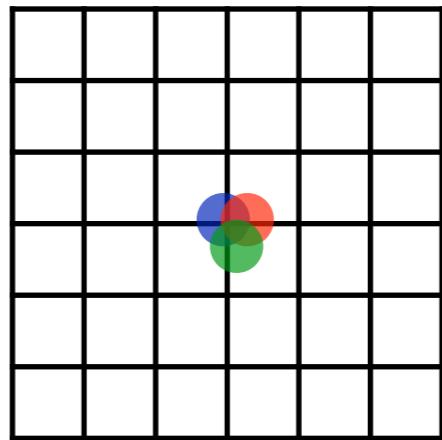
Reducing single nucleon inelastic states: Matrix Prony (poor man's GEVP)



Single nucleon correlator

NPLQCD (2009)

Reducing single nucleon inelastic states: Matrix Prony (poor man's GEVP)



Single nucleon correlator

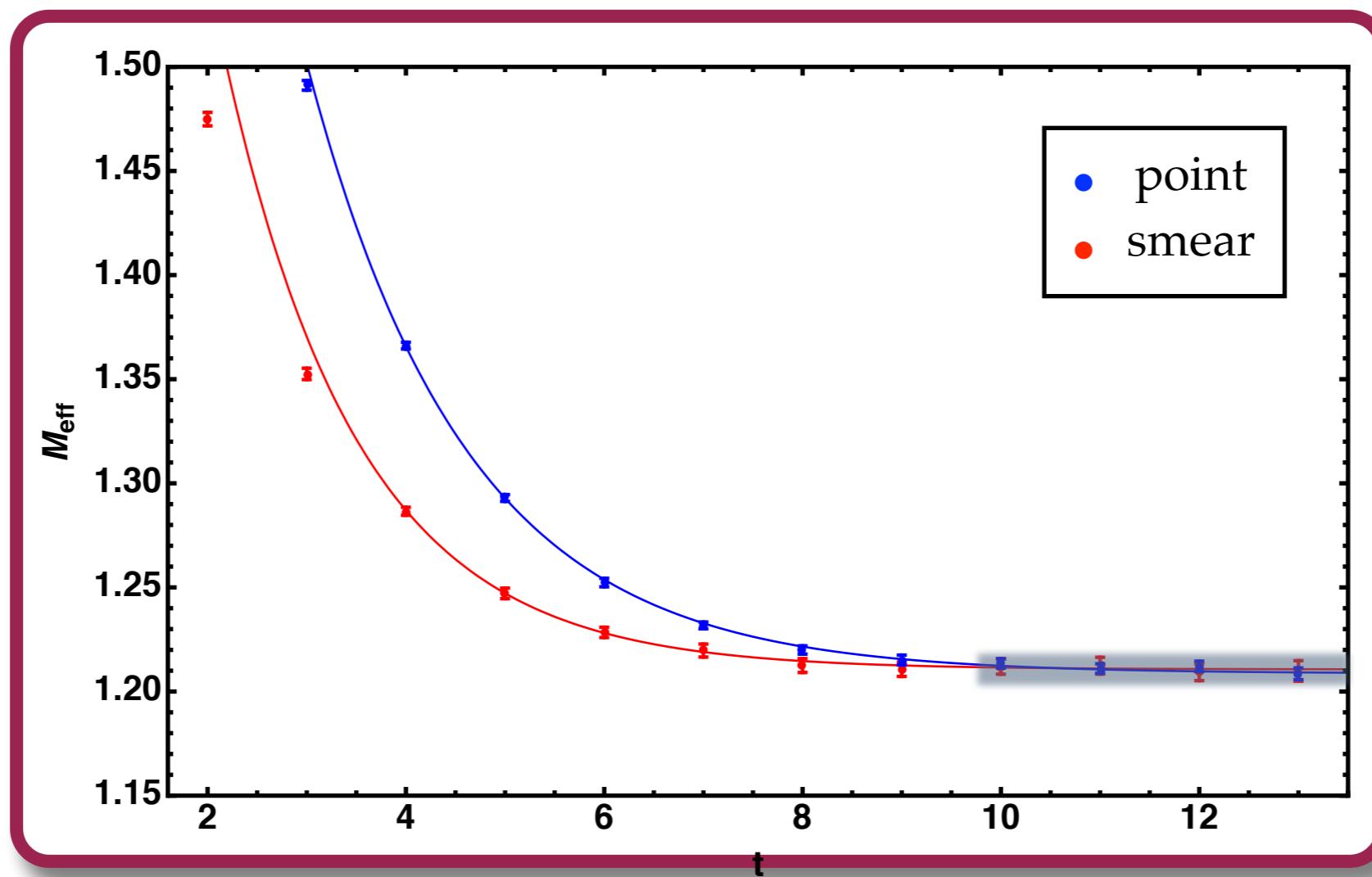
NPLQCD (2009)

Reducing single nucleon inelastic states: Matrix Prony (poor man's GEVP)

$$C_0(t + t_0) + \alpha C(t) = 0$$

$$\alpha = -e^{-E_0 t_0}$$

$$E_0 = -\frac{1}{t_0} \ln \frac{C(t + t_0)}{C(t)}$$



Single nucleon correlator

NPLQCD (2009)

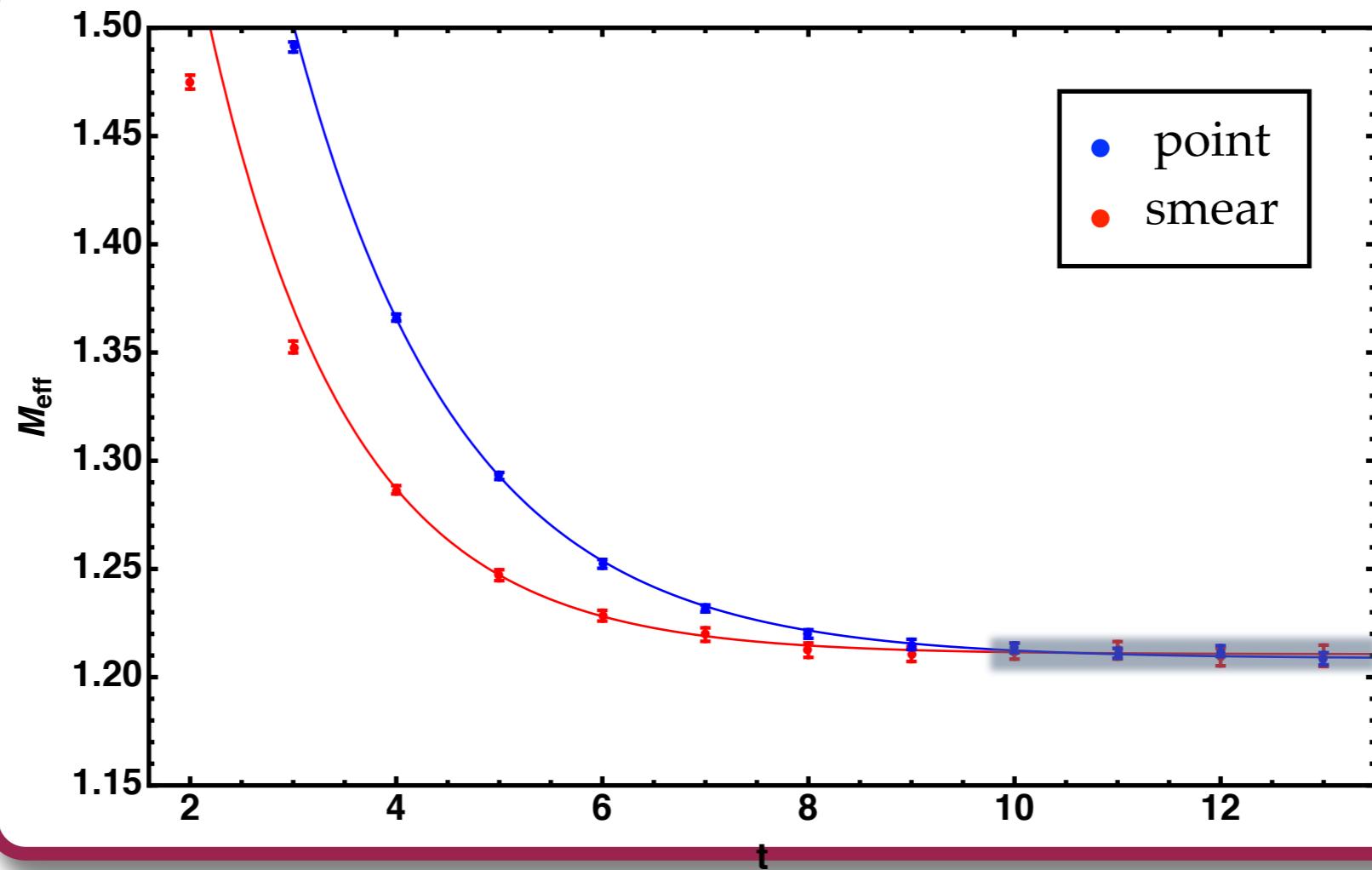
Reducing single nucleon inelastic states: Matrix Prony (poor man's GEVP)

$$MC(t + t_0) - VC(t) = 0$$

$$C(t) = \sum_{n=1}^N \alpha_n u_n \lambda_n^{-t} \quad \lambda = e^{E_n}$$

$$Mu = \lambda^{t_0} Vu$$

$$M = \left[\sum_{\tau=t}^{t+t_W} C(\tau + t_0) C(\tau)^T \right]^{-1} \quad V = \left[\sum_{\tau=t}^{t+t_W} C(\tau) C(\tau)^T \right]^{-1}$$



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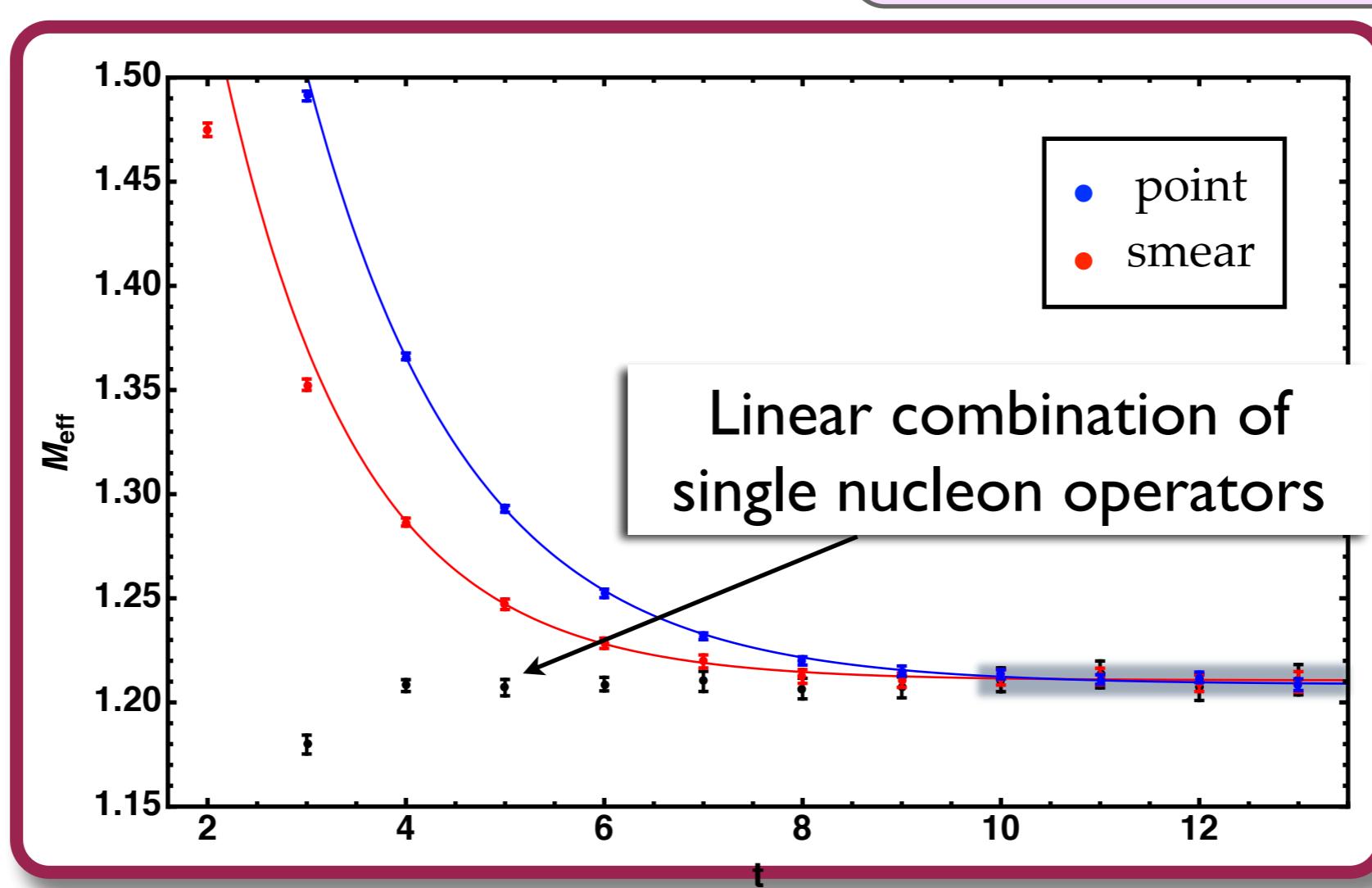
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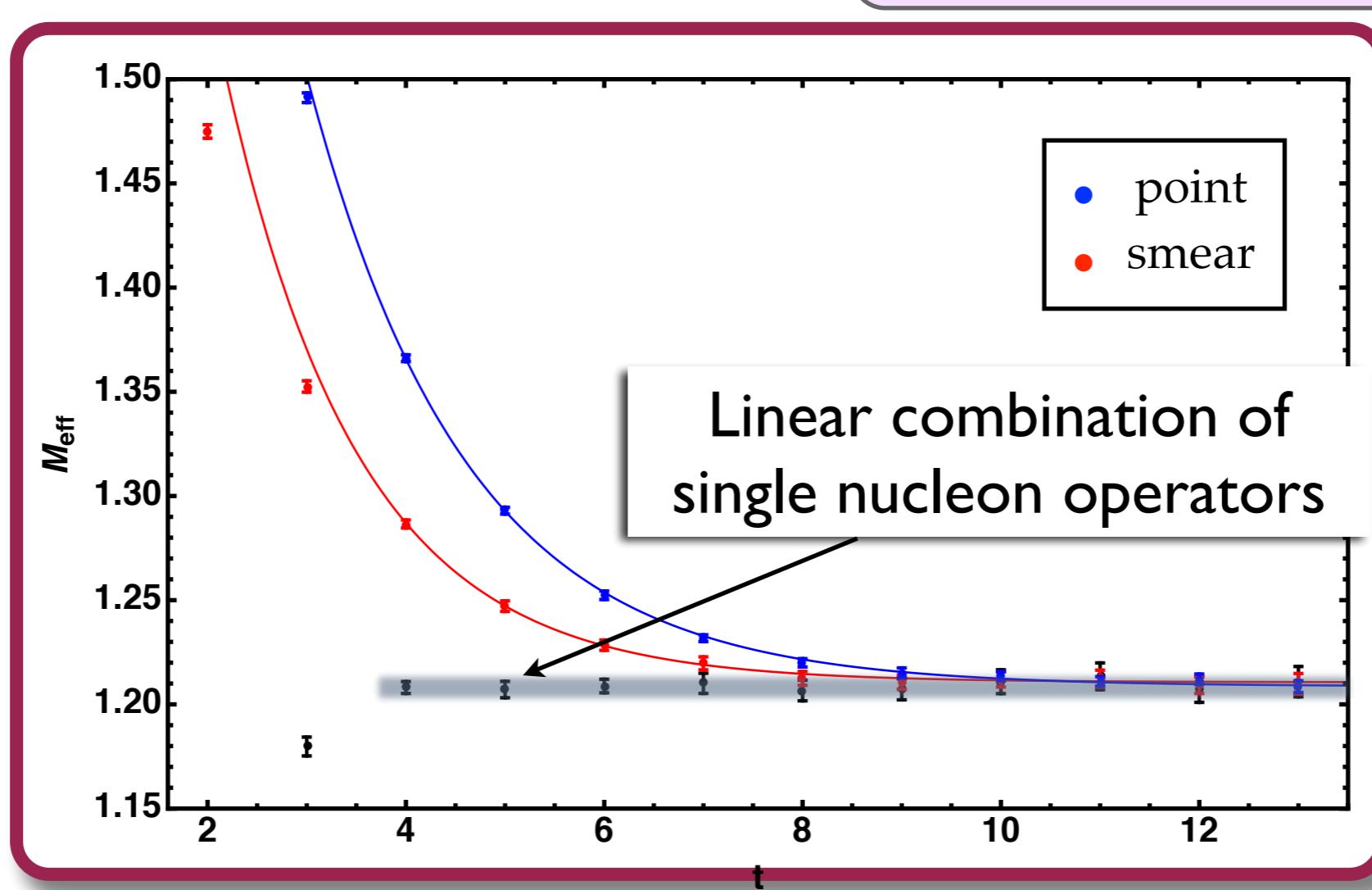
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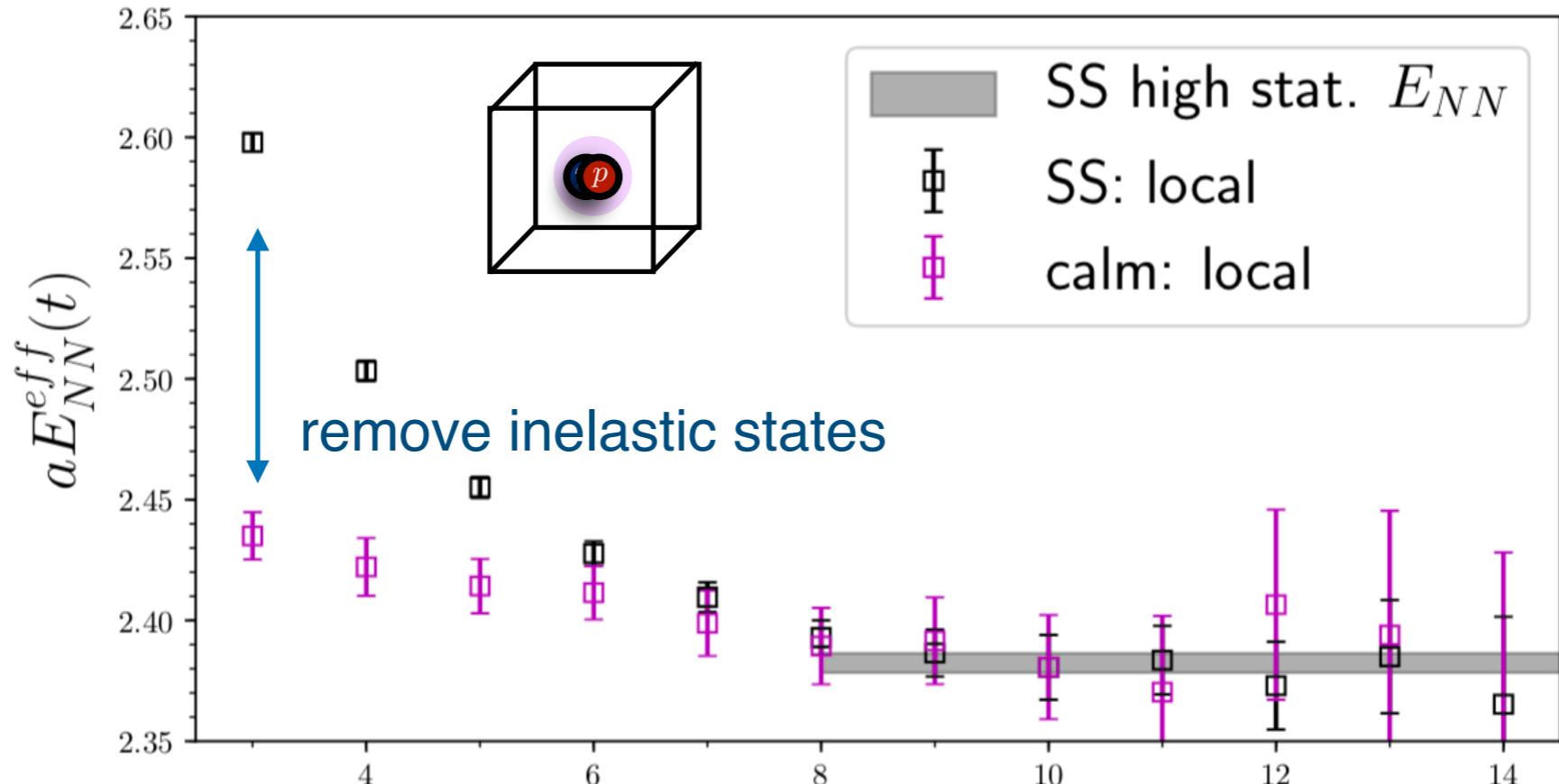


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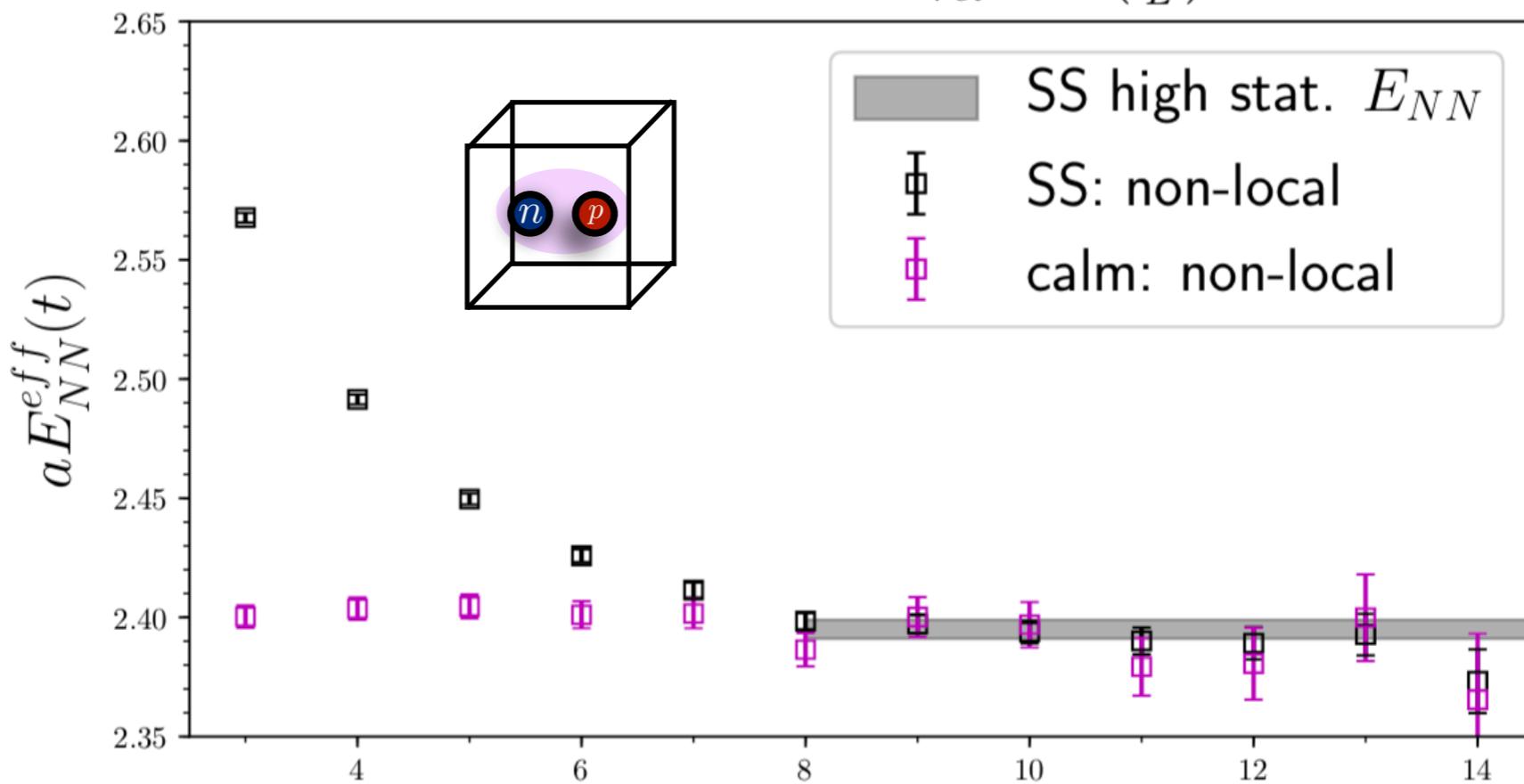
NPLQCD (2009)

$$NN : T_1^+ : {}^3S_1 : p_{rel}^2 = 0 \left(\frac{2\pi}{L}\right)^2$$

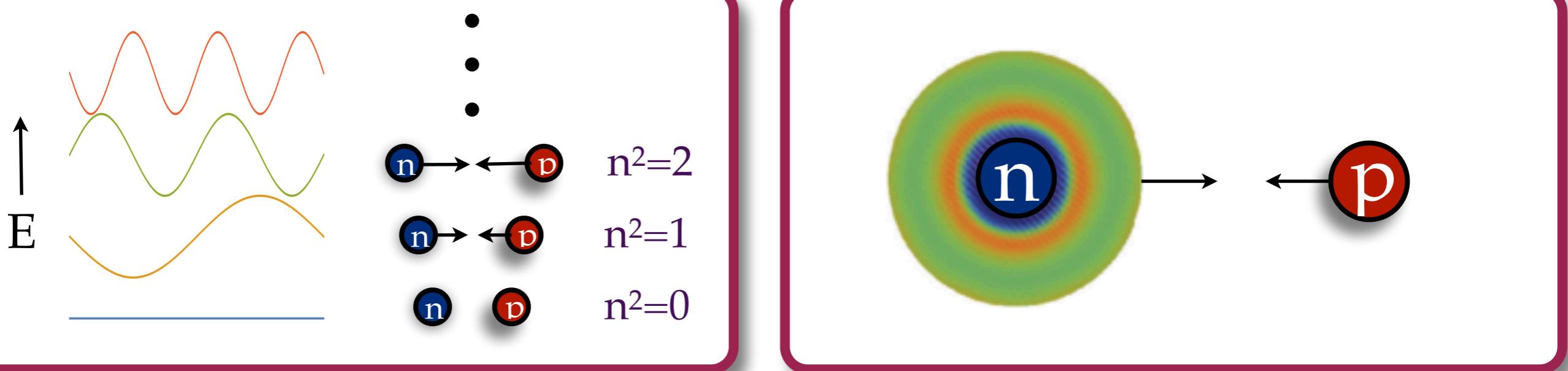
remove
elastic
states



$$NN : T_1^+ : {}^3S_1 : p_{rel}^2 = 0 \left(\frac{2\pi}{L}\right)^2$$



MP method for NN



- NPLQCD first used MP directly on NN correlators
- Works best as a two-step process: determine single-nucleon op, then minimize two-body elastic excited states
- Prony often doesn't work well for more than 2 ops:
 - excited states extracted are unreliable
 - may be able to do two stages of Prony to further reduce elastic excited states

The future? GEVP approaches

Variational basis of interpolating operators: $O_i(x_0)$

Define the states: $|\tilde{\phi}_i\rangle = \hat{O}_i|0\rangle$ and $|\phi_i\rangle = e^{-t_0 \hat{H}/2}|\tilde{\phi}_i\rangle$

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Variational principle ($t > t_0$):

$$\lambda_1(t, t_0) = \underset{\{\alpha_i\}}{\text{Max}} \frac{\langle \phi | e^{-(t-t_0)\hat{H}} | \phi \rangle}{\langle \phi | \phi \rangle}, \quad |\phi\rangle = \sum_{i=1}^N \alpha_i |\phi_i\rangle$$

Eigenvalue: $\lambda_1(t, t_0) \approx e^{-E_1(t-t_0)}$

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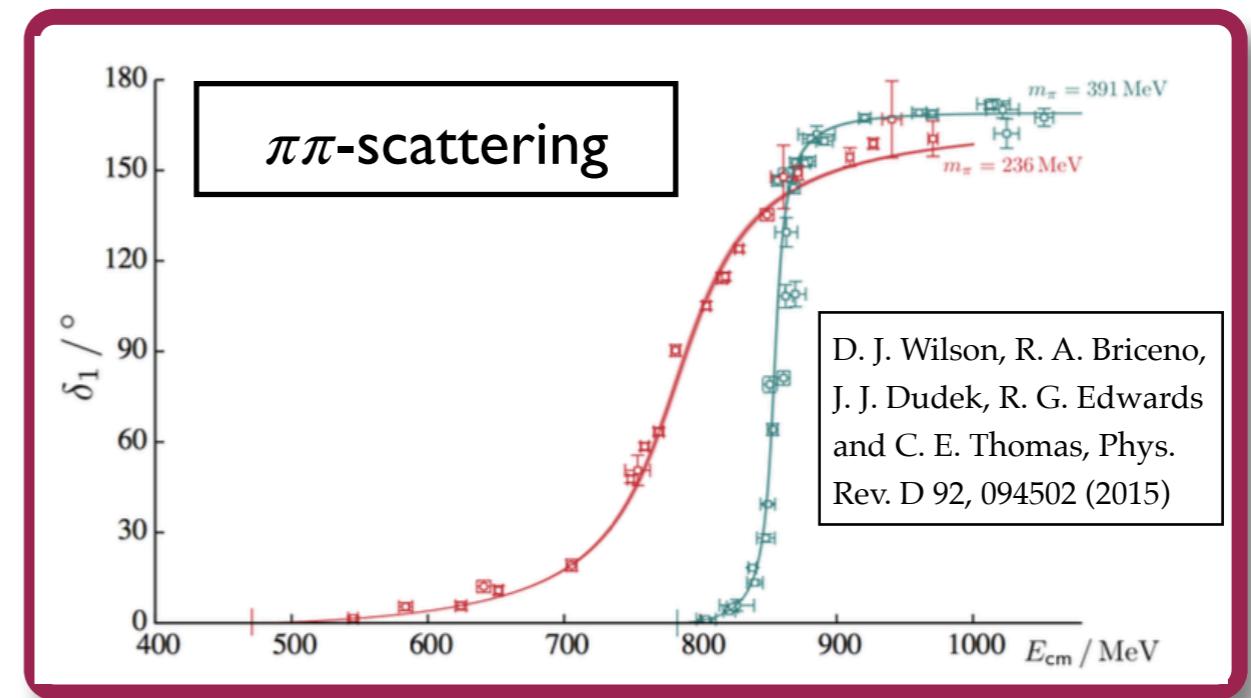
Eigenvalue: $\lambda_1(t, t_0) \approx e^{-E_1(t-t_0)}$

Largest eigenvalue of a
GEVP, which can be used to
determine multiple states:

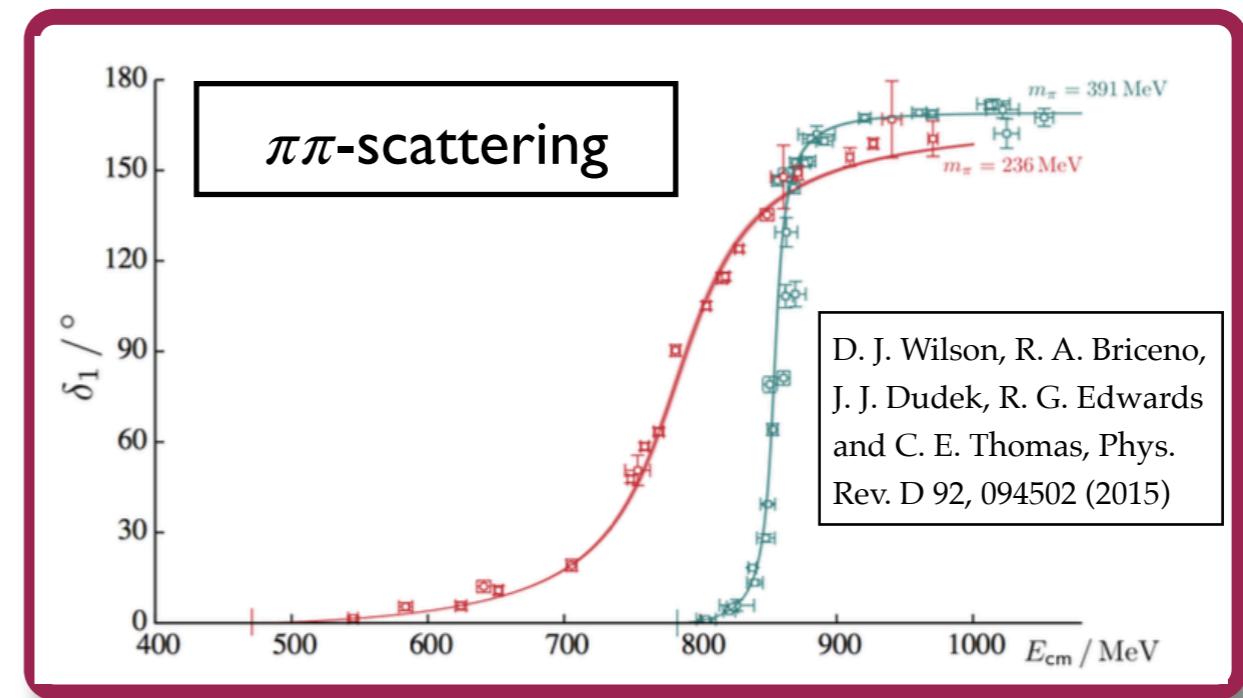
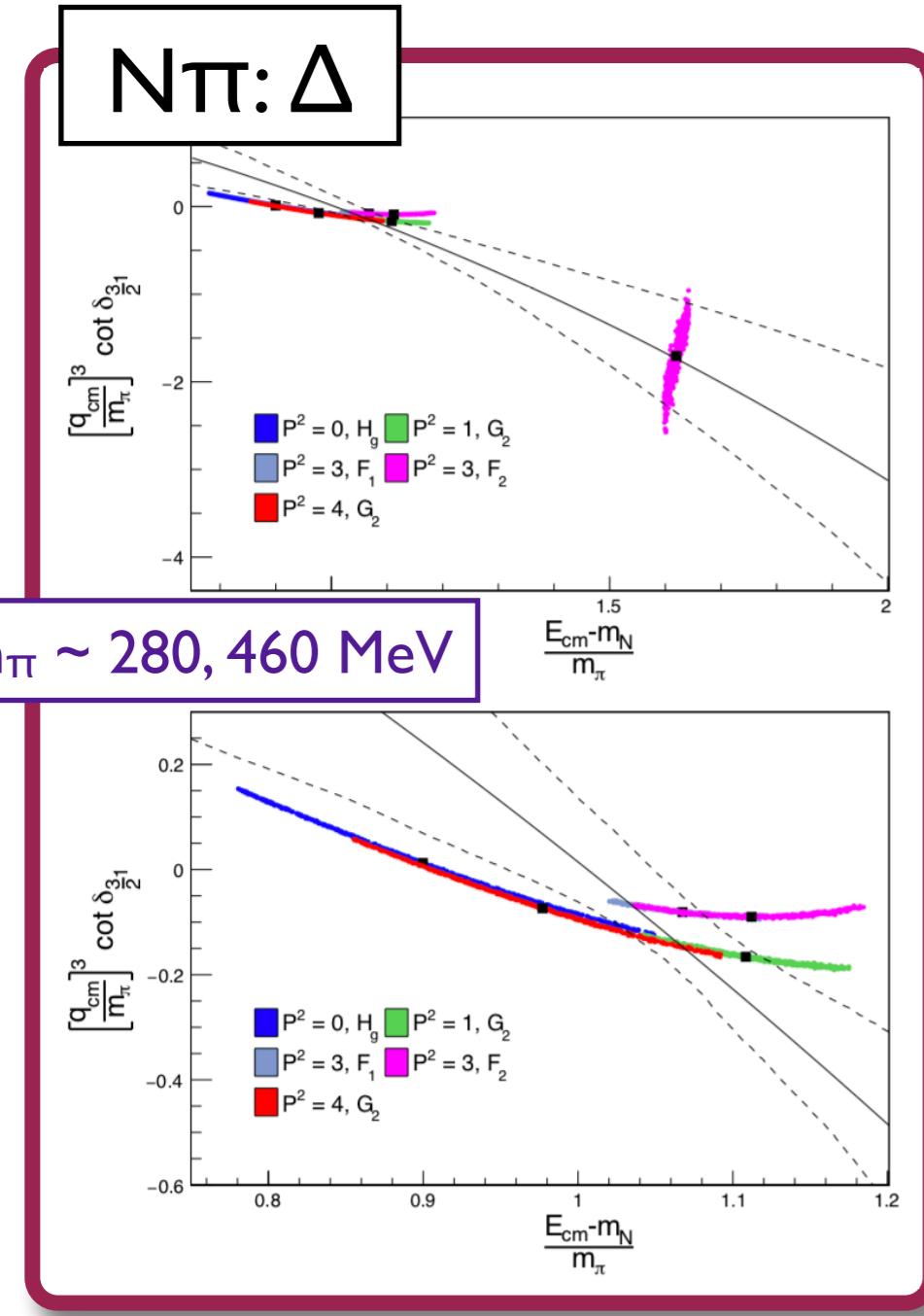
$$C_{ij}(t) = \langle \hat{O}_i(t) \hat{O}_j^\dagger(0) \rangle$$

$$C(t) v_n(t, t_0) = \lambda_n(t, t_0) C(t_0) v_n(t, t_0)$$
$$n = 1, \dots, N$$

The future? GEVP approaches

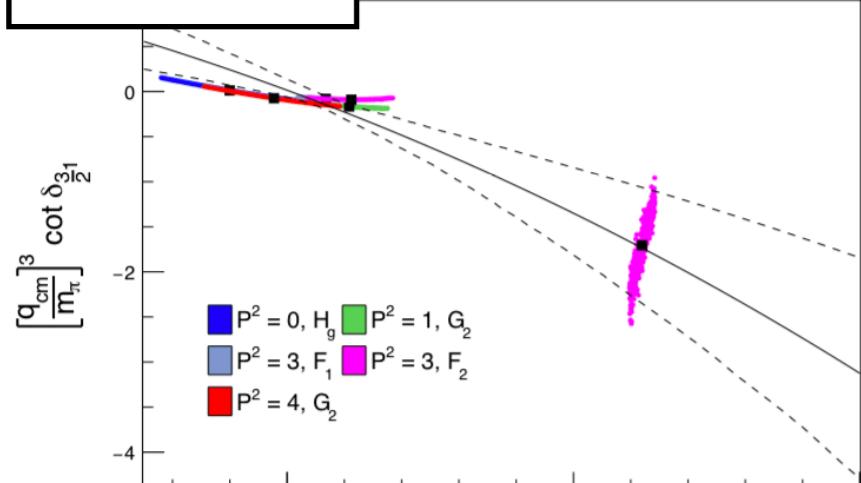


The future? GEVP approaches

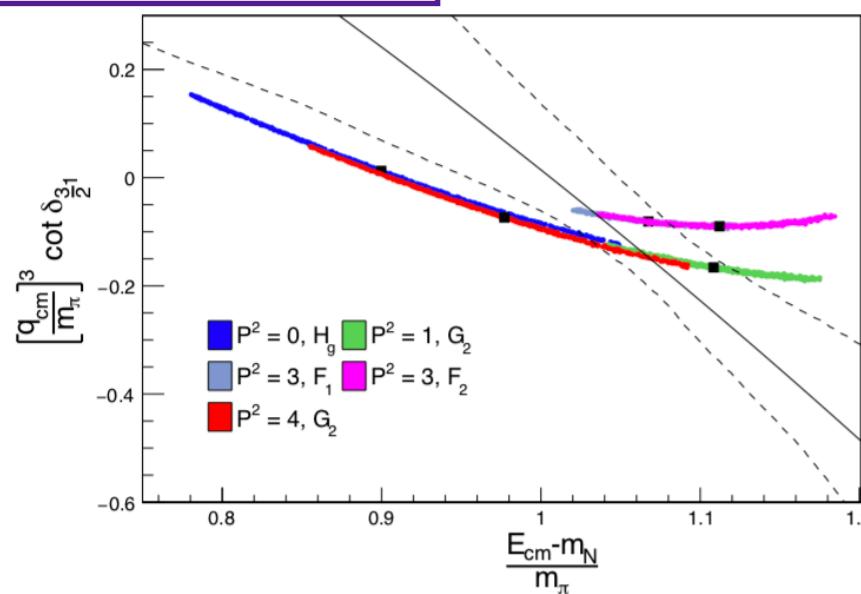


The future? GEVP approaches

$N\pi: \Delta$

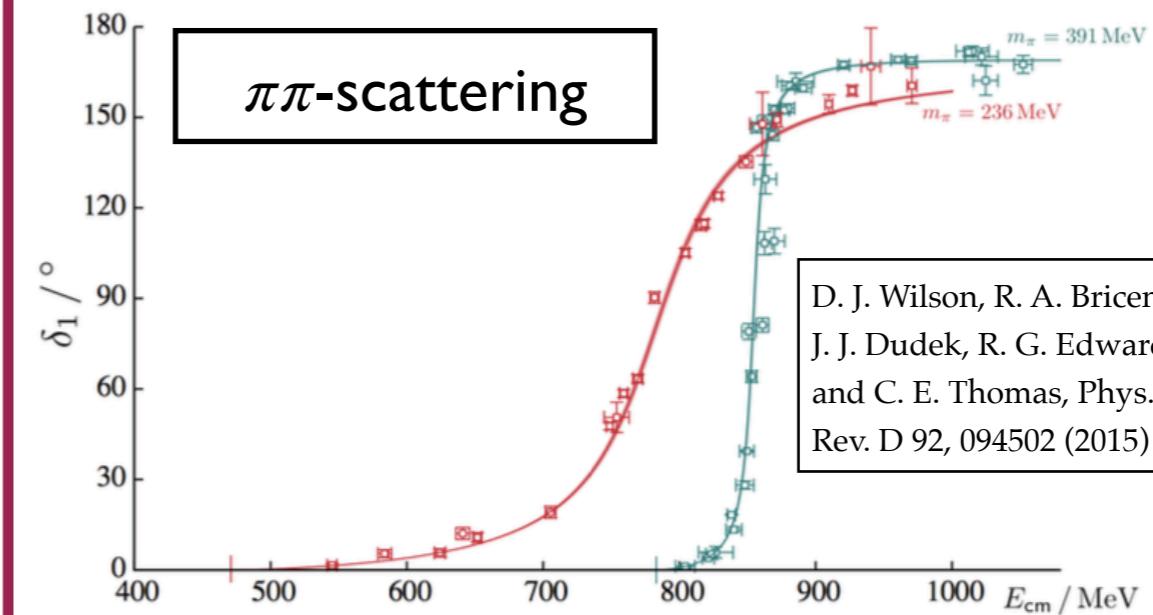


$m_\pi \sim 280, 460 \text{ MeV}$

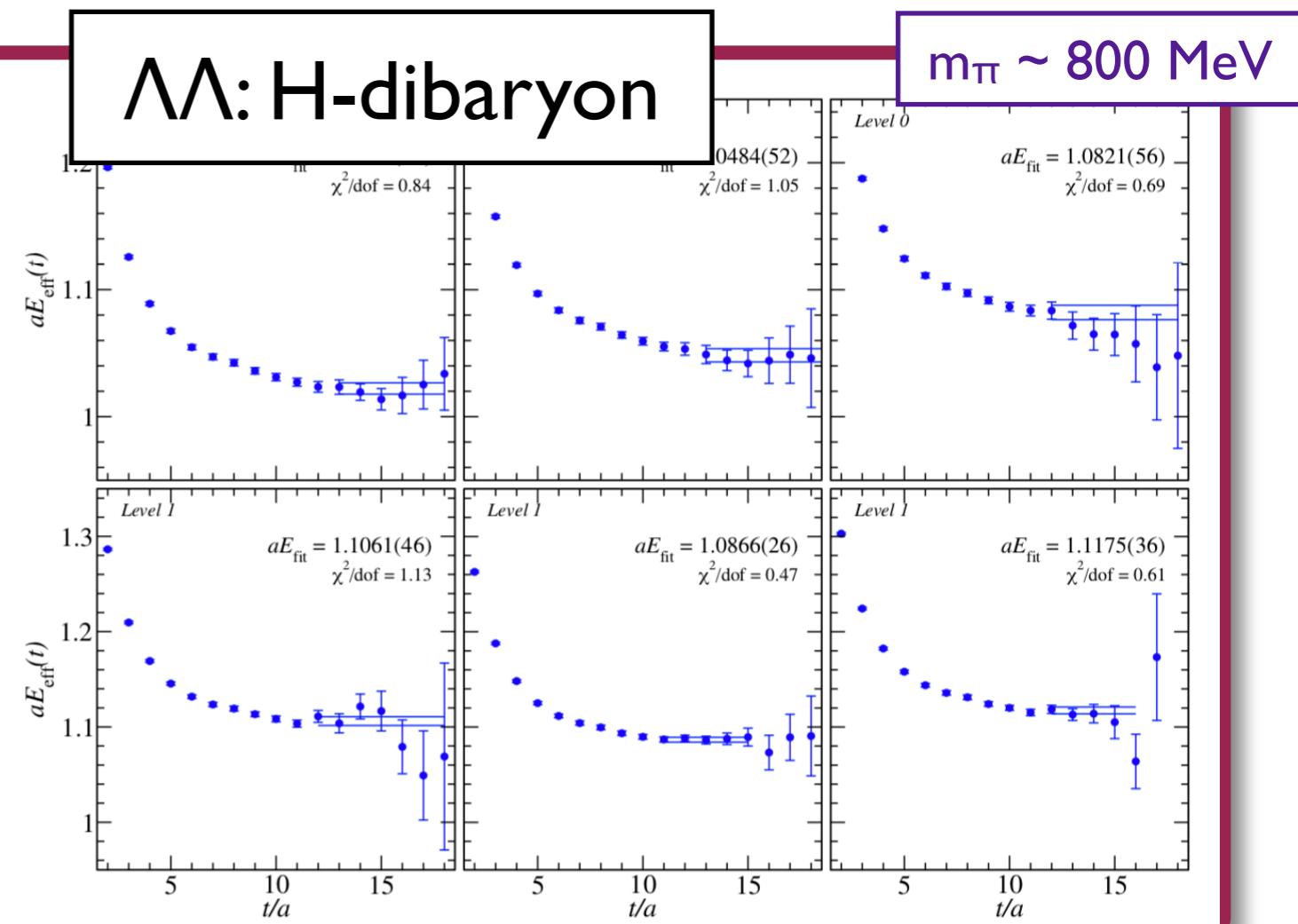


Andersen, Bulava, Horz, Morningstar (2018)

$\pi\pi$ -scattering



$\Lambda\Lambda: H$ -dibaryon



Hanlon, Francis, Green, Junnarkar, Wittig (2018)

The future? GEVP approaches NN?



The future? GEVP approaches NN?

- Why is GEVP so much more expensive?



The future? GEVP approaches NN?

- Why is GEVP so much more expensive?
 - Generally requires large basis of operators properly spanning the low-lying eigenstates



The future? GEVP approaches NN?



- Why is GEVP so much more expensive?
 - Generally requires large basis of operators properly spanning the low-lying eigenstates
 - Omission of a given operator type (e.g. local operators for channels containing a bound state or resonance) can give distorted energy levels

The future? GEVP approaches NN?



- Why is GEVP so much more expensive?
 - Generally requires large basis of operators properly spanning the low-lying eigenstates
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 - Positive definite = no fake plateaus!

The future? GEVP approaches NN?



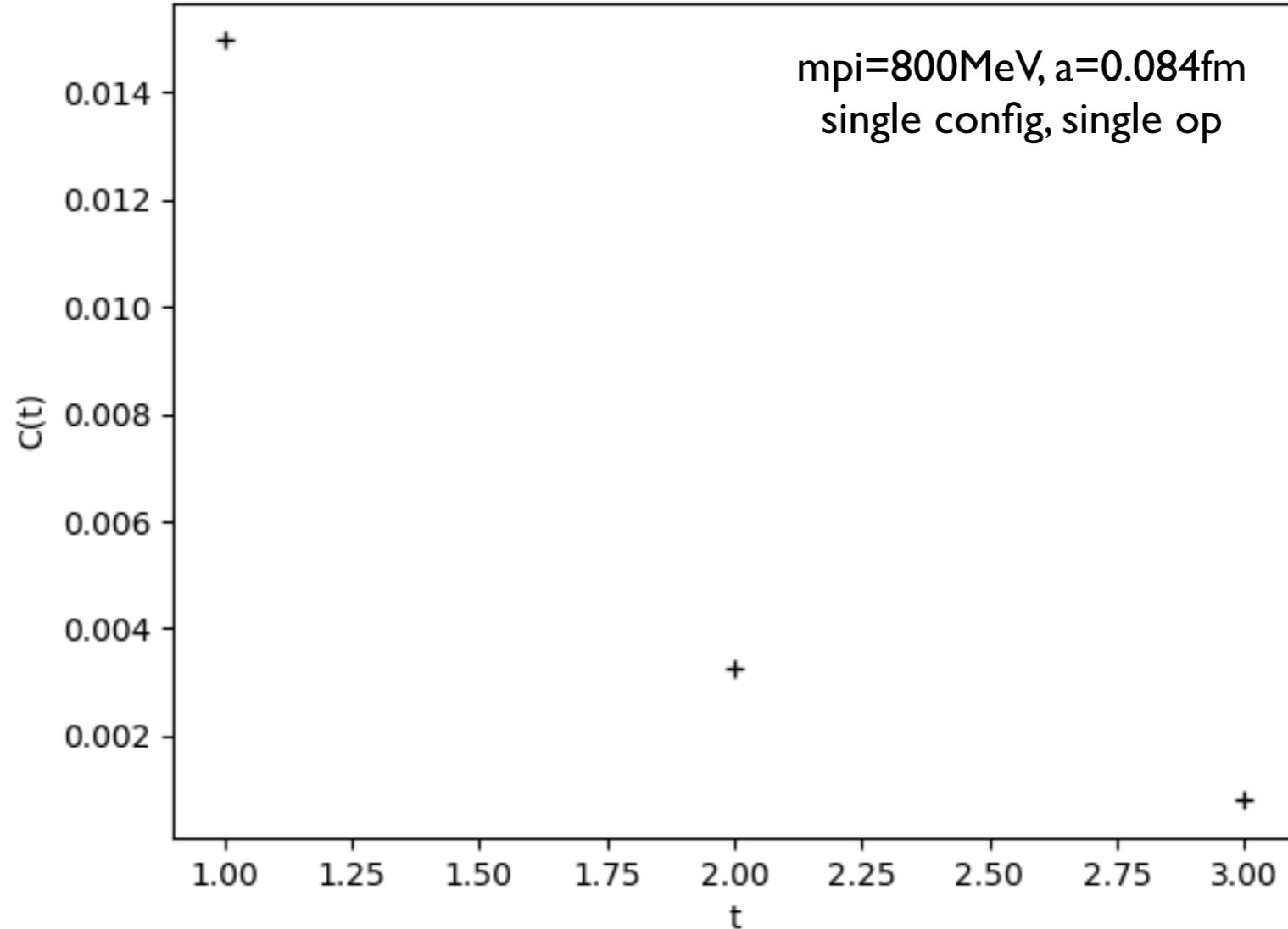
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The future? GEVP approaches NN?



- Why is GEVP so much more expensive?
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 - Omission of a given operator type (e.g. local operators for channels containing a bound state or resonance) can give distorted energy levels
 - Requires symmetric matrix of correlation functions
 - Positive definite = no fake plateaus!
 - Momentum \rightarrow momentum: cost $O(V)$
 - sLapH stochastically projects onto low momentum states, easing this scaling with V

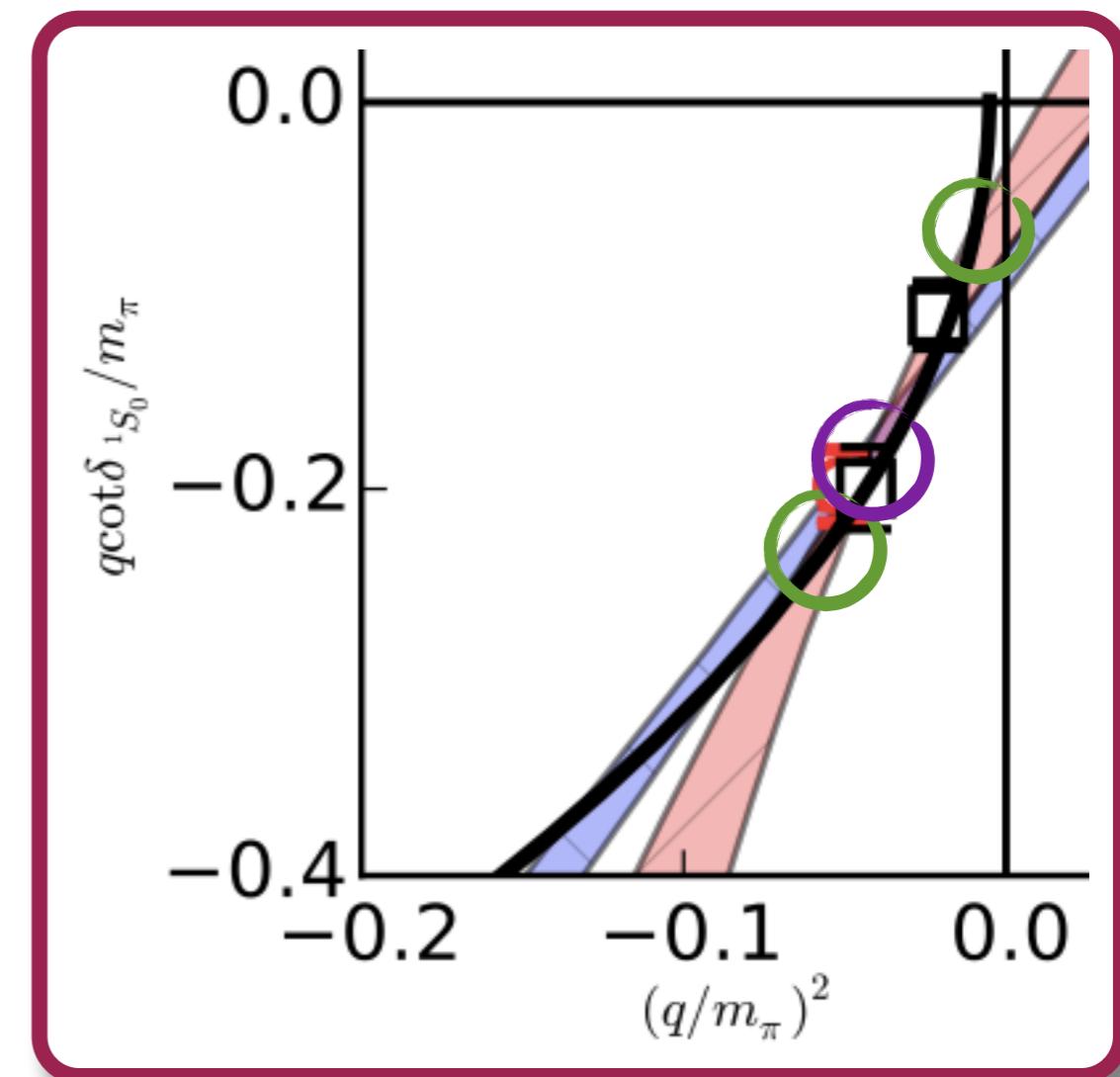
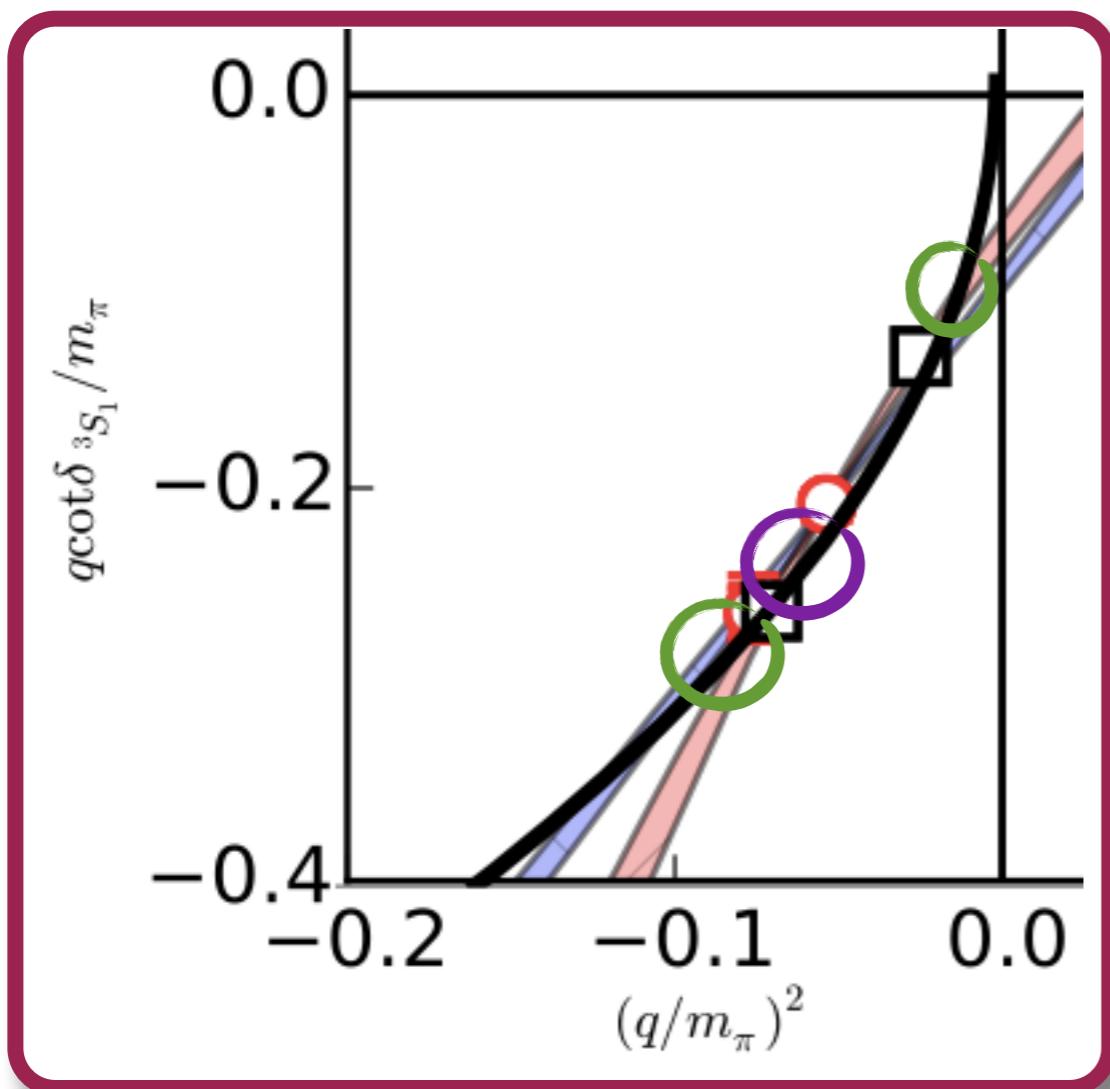
NN scattering with sLapH



+ C. Andersen, J. Bulava, A. Hanlon, D. Howarth, B. Hörz, C. Morningstar

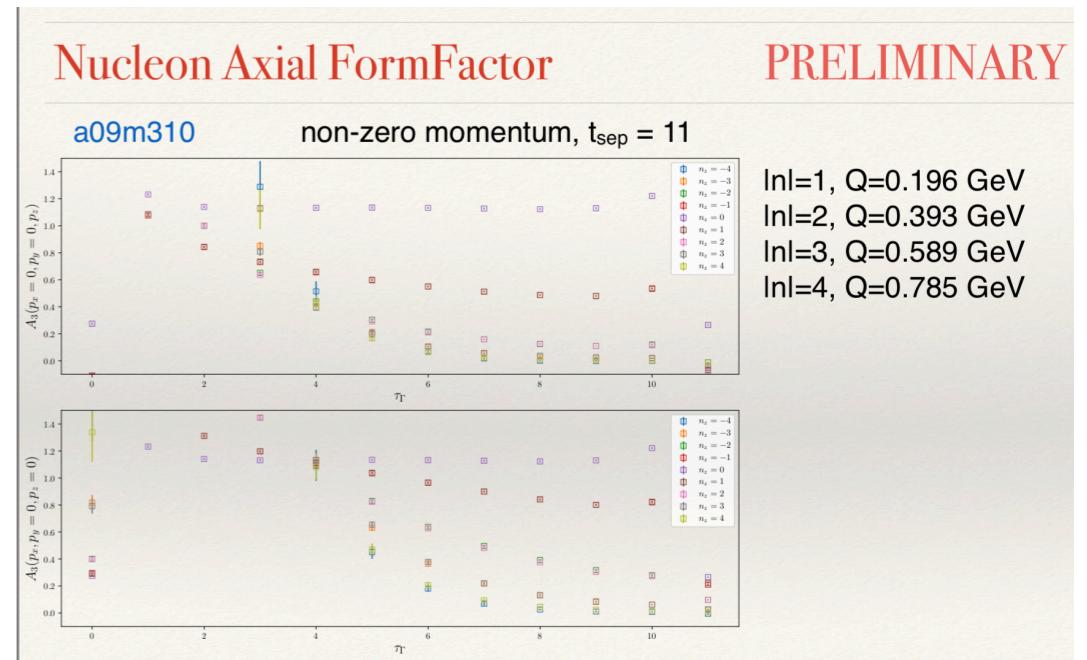
Fully resolving this puzzle likely requires GEVP including both momentum space and local ops

We are currently performing a comparison of methods (HAL potential, Luscher using both MP and sLapH) on same ensembles at 800 MeV

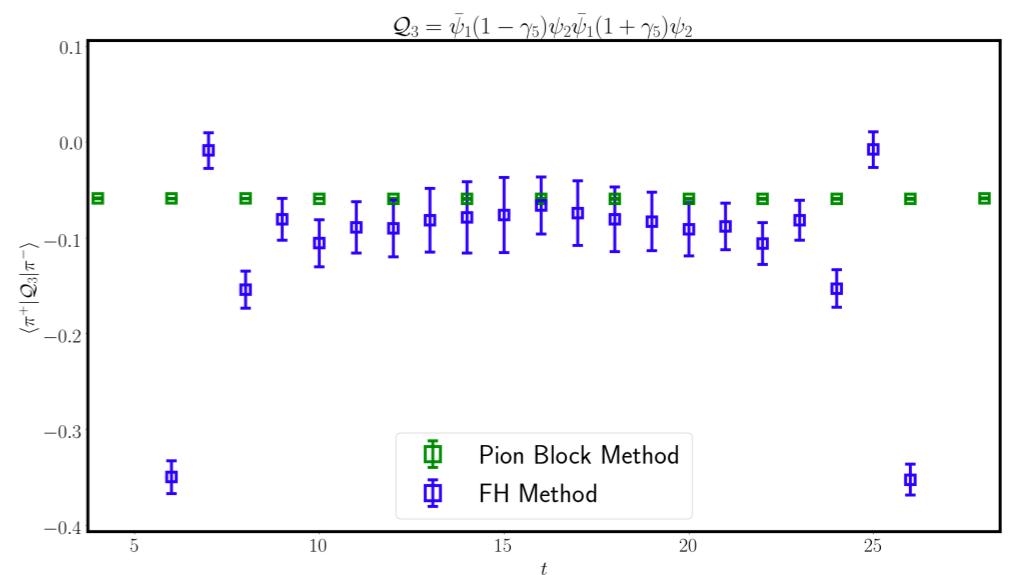


Other progress

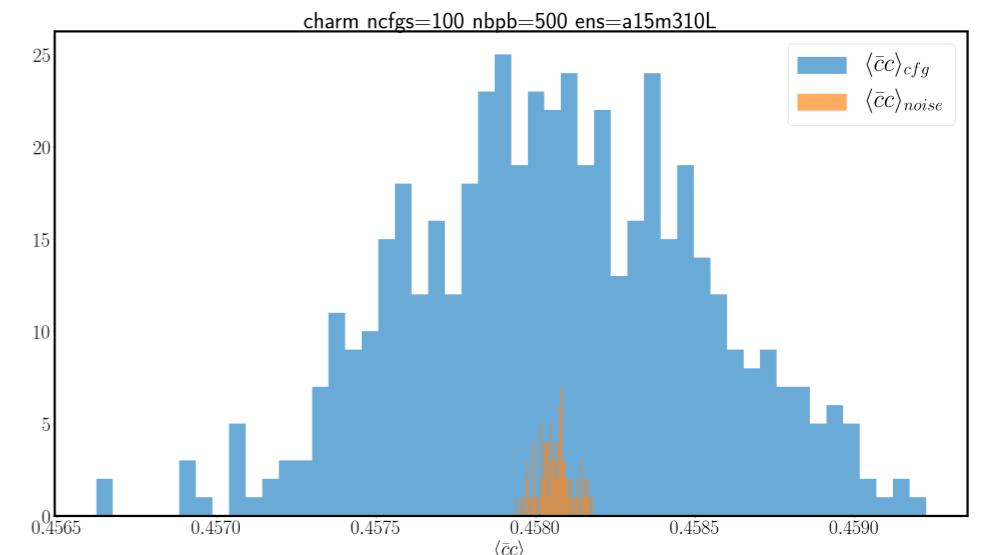
- Nucleon axial form factors



- Feynman-Hellmann method for computing 4-quark MEs



- Charm content of the nucleon (DM MEs) $\langle N | \bar{c}c | N \rangle$



- RIKEN/LBL: C.C. Chang
- RIKEN: E. Rinaldi
- NERSC: T. Kurth
- nVidia: M.A. Clark
- LBL/UCB: A. Walker-Loud, B. Hörz
- Glasgow: C. Bouchard
- LLNL: P. Vranas, D. Howarth
- Carnegie Mellon: C. Morningstar
- SDSU: J. Bulava, C. Andersen
- UMD: E. Berkowitz
- Mainz: A. Hanlon
- UNC: H. Monge-Camacho, AN

