Neutrino properties and double beta decay

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TC-May20

CMU Deliverables

Peer-reviewed journals:

1. F. Ahmed and M. Horoi, “Interference Effects for $0\nu\beta\beta$ Decay in the Left-Right Symmetric Model”, Phys. Rev. C 101, 035504 (2020), https://doi.org/10.1103/PhysRevC.101.035504.

2. M. Horoi, "On the MSW-like neutrino mixing effects in atomic weak interactions and double beta decays", EPJA 56, 39 (2020), https://doi.org/10.1140/epja/s10050-020-00042-x .

3. C. F. Jiao, M. Horoi, and A. Neacsu, “Neutrinoless double-decay of ^{124}Sn, ^{130}Te, and ^{136}Xe in the Hamiltonian-based generator-coordinate method”, Phys. Rev. C 98, 064324, https://doi.org/10.1103/PhysRevC.98.064324.

4. M. Horoi and A. Neacsu, “Shell model study of using an effective field theory for disentangling several contributions to neutrinoless double-beta decay”, Phys. Rev. C 98, 035502 (2018), https://doi.org/10.1103/PhysRevC.98.035502.

5. F. Ahmed, A. Neacsu, and M. Horoi, “Interference between light and heavy neutrinos for $0\nu\beta\beta$ decay in the left–right symmetric model”, Physics Letters B 769, 299–304 (2017), https://doi.org/10.1016/j.physletb.2017.03.066.

6. A. Neacsu and M. Horoi, “Shell Model Studies of Competing Mechanisms to the Neutrinoless Double-Beta Decay in ^{124}Sn, ^{130}Te, and ^{136}Xe”, Advances in High Energy Physics 2016, 1903767 (2016), https://doi.org/10.1155/2016/1903767.

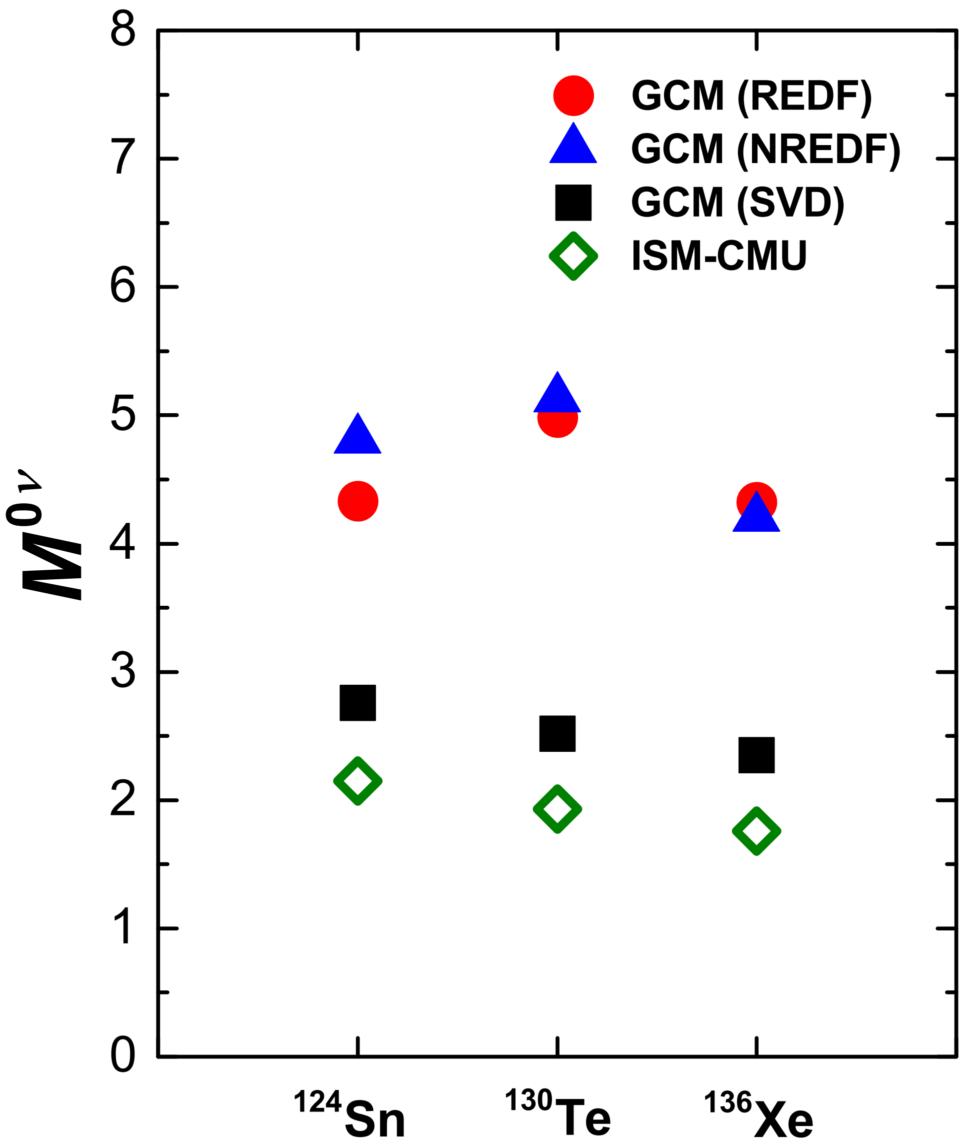
Peer-reviewed Proceedings:

7. M. Horoi, “Neutrinoless double beta decay of atomic nuclei”, AIP Proceedings **2165**, 020012 (2019).

8. M. Horoi, “Nuclear Structure for Double Beta Decay”, in 12th INTERNATIONAL SPRING SEMINAR ON NUCLEAR PHYSICS, Ischia, May 15-19, 2017, volume 966 of J. Phys.: Conf. Series, page 012009, (2018), https://doi.org/10.1088/1742-6596/966/1/012009.

9. M. Horoi, “Towards a complete description of the neutrinoless double beta decay”, in Matrix Elements for the Double beta decay EXperiments: MEDEX’17, Prague, June 1-6, 2017, volume 1894 of AIP Conference Proceedings, page 020011, (2017), https://doi.org/10.1063/1.5007636.

3. C. F. Jiao, M. Horoi, and A. Neacsu, "Neutrinoless double-decay of ^{124}Sn, ^{130}Te, and ^{136}Xe in the Hamiltonian-based generator-coordinate method", Phys. Rev. C 98, 064324, https://doi.org/10.1103/PhysRevC.98.064324.



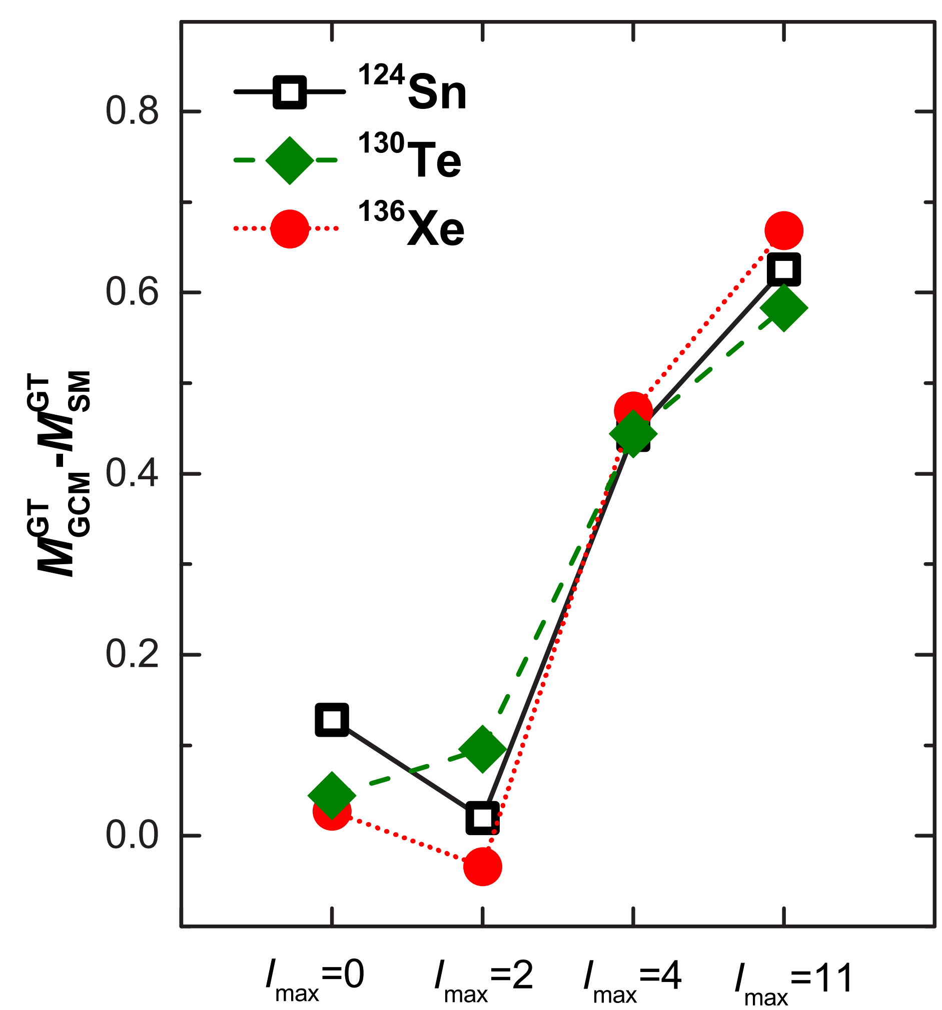


FIG. 5. The differences of Gamow-Teller part of NMEs between our GCM and SM calculations against the pair-spin I for ^{124}Sn, ^{130}Te, and ^{136}Xe.

GCM and Shell Model give similar NME if the same model spaces and Hamiltonians are used

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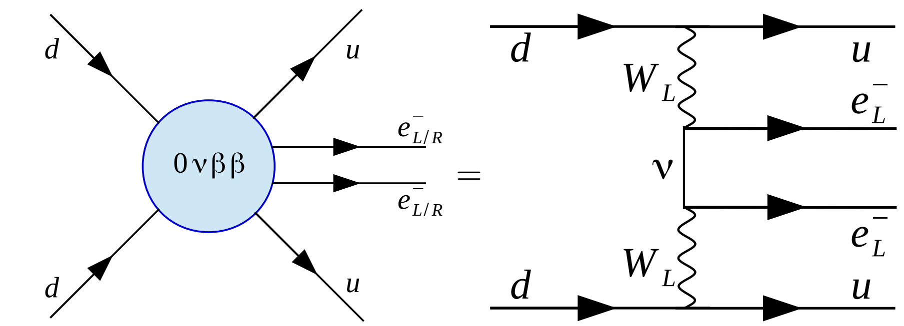
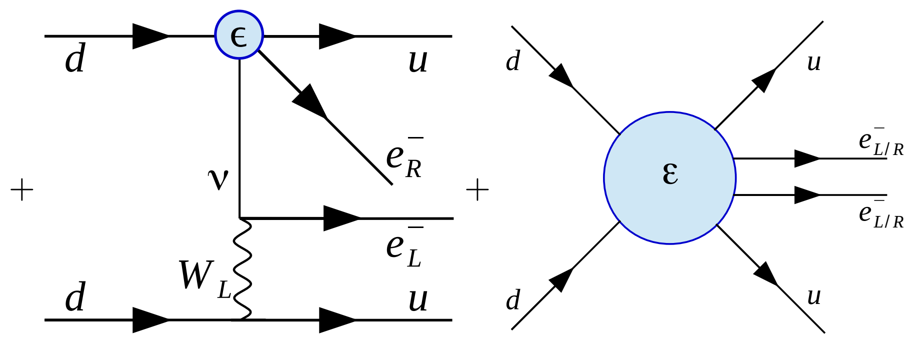
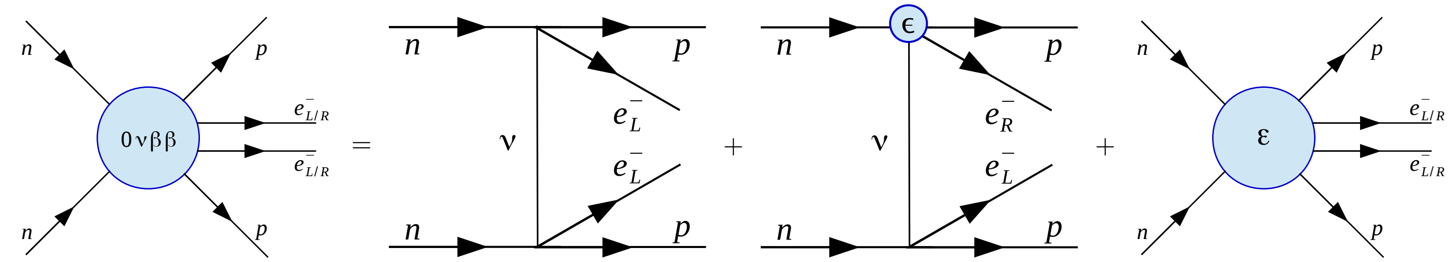


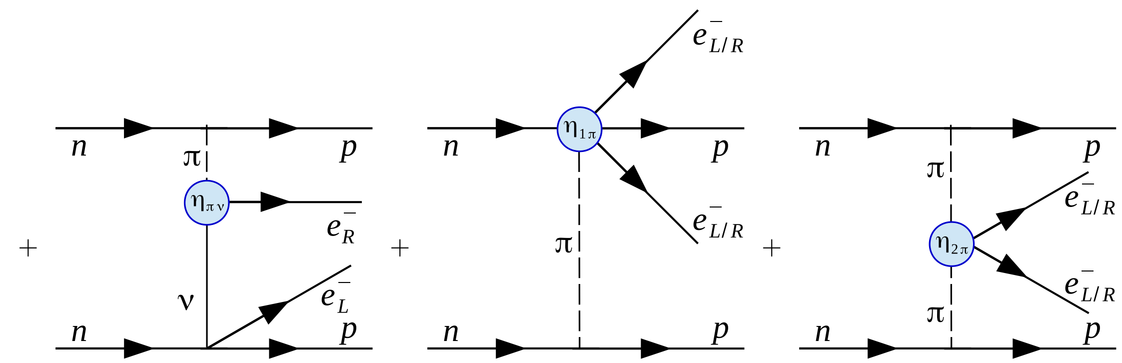
(a) The generic $0\nu\beta\beta$ decay diagram at the quark-level. (b) Light left-handed neutrino exchange diagram.



(c) The long-range part of the $0\nu\beta\beta$ diagram. (d) The short-range part of the $0\nu\beta\beta$ diagram.



(a) The generic $0\nu\beta\beta$ decay process nucleon-level diagram. (b) Light left-handed neutrino exchange diagram. (c) The The nucleon-nucleon long-range $\mathcal{L}_6$ mode. (d) The nucleon-nucleon short-range part of the $0\nu\beta\beta$ diagram.



(e) The pion-neutrino long-range diagram. (f) The one-pion long-range diagram. (g) The two-pion long-range diagram.

$$\mathcal{L}_6 = \frac{G_F}{\sqrt{2}}\left[j^{\mu}_{V-A} J^{\dagger}_{V-A,\mu} + \sum_{\alpha,\beta}^{*} \epsilon^{\beta}_{\alpha} j_{\beta} J^{\dagger}_{\alpha} \right]$$

$$\left[T^{0\nu}_{1/2}\right]^{-1} = g_A^4 \left[\sum_i |\mathcal{E}_i|^2 \mathcal{M}_i^2 + \mathrm{Re}\left[\sum_{i\neq j} \mathcal{E}_i \mathcal{E}_j \mathcal{M}_{ij} \right] \right]$$

$$\mathcal{L}_9 = \frac{G_F^2}{2m_p}\Big[\varepsilon_1 JJj + \varepsilon_2 J^{\mu\nu} J_{\mu\nu} j + \varepsilon_3 J^{\mu} J_{\mu} j + \varepsilon_4 J^{\mu} J_{\mu\nu} j^{\nu} + \varepsilon_5 J^{\mu} J j_{\mu} \Big],$$

$$\mathcal{E}_{2-7} = \{\epsilon^{V+A}_{V-A},\ \epsilon^{V+A}_{V+A},\ \epsilon^{S+P}_{S\pm P},\ \epsilon^{TR}_{TR},\ \eta_{\pi\nu}\}$$

$$\mathcal{E}_{8-15} = \{\varepsilon_1,\ \varepsilon_2,\ \varepsilon_3^{LLz(RRz)},\ \varepsilon_3^{LRz(RLz)},\ \varepsilon_4,\ \varepsilon_6,\ \eta_{1\pi},\ \eta_{2\pi}\}$$

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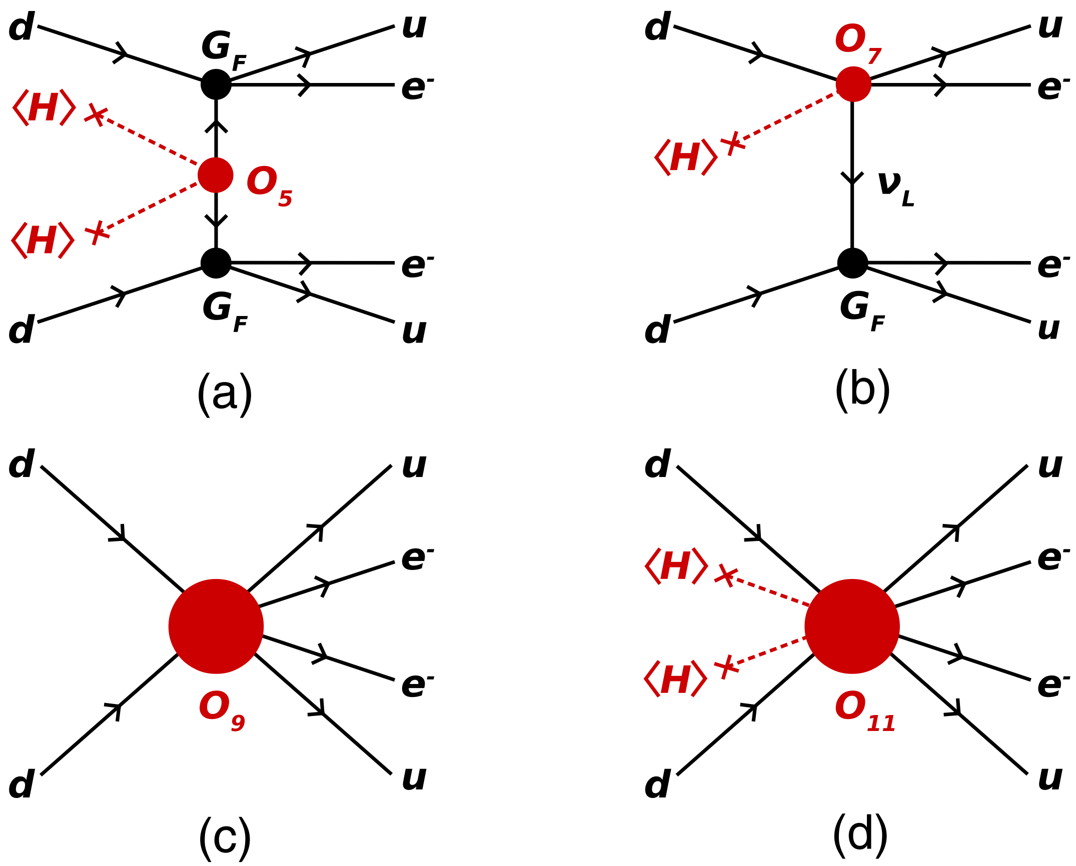
4. M. Horoi and A. Neacsu, "Shell model study of using an effective field theory for disentangling several contributions to neutrinoless double-beta decay", Phys. Rev. C 98, 035502 (2018), https://doi.org/10.1103/PhysRevC.98.035502.

PHYSICAL REVIEW D **92,** 036005 (2015)

$$\mathcal{L}_D = \frac{g}{(\Lambda_D)^{D-4}} \mathcal{O}_D$$



$$m_e \bar{\epsilon}_5 = \frac{g^2 (yv)^2}{\Lambda_5}, \qquad \frac{G_F \bar{\epsilon}_7}{\sqrt{2}} = \frac{g^3 (yv)}{2(\Lambda_7)^3},$$

$$\frac{G_F^2 \bar{\epsilon}_9}{2m_p} = \frac{g^4}{(\Lambda_9)^5}, \qquad \frac{G_F^2 \bar{\epsilon}_{11}}{2m_p} = \frac{g^6 (yv)^2}{(\Lambda_{11})^7}$$

TABLE VIII. The BSM effective scale (in GeV) for different dimension-D operators at the present ^{136}Xe half-life limit (Λ_D^0) and for $T_{1/2} \approx 1.1 \times 10^{28}$ years (Λ_D).

$\mathcal{O}_D$	$\bar{\epsilon}_D$	$\Lambda_D^0(y=1)$	$\Lambda_D^0(y=y_e)$	$\Lambda_D(y=y_e)$
$\mathcal{O}_5$	$2.8 \cdot 10^{-7}$	$2.12 \cdot 10^{14}$	1904	19044
$\mathcal{O}_7$	$2.0 \cdot 10^{-7}$	$3.75 \cdot 10^{4}$	541	1165
$\mathcal{O}_9$	$1.5 \cdot 10^{-7}$	$2.47 \cdot 10^{3}$	2470	3915
$\mathcal{O}_{11}$	$1.5 \cdot 10^{-7}$	$1.16 \cdot 10^{3}$	31	43

$$\eta_N \propto \frac{1}{m_{W_R}^4 m_N}$$

$g \approx 1 \quad v = 174\ GeV \qquad y_e = 3 \times 10^{-6}$ *electron mass Yukawa*

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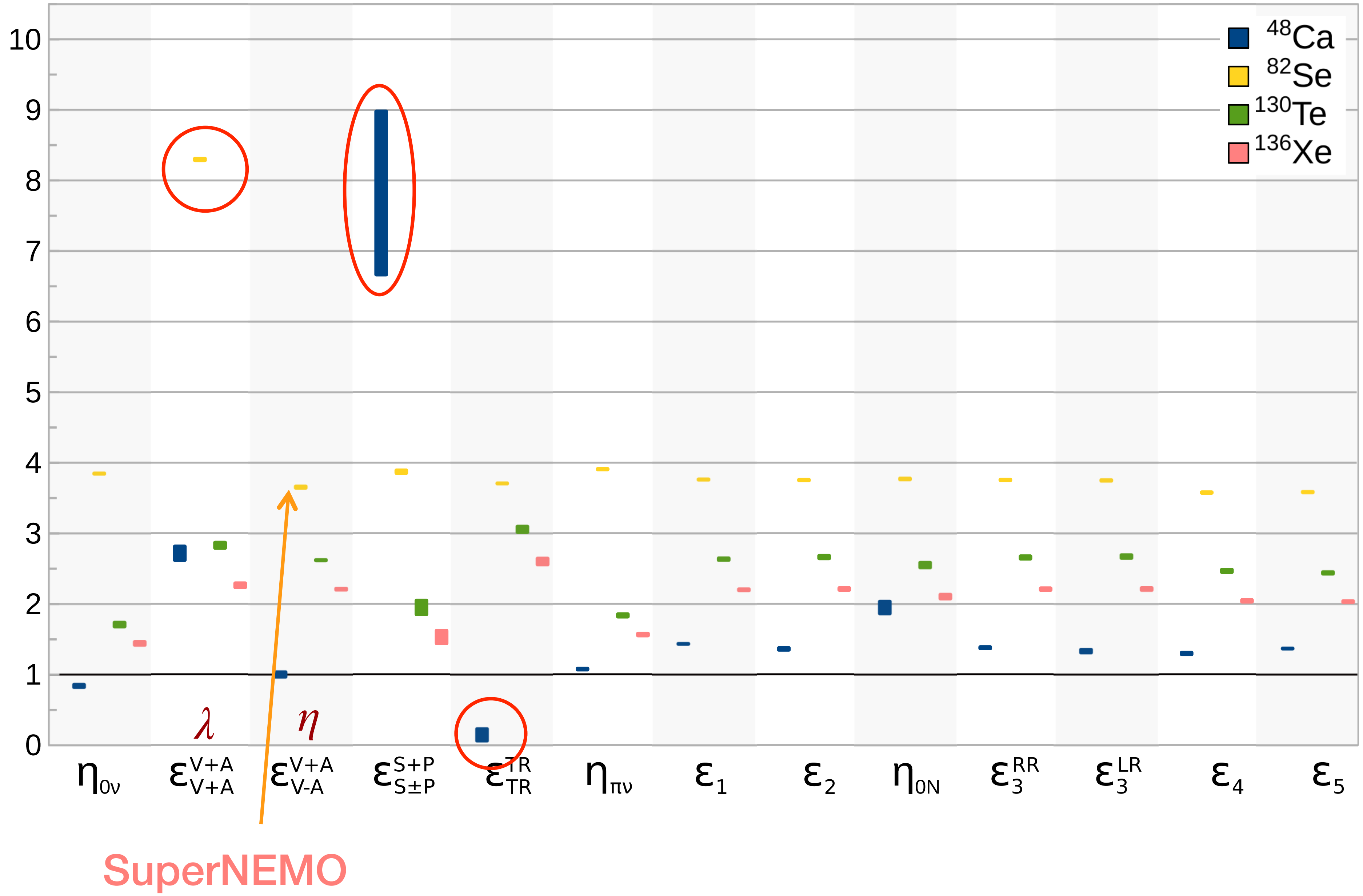
One coupling dominance: which one?

$$\left[T^{0\nu}_{1/2}\right]^{-1} = g_A^4 \left[\sum_i |\mathcal{E}_i|^2 \mathcal{M}_i^2 + \mathrm{Re}\left[\sum_{i\neq j} \mathcal{E}_i \mathcal{E}_j^* \mathcal{M}_{ij}\right]\right]$$

T[^{76}Ge]/T[^{A}Z]

CMU Hamiltonians



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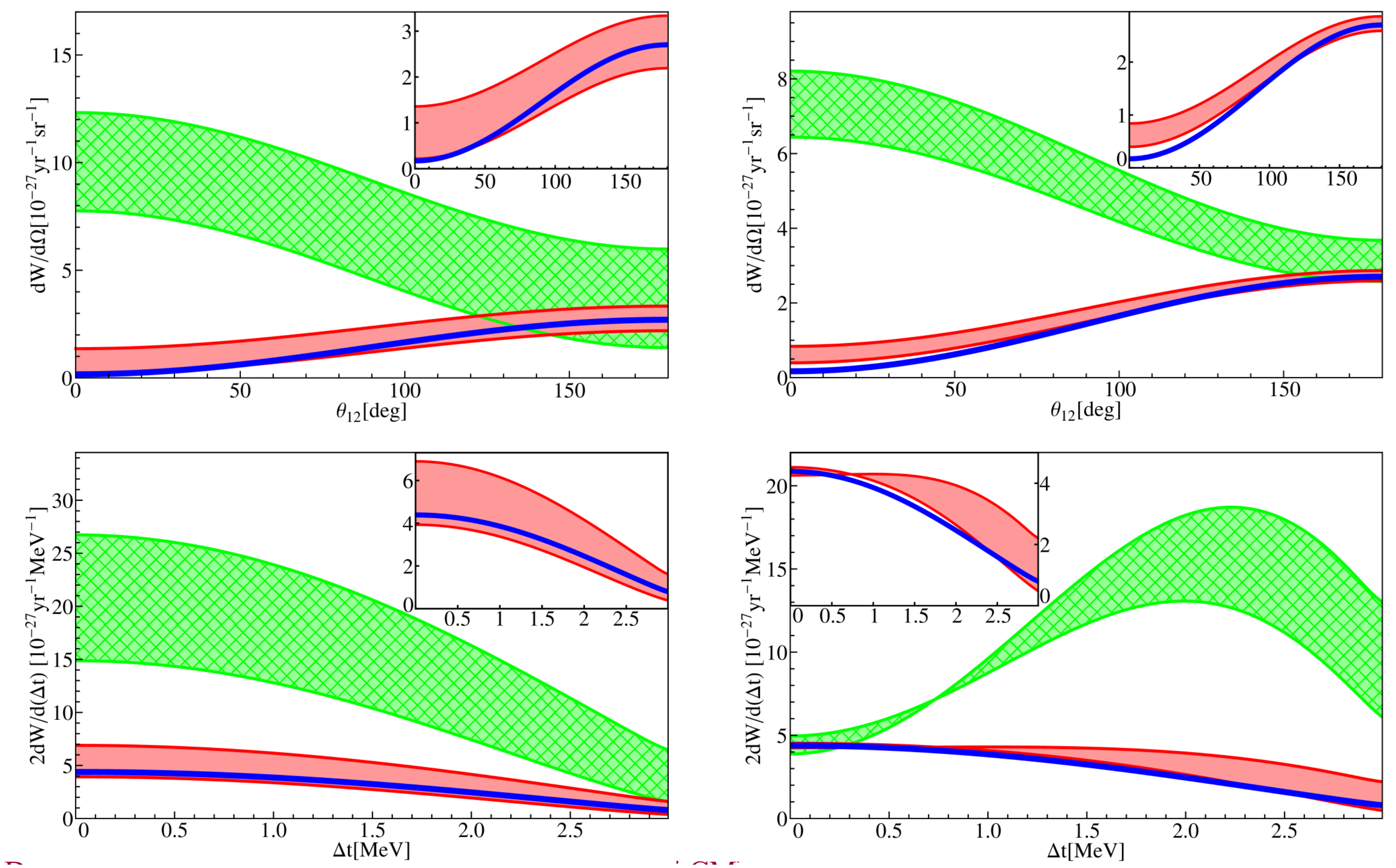
M. Horoi and A. Neacsu, “ Analysis of mechanisms that could contribute to neutrinoless double-beta decay “, Phys. Rev D 93, 113014 (2016).

λ and η mechanisms (^{82}Se): look for green

$<\lambda>$ dominates $\qquad$ $<\eta>$ dominates



$$\frac{d^2W^{0\nu}_{0^+\rightarrow 0^+}}{d\epsilon_1 d\cos\theta_{12}} = \frac{a_{0\nu}\omega_{0\nu}(\epsilon_1)}{2\,(m_eR)^2}\left[A(\epsilon_1)+B(\epsilon_1)\cos\theta_{12}\right]$$

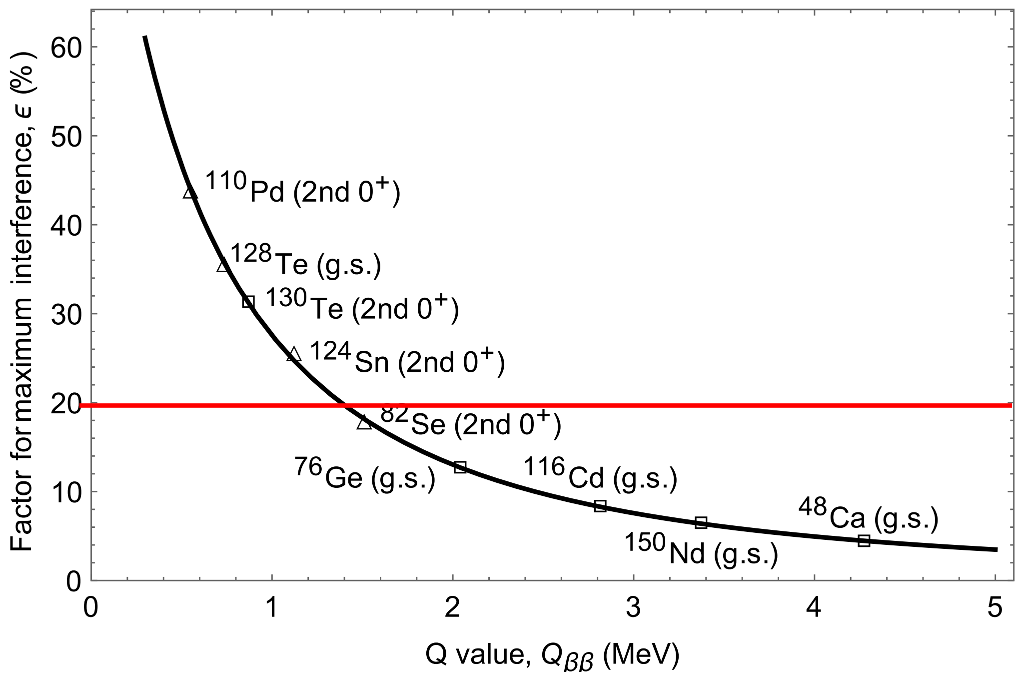
$$\frac{2dW^{0\nu}_{0^+\rightarrow 0^+}}{d(\Delta t)} = \frac{2a_{0\nu}}{(m_eR)^2}\frac{\omega_{0\nu}(\Delta t)}{m_ec^2}A(\Delta t)$$

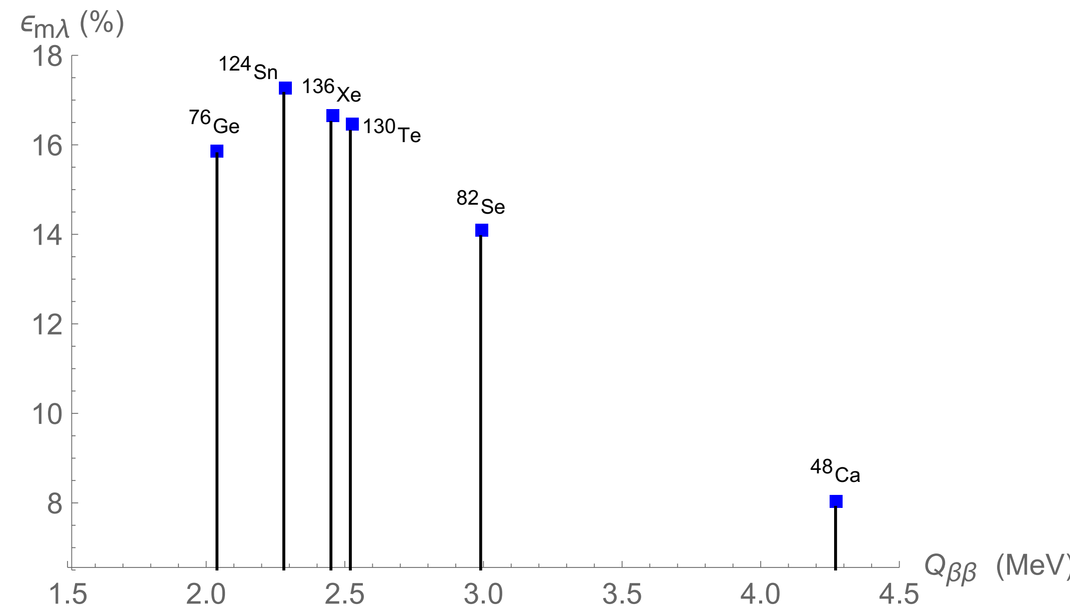
$$t = \varepsilon_{e1} - \varepsilon_{e2}$$

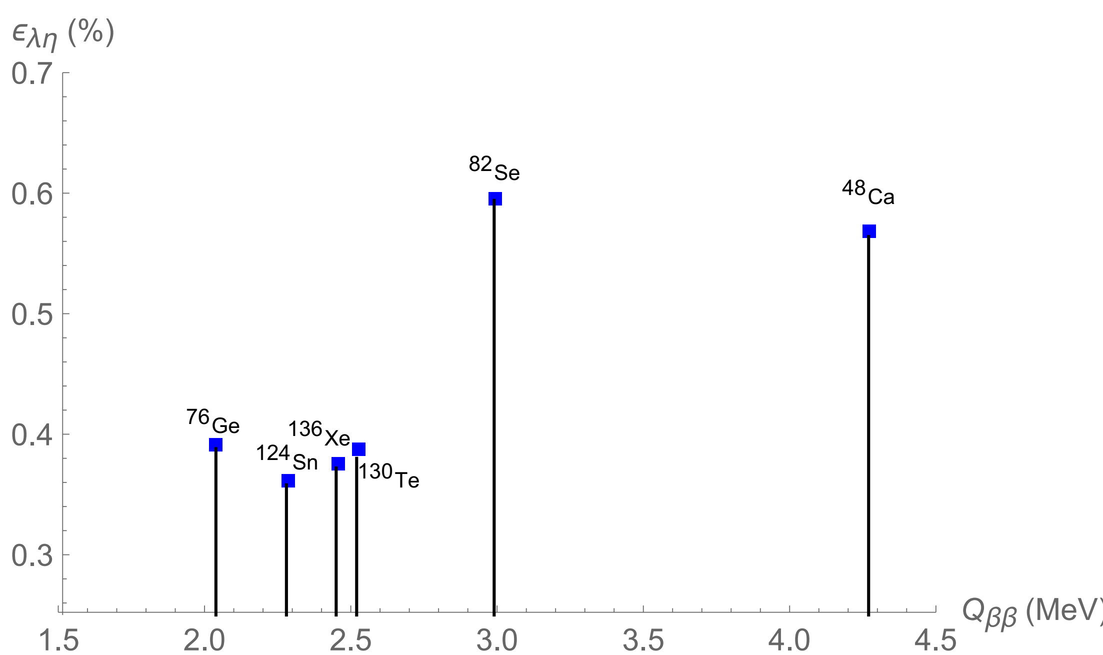
1. F. Ahmed and M. Horoi, “Interference Effects for 0νββ Decay in the Left-Right Symmetric Model”, Phys. Rev. C 101, 035504 (2020), https://doi.org/10.1103/PhysRevC.101.035504.

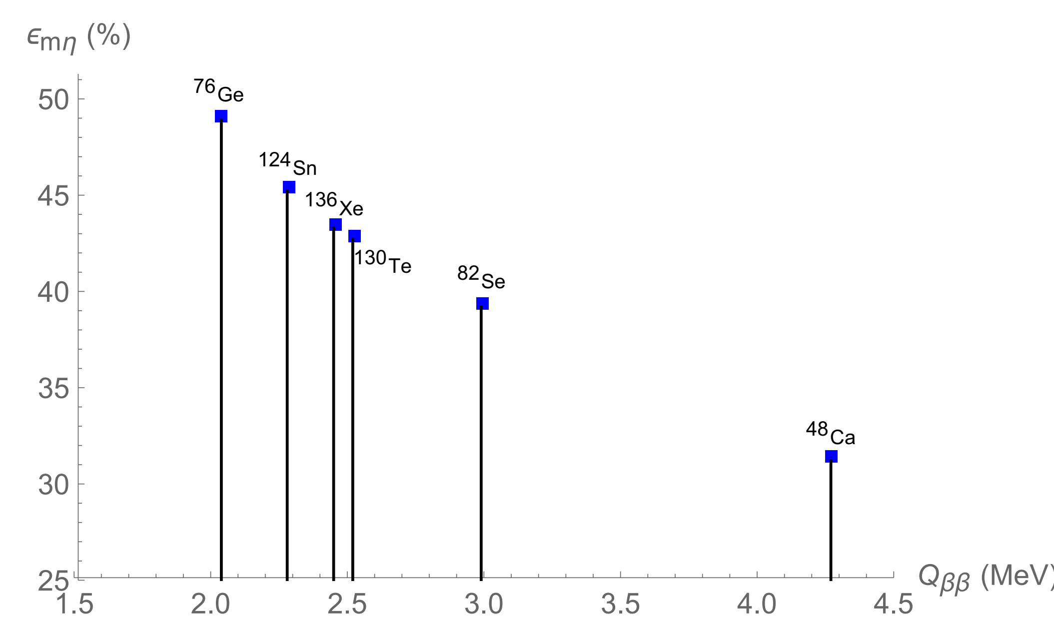
5. F. Ahmed, A. Neacsu, and M. Horoi, “Interference between light and heavy neutrinos for 0νββ decay in the left–right symmetric model”, Physics Letters B 769, 299–304 (2017), https://doi.org/10.1016/j.physletb.2017.03.066.

ϵ_{mN}









$$\left[T_{1/2}^{0\nu}\right]^{-1} \simeq g_A^4(1+\alpha)C_i|\eta_i|^2[1+\epsilon_{ij}\cos(\phi_i-\phi_j)]$$

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2. M. Horoi, "On the MSW-like neutrino mixing effects in atomic weak interactions and double beta decays", EPJA 56, 39 (2020), https://doi.org/10.1140/epja/s10050-020-00042-x .

M. Horoi and A Zettel, “Effects of Atomic-Scale Electron Density Profile and a Fast and Efficient Iteration Algorithm for Matter Effect of Neutrino Oscillation“, Universe 6, 16 (2020).

Atomic nucleus is a high electron density medium:

Consider 2 electrons in the lowest s-orbital of an Hydrogen-like atom

Electron density near nucleus:

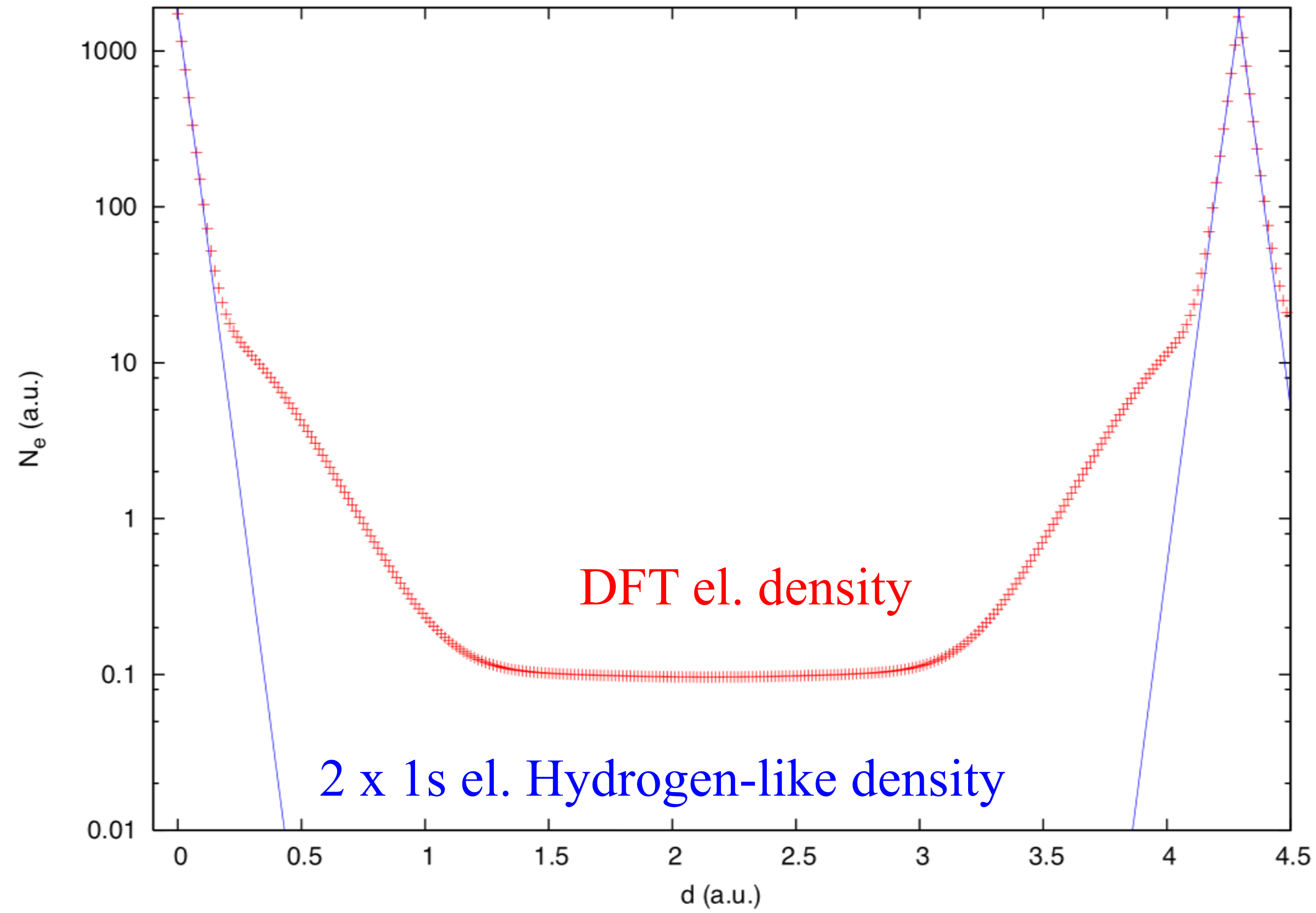
$$N_e(r) \approx \frac{2}{\pi}\left(\frac{Z}{a_B}\right)^3 e^{-2rZ/a_B}$$

Electron density inside nucleus:

$$N_e(0) \approx \frac{2}{\pi}\left(\frac{Z}{a_B}\right)^3$$

$$\rho_{Sun\,core} \approx 150\ g/cm^3$$

Si$_2$ dimer



Equivalent matter density: $\rho = m_N N_e = 1.67\times10^6 \frac{2}{\pi}\left(\frac{Z}{53}\right)^3$ *in* $g/cm^2 >> \rho_{Sun}$

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Neutrinoless double beta decay in vacuum

$$A_{0\beta\beta} \propto NP = \langle 0| T\left[\psi_{eL}(x_1)\psi^T_{eL}(x_2)\right]|0\rangle \qquad \psi_e(x) = \sum_{a=1}^{N(3)} U_{ea}\psi_a(x)$$

$$NP = \sum_{a=1}^{3} U_{ea}^2 \langle 0| T\left[\psi_{aL}(x_1)\psi^T_{aL}(x_2)\right]|0\rangle$$

$$= \sum_{a=1}^{3} U_{ea}^2 \left[-i\int \frac{d^4p}{(2\pi)^4}\frac{m_a e^{-ip(x_1-x_2)}}{p^2-m_a^2+i\epsilon} P_L \mathcal{C}\right] \longrightarrow \frac{1}{T_{1/2}} = G(Z,Q)\left|M_{0\nu}\right|^2 \left|\sum_{a=1}^{3} U_{ea}^2 m_a\right|^2 / m_e^2$$

$$P_L = \frac{1}{2}\left(1-\gamma^5\right) \qquad \hat{\psi}(x) = C\psi^*(x)$$

$P_L C$ product is further used to process the electron current, and one finally gets:

Neutrinoless double beta decay of atomic nuclei

- the in-matter propagator still contains the vacuum PMNS matrix and masses!
- The formalism allows the extension of this result if sterile neutrinos are present (a = 1…4,(5))
- The propagators for long range 0νββ diagrams seem to remain unchanged (work not finished yet)

In atomic nuclei NP = In vacuum NP

Vacuum result stands : $m_{\beta\beta} = \left|\sum_{a=1}^{3} U_{ea}^2 m_a\right|$

2. M. Horoi, "On the MSW-like neutrino mixing effects in atomic weak interactions and double beta decays", EPJA 56, 39 (2020), https://doi.org/10.1140/epja/s10050-020-00042-x .

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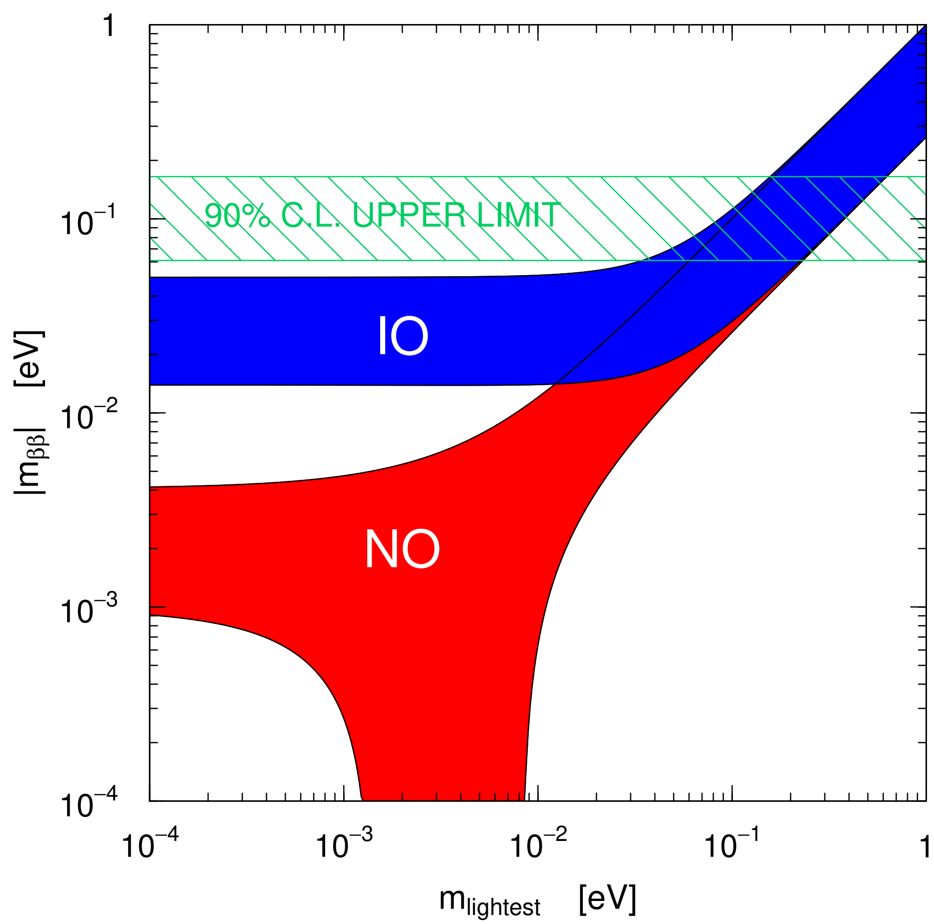
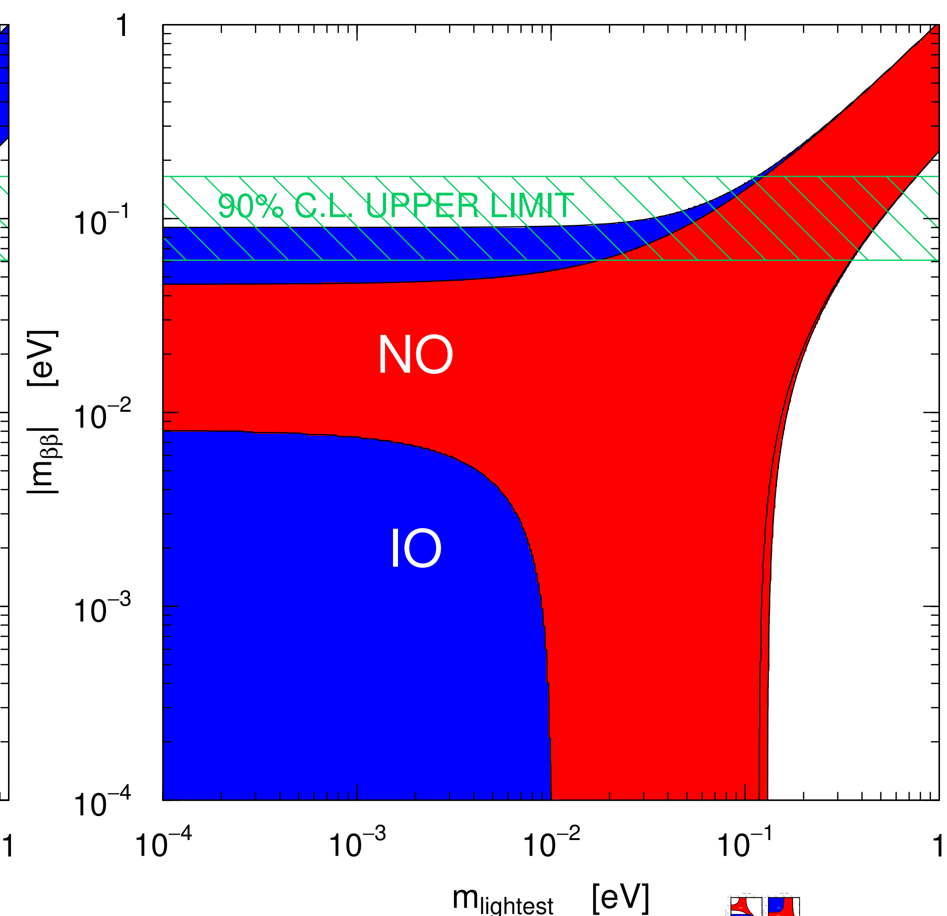
3 neutrino flavors



3+1(sterile) neutrino flavors



$$\left|m_{\beta\beta}\right| = \left|\sum_{k=1}^{3} m_k U_{ek}^2\right| = \left|c_{12}^2 c_{13}^2 m_1 + c_{13}^2 s_{12}^2 m_2 e^{i\phi_2} + s_{13}^2 m_3 e^{i\phi_3}\right| \quad \Leftarrow T_{1/2}^{-1}(0\nu) = G^{0\nu}(Q_{\beta\beta})\left[M^{0\nu}(0^+)\right]^2 (\eta_{0\nu})^2$$

$$\phi_2 = \alpha_2 - \alpha_1 \qquad \phi_3 = -\alpha_1 - 2\delta \qquad \eta_{0\nu} = \frac{\left|m_{\beta\beta}\right|}{m_e}$$

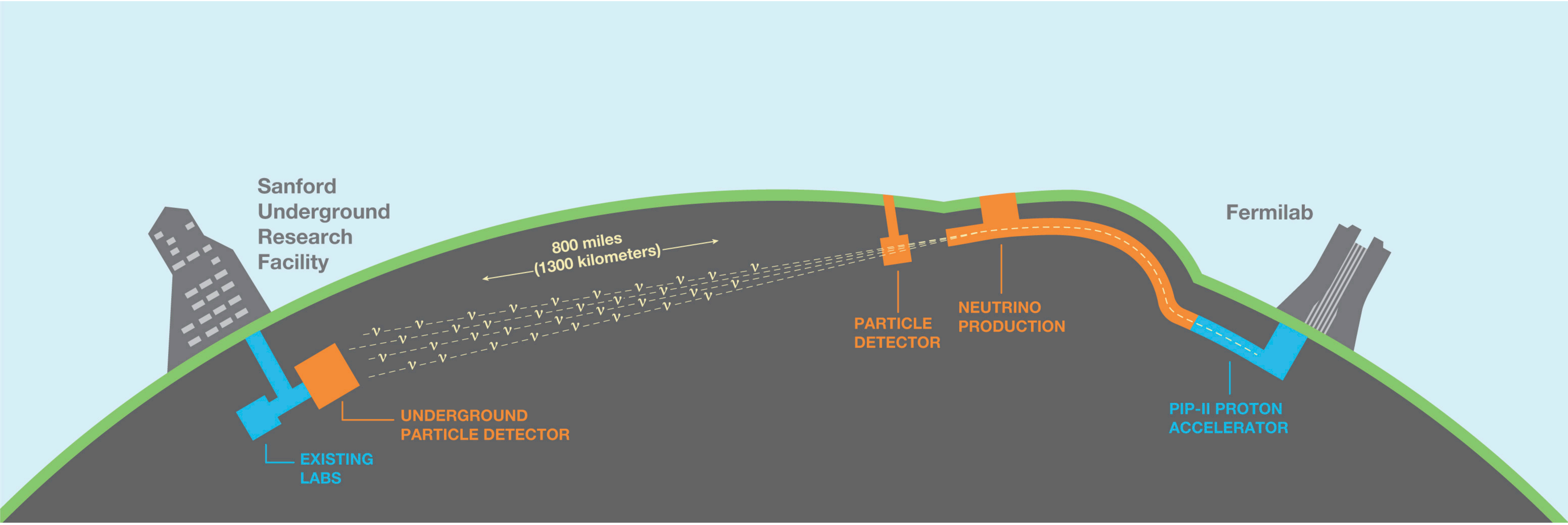
2. M. Horoi, "On the MSW-like neutrino mixing effects in atomic weak interactions and double beta decays", EPJA 56, 39 (2020), https://doi.org/10.1140/epja/s10050-020-00042-x .

M. Horoi and A Zettel, “Effects of Atomic-Scale Electron Density Profile and a Fast and Efficient Iteration Algorithm for Matter Effect of Neutrino Oscillation“, Universe 6, 16 (2020).

$$m_{\beta\beta} = \sum_{k=0}^{N} U_{ek}^2 m_k$$

Do we really know U_{ek} ?

DUNE/LBNF

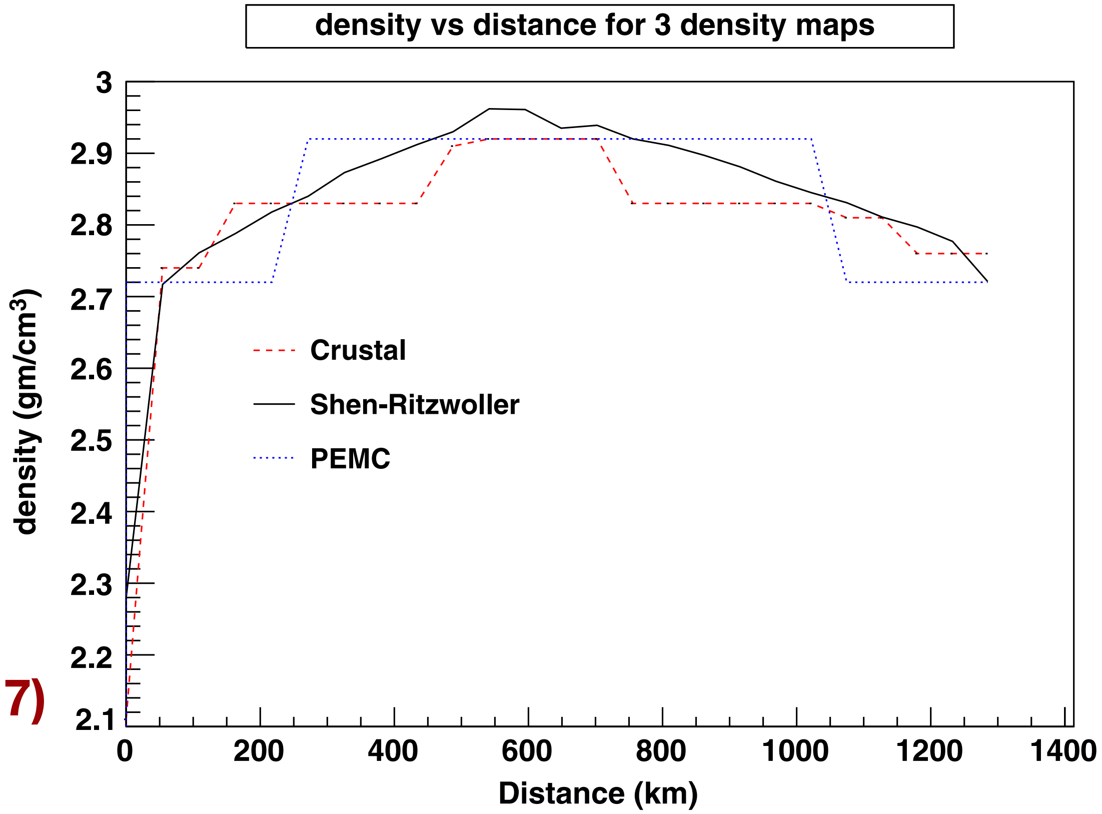


Neutrino oscillations through matter

Matter acts as an optical potential

Goals: $U_{PMNS}\left(\theta_{12}, \theta_{13}, \theta_{23}, \delta_{CP}\right)$

$\left(m_1, m_2, m_3\right)\ vs\ \left(m_3, m_1, m_2\right)$



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B. Roe, Phys. Rev. D 95113004 (2017)

Neutrinos traveling in matter:

Coupled Dirac equations for neutrino mass-eigenstates:

$$i\frac{d}{dt}\begin{pmatrix}\psi_1\\ \psi_2\\ \psi_3\end{pmatrix} = \left[\begin{pmatrix}p_x\alpha_x + m_1\beta & 0 & 0\\ 0 & p_x\alpha_x + m_2\beta & 0\\ 0 & 0 & p_x\alpha_x + m_3\beta\end{pmatrix} + U^\dagger\begin{pmatrix}V_e(x)+V_N & 0 & 0\\ 0 & V_N & 0\\ 0 & 0 & V_N\end{pmatrix}U\right]\begin{pmatrix}\psi_1\\ \psi_2\\ \psi_3\end{pmatrix}$$

In-matter neutrino optical potential:

$$V_e(eV) = \pm\sqrt{2}G_F N_e \approx \pm 1.26\times 10^{-37} N_e\ (cm^{-3})$$

$$V_N(eV) \approx \mp G_F N_n/\sqrt{2} \approx \mp 6.3\times 10^{-38} N_n\ (cm^{-3})$$

Reduction to a "time-dependent" Schroedinger-like equation for amplitudes:

$$i\frac{d}{dx}\begin{pmatrix}\nu_e\\ \nu_\mu\\ \nu_\tau\end{pmatrix} = \left[U\begin{pmatrix}0 & 0 & 0\\ 0 & \Delta m^2_{21}/2E & 0\\ 0 & 0 & \Delta m^2_{31}/2E\end{pmatrix}U^\dagger + \begin{pmatrix}V_e(x) & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & 0\end{pmatrix}\right]\begin{pmatrix}\nu_e\\ \nu_\mu\\ \nu_\tau\end{pmatrix} \equiv H(x)\begin{pmatrix}\nu_e\\ \nu_\mu\\ \nu_\tau\end{pmatrix}$$

Condition:

$$\lambda \ll |\ V(x)/(dV/dx)\ |$$

Amplitudes:

$$\psi_f = \sum_{a=1,2,3}\psi_a = \sum_{a=1,2,3}\nu_{f,a}\phi_a$$

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Neutrinos traveling in matter:

Constant electron density: the eigenvalues method

$$HU^m = U^m M \rightarrow \left[U \begin{pmatrix} 0 & 0 & 0 \\ 0 & \Delta m_{21}^2/2E & 0 \\ 0 & 0 & \Delta m_{31}^2/2E \end{pmatrix} U^\dagger + \begin{pmatrix} V_e & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \right] U^m = U^m \begin{pmatrix} 0 & 0 & 0 \\ 0 & \Delta M_{21}^2/2E & 0 \\ 0 & 0 & \Delta M_{31}^2/2E \end{pmatrix}$$

The flavor oscillation probability becomes:

$$P_{\alpha\rightarrow\beta} = \delta_{\alpha\beta} - 4\sum_{i>j} Re\left({U^m}^*_{\alpha i} U^m{}_{\beta i} U^m{}_{\alpha j} {U^m}^*_{\beta j}\right) sin^2\left(\frac{\Delta M_{ij}^2 L}{4E}\right) + 2\sum_{i>j} Im\left({U^m}^*_{\alpha i} U^m{}_{\beta i} U^m{}_{\alpha j} {U^m}^*_{\beta j}\right) sin^2\left(\frac{\Delta M_{ij}^2 L}{2E}\right)$$

Take the case of two flavors: $U = \begin{pmatrix} cos\theta & sin\theta \\ -sin\theta & cos\theta \end{pmatrix} \rightarrow U^m = \begin{pmatrix} cos\theta_m & sin\theta_m \\ -sin\theta_m & cos\theta_m \end{pmatrix}$

$$\Delta M_{21}^2 = \Delta m_{21}^2 \sqrt{\left(cos2\theta - 2V_e E/\Delta m_{21}^2\right)^2 + sin^2 2\theta} \qquad sin2\theta_m = \frac{\Delta m_{21}^2}{\Delta M_{21}^2} sin2\theta$$

$$P_{e\rightarrow\mu,\tau} = sin^2 2\theta_m sin^2\left(\frac{\Delta M_{21}^2 L}{4E}\right)$$

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Neutrinos traveling in matter:

Constant electron density: the eigenvalues method

$$HU^m = U^m M \rightarrow \left[U \begin{pmatrix} 0 & 0 & 0 \\ 0 & \Delta m^2_{21}/2E & 0 \\ 0 & 0 & \Delta m^2_{31}/2E \end{pmatrix} U^\dagger + \begin{pmatrix} V_e & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \right] U^m = U^m \begin{pmatrix} 0 & 0 & 0 \\ 0 & \Delta M^2_{21}/2E & 0 \\ 0 & 0 & \Delta M^2_{31}/2E \end{pmatrix}$$

3 flavors: no compact solution

Perturbations approach: to get an idea here is one of them

- H. Minakata and S. J. Parke, JHEP 01, 180 (2016)

$$\begin{aligned}
&P(\nu_\alpha \rightarrow \nu_\beta) = \delta_{\alpha\beta} \\
&+ 4 \left[\{A^{\alpha\beta}_{+-}\}\ s^2_\phi c^2_\phi + \epsilon\ \{B^{\alpha\beta}_{+-}\}\ (J_r \cos\delta)\ \frac{(\Delta m^2_{\text{ren}})^2 \{(\lambda_+ - \lambda_-) - (\Delta m^2_{\text{ren}} - a)\}}{(\lambda_+ - \lambda_-)^2 (\lambda_+ - \lambda_0)} \right] \sin^2 \frac{(\lambda_+ - \lambda_-)L}{4E} \\
&+ 4 \left[\{A^{\alpha\beta}_{+0}\}\ c^2_\phi\ + \epsilon\ \{B^{\alpha\beta}_{+0}\}\ (J_r \cos\delta / c^2_{13})\ \frac{\Delta m^2_{\text{ren}} \{(\lambda_+ - \lambda_-) - (\Delta m^2_{\text{ren}} + a)\}}{(\lambda_+ - \lambda_-)(\lambda_+ - \lambda_0)} \right] \sin^2 \frac{(\lambda_+ - \lambda_0)L}{4E} \\
&+ 4 \left[\{A^{\alpha\beta}_{-0}\}\ s^2_\phi\ + \epsilon\ \{B^{\alpha\beta}_{-0}\}\ (J_r \cos\delta / c^2_{13})\ \frac{\Delta m^2_{\text{ren}} \{(\lambda_+ - \lambda_-) + (\Delta m^2_{\text{ren}} + a)\}}{(\lambda_+ - \lambda_-)(\lambda_- - \lambda_0)} \right] \sin^2 \frac{(\lambda_- - \lambda_0)L}{4E} \\
&+ 8\epsilon\ J_r\ \frac{(\Delta m^2_{\text{ren}})^3}{(\lambda_+ - \lambda_-)(\lambda_+ - \lambda_0)(\lambda_- - \lambda_0)}\ \sin \frac{(\lambda_+ - \lambda_-)L}{4E}\ \sin \frac{(\lambda_- - \lambda_0)L}{4E} \\
&\qquad \times \left[\{C^{\alpha\beta}\}\ \cos\delta\ \cos \frac{(\lambda_+ - \lambda_0)L}{4E} + \{S^{\alpha\beta}\}\ \sin\delta\ \sin \frac{(\lambda_+ - \lambda_0)L}{4E} \right], \qquad (3.17)
\end{aligned}$$

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Neutrinos traveling in matter:

Integration method: rewrite the time-dependent Schroedinger eq. in dimensionless form

$$i\frac{d}{ds}\begin{pmatrix}\nu_e\\ \nu_\mu\\ \nu_\tau\end{pmatrix} = \left[U\begin{pmatrix}0&0&0\\0&\alpha&0\\0&0&\gamma\end{pmatrix}U^\dagger + \begin{pmatrix}A(s)&0&0\\0&0&0\\0&0&0\end{pmatrix}\right]\begin{pmatrix}\nu_e\\ \nu_\mu\\ \nu_\tau\end{pmatrix} \equiv H(s)\begin{pmatrix}\nu_e\\ \nu_\mu\\ \nu_\tau\end{pmatrix} \equiv \left[UD_1U^\dagger + D_2\right]\begin{pmatrix}\nu_e\\ \nu_\mu\\ \nu_\tau\end{pmatrix}$$

Definition of the dimensionless variables: $\alpha = \delta m^2_{21}/|\delta m^2_{31}|$ $\gamma = \delta m^2_{31}/|\delta m^2_{31}|$

$s = x/x_u$

$x_u = (2E\hbar c)/|\delta m^2_{31}|$ the unit length

$A(s) = 2EV_e(x)/|\delta m^2_{31}| \propto N_e(x)$

The S-matrix and probability: $S(s) = Te^{-i\int_0^s H(s')ds'}$

$$P_{\beta\to\alpha} = \left|S_{\alpha\beta}(s)\right|^2$$

An iterations approach: $S(s) = \prod_{i=1}^{N} S(\Delta s_i)$

The piece-wise S-matrix formula: $S(\Delta s_i) = e^{-i\Delta s_i D_2(s_i)}Ue^{-i\Delta s_i D_1}U^\dagger$

TC-May20

Neutrinos traveling in matter:

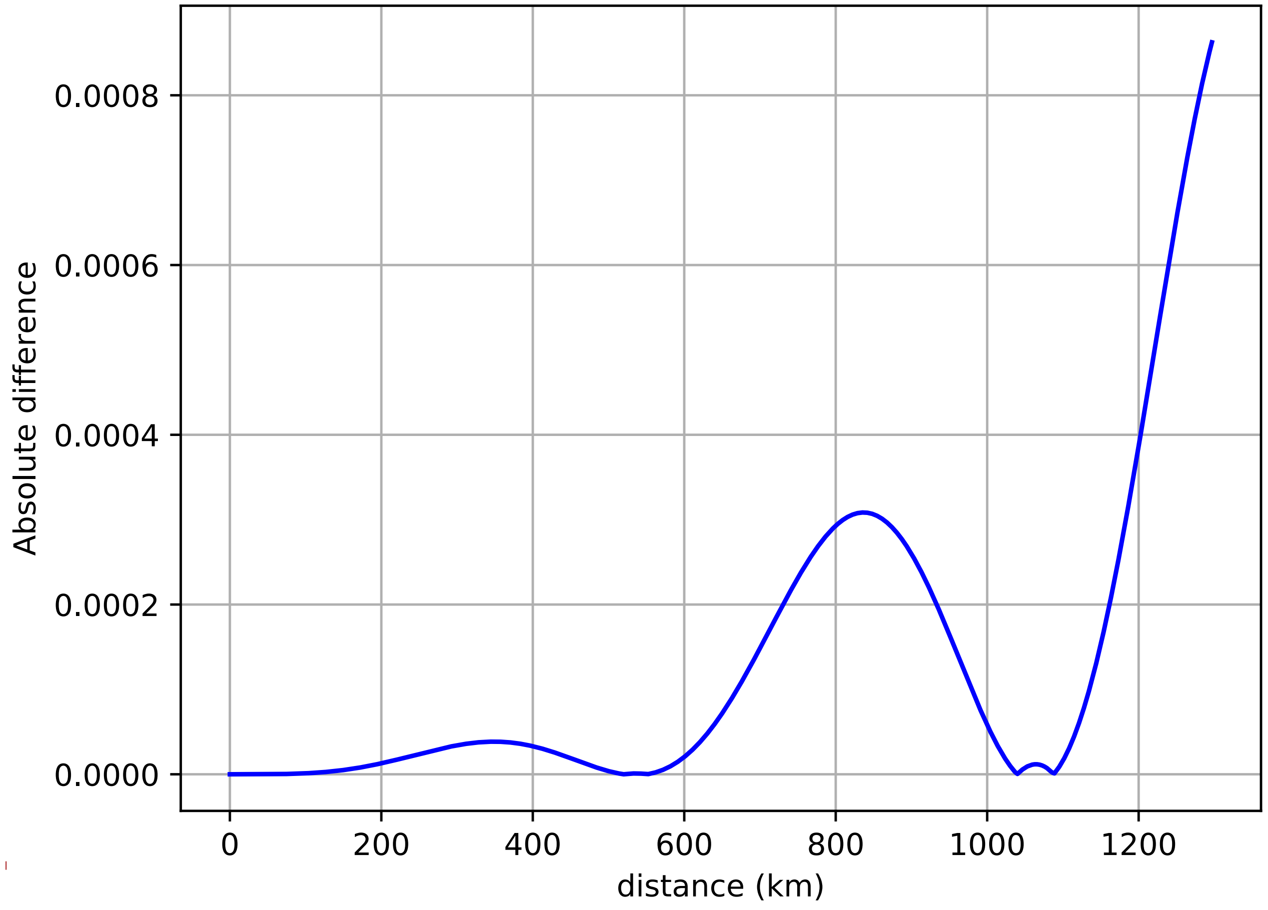
The iterations approach:

$$S(s) = \prod_{i=1}^{N} S(\Delta s_i) \qquad P_{\beta\to\alpha} = \left| S_{\alpha\beta}(s) \right|^2$$

$$\boxed{S(\Delta s_i) = U_A(s_i) U U_f U^\dagger}$$

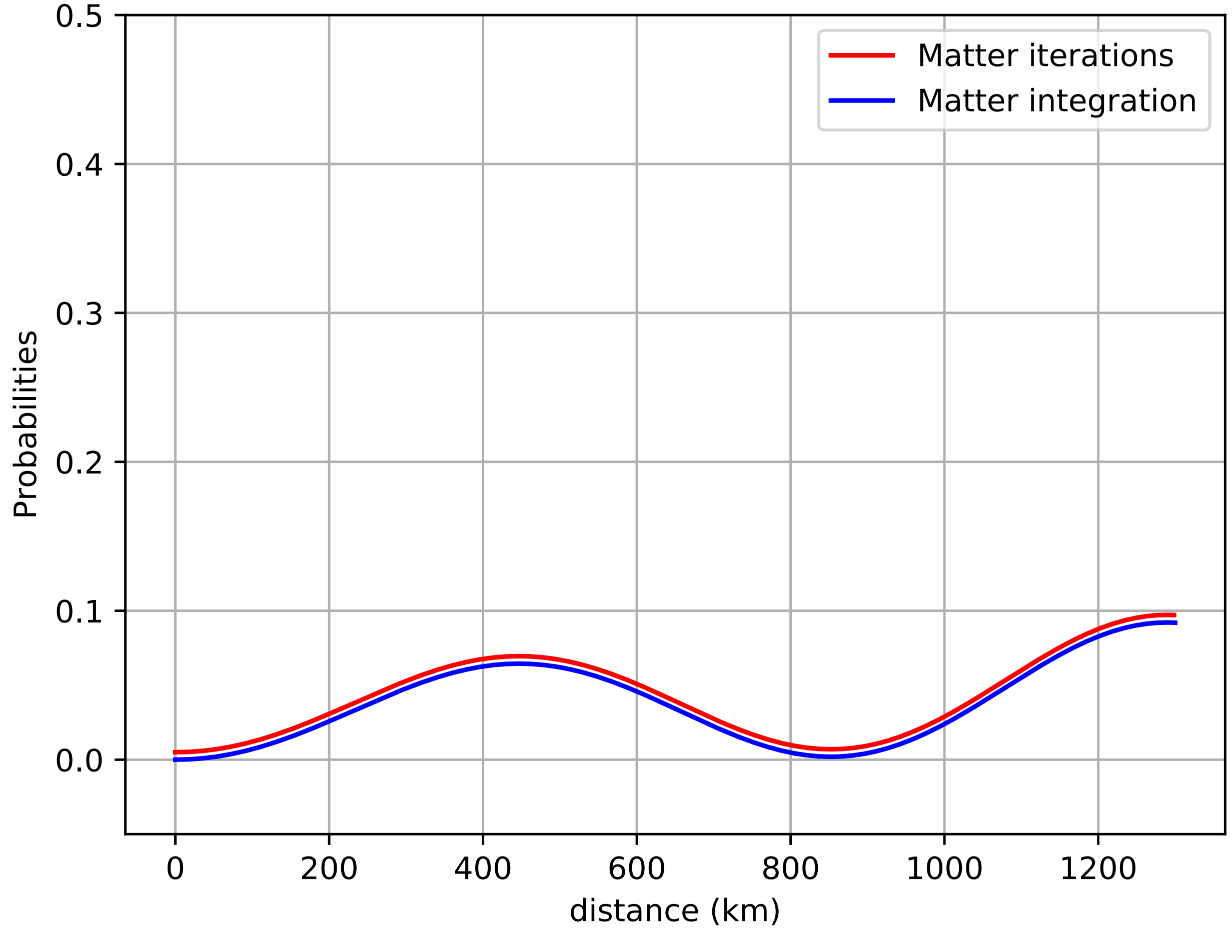
$$U_A(s_i) \equiv e^{-i\Delta s_i D_2(s_i)} = \begin{pmatrix} e^{-i\Delta s_i A(s_i)} & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \qquad U_f \equiv e^{-i\Delta s_i D_1} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{-i\Delta s_i \alpha} & 0 \\ 0 & 0 & e^{-i\Delta s_i \gamma} \end{pmatrix}$$

$N \approx 15$ is enough for good accuracy:



$P_{\nu_\mu \to \nu_e}$

Method works well for the Earth's crust variable density:



Neutrinos in atomic nuclei

Atomic nucleus is a high electron density medium:

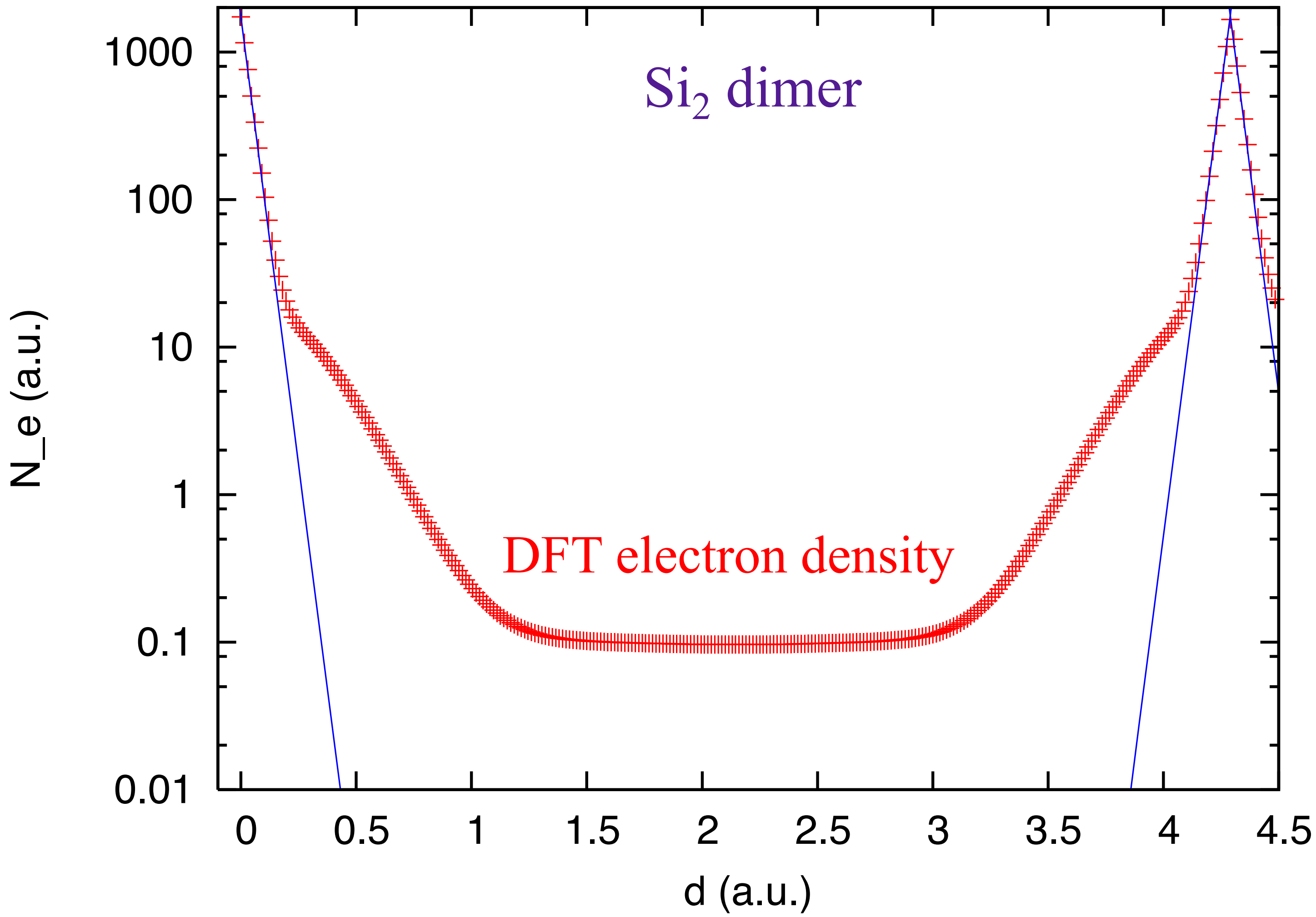
Consider 2 electrons in the lowest s-orbital of an Hydrogen-like atom

Electron density near nucleus:

$$N_e(r) \approx \frac{2}{\pi}\left(\frac{Z}{a_B}\right)^3 e^{-2rZ/a_B}$$

Electron density inside nucleus:

$$N_e(0) \approx \frac{2}{\pi}\left(\frac{Z}{a_B}\right)^3$$



2 x 1s el. Hydrogen-like density

$$\rho_{Suncore} \approx 150\ g/cm^3$$

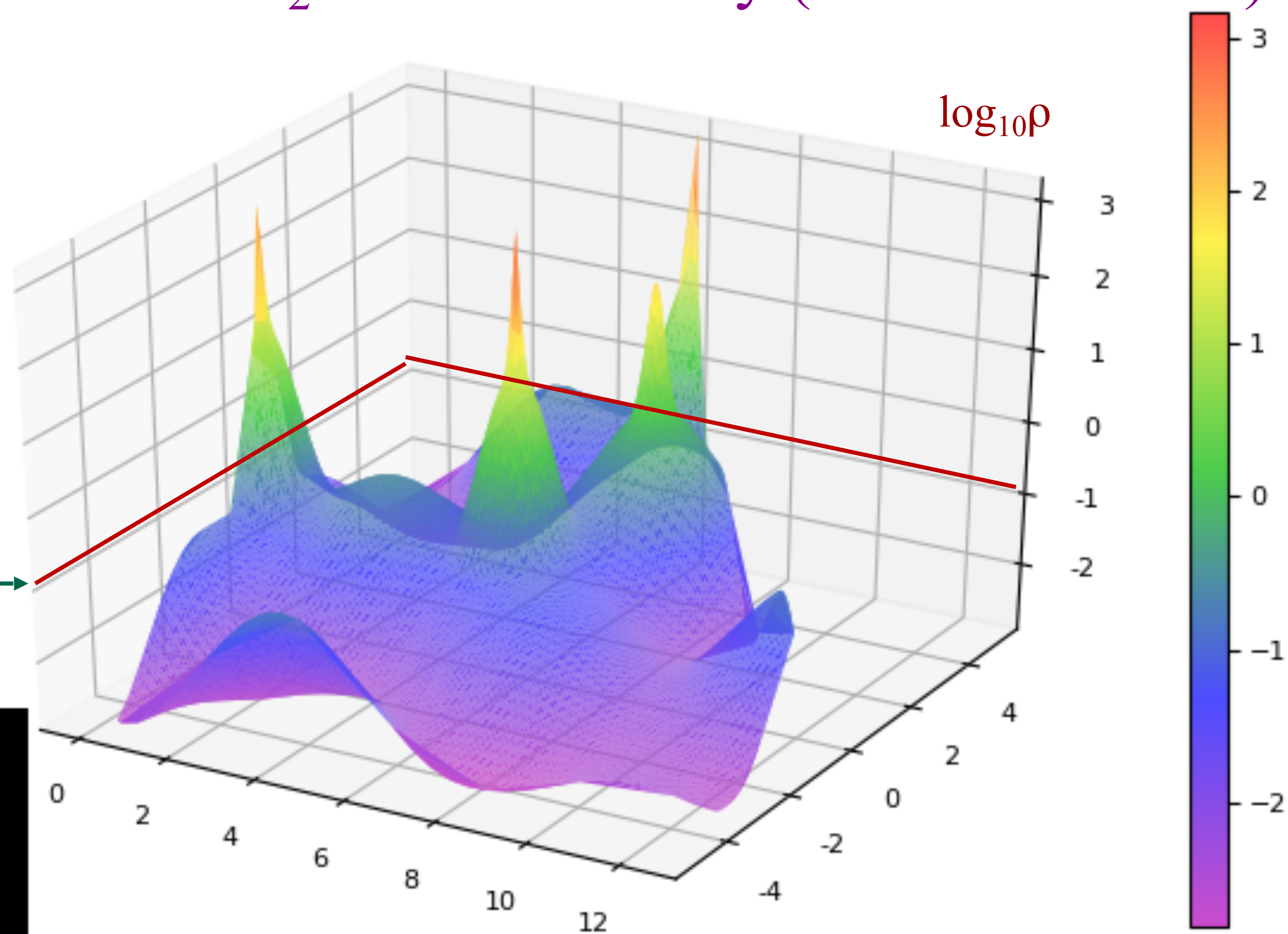
Equivalent matter density: $\rho = m_N N_e = 1.67 \times 10^6 \frac{2}{\pi}\left(\frac{Z}{53}\right)^3$ *in* $g/cm^2 >> \rho_{Sun}$

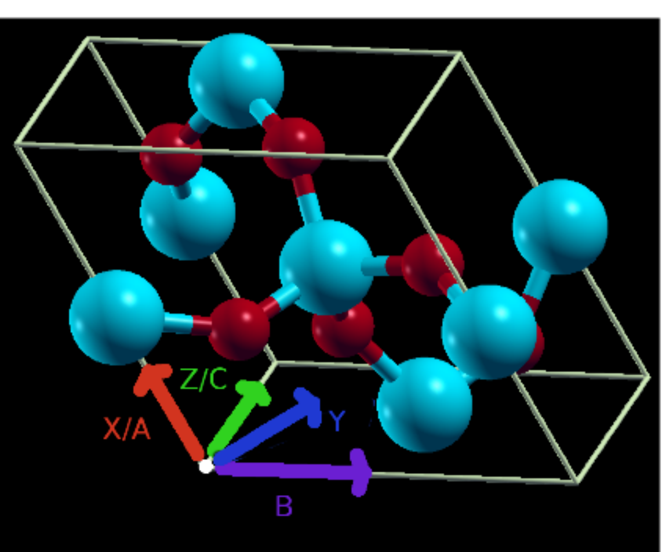
Matter effects in neutrino oscillations

Electron density unevenly distributed in condensed matter: spikes

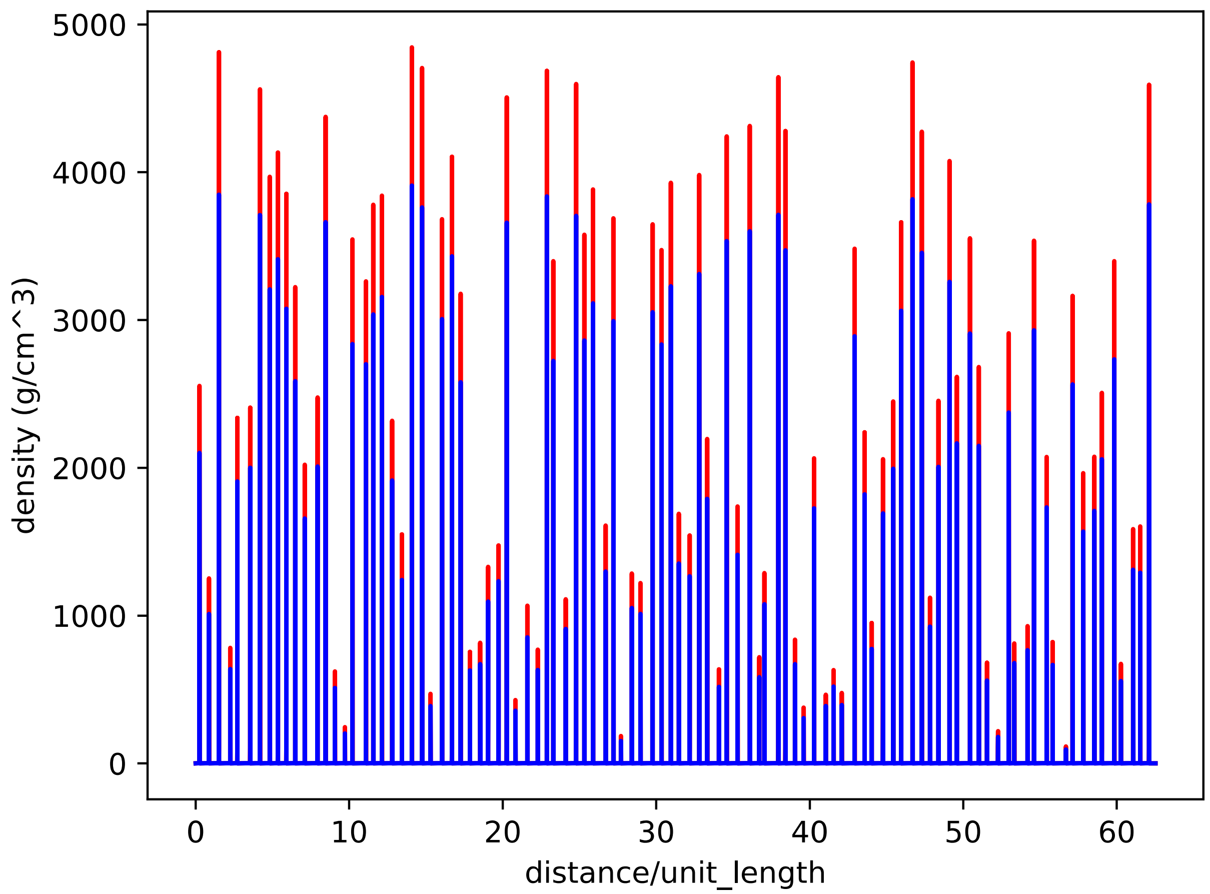
DFT calculations of SiO_2 electron density (all atomic units)

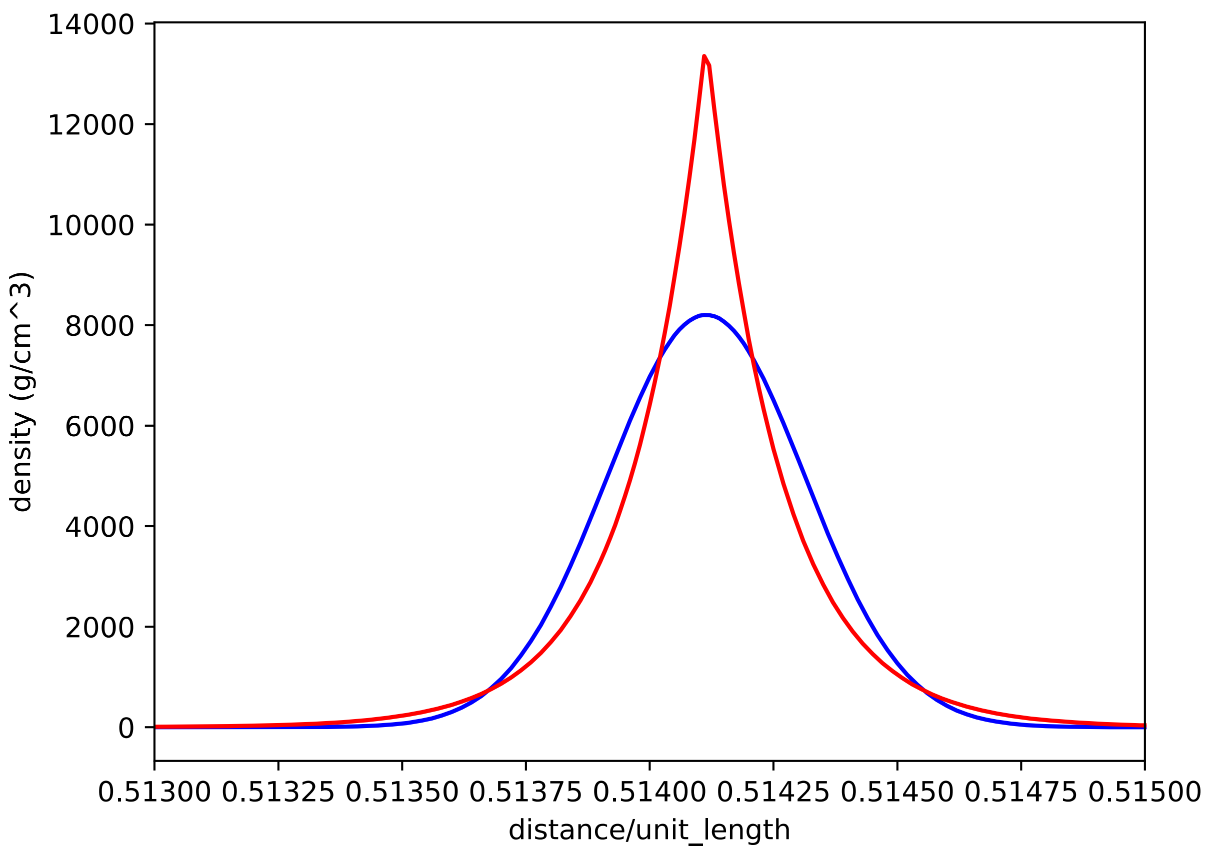
Average flat density used in matter effects

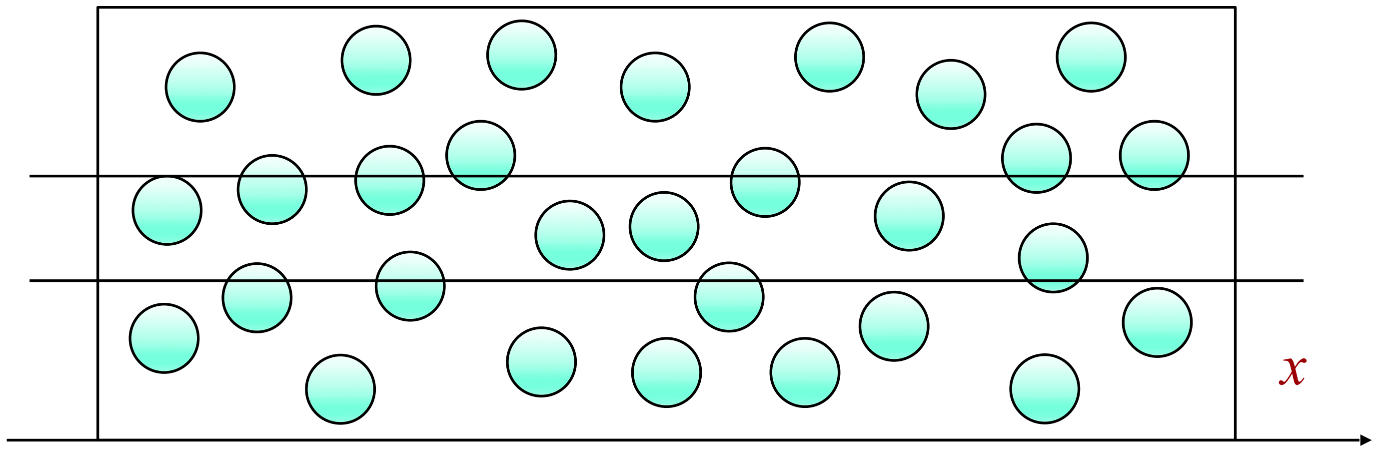




Matter density model







- Different spike shapes produce the same result
- The 3D topology of atoms can be simulated in 1D with random spikes
- Actual density is a mixture: $\rho_{mixed\_spikes}=0.6\rho_{spikes}+0.4\rho_{flat}$
- $\rho_{ave}=\rho_{flat}=3.8\ g/cm^3$ (PREM)

Are there any effects of the spikes in the electron density?

Apparently yes for very long baseline in neutrino oscillations!

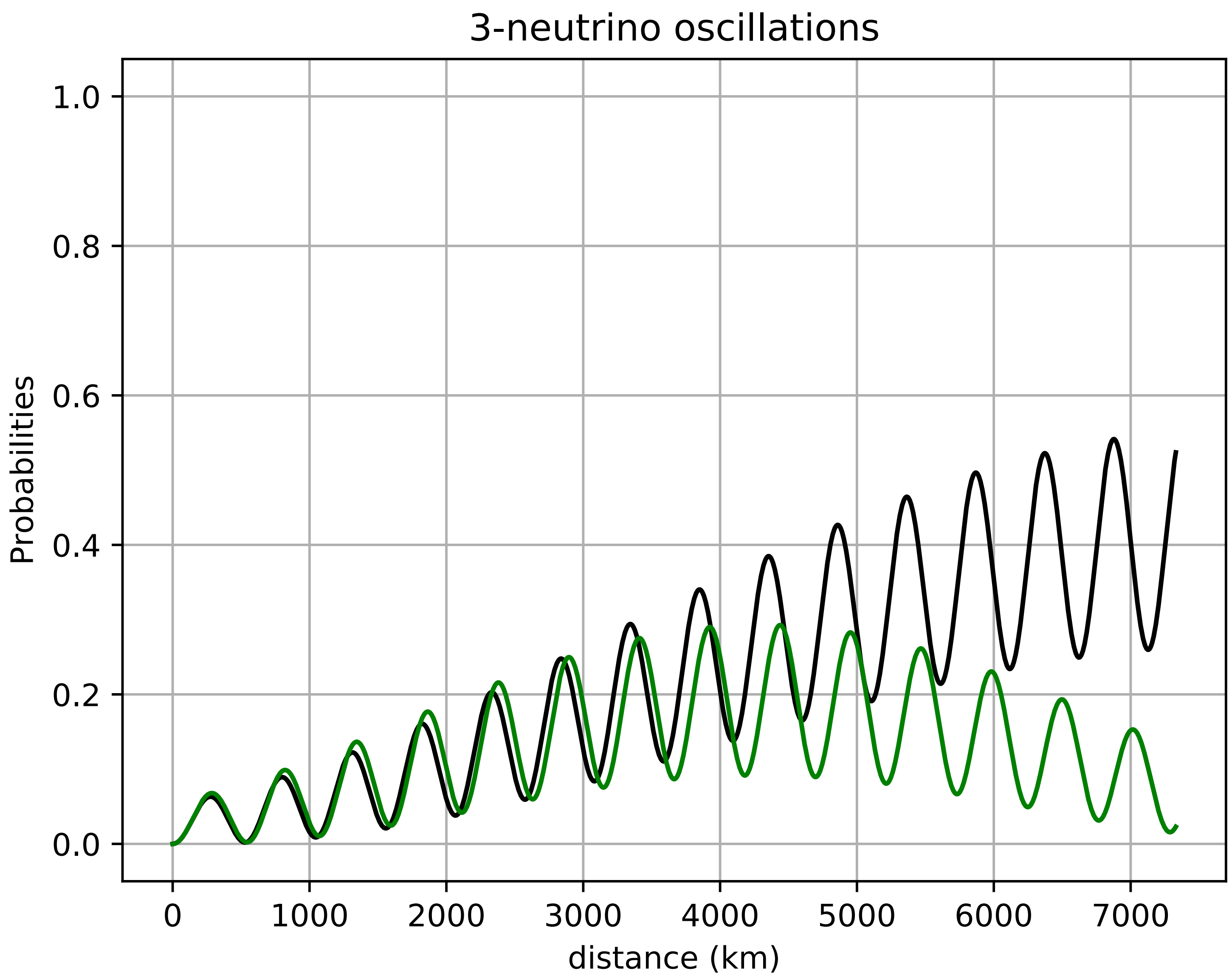
$$E_{\nu_\mu} = 0.50\ GeV$$

$$i\frac{d}{ds}\begin{pmatrix}\nu_e\\ \nu_\mu\\ \nu_\tau\end{pmatrix} = \left[U\begin{pmatrix}0&0&0\\0&\alpha&0\\0&0&\gamma\end{pmatrix}U^\dagger + \begin{pmatrix}A(s)&0&0\\0&0&0\\0&0&0\end{pmatrix}\right]\begin{pmatrix}\nu_e\\ \nu_\mu\\ \nu_\tau\end{pmatrix} \equiv H(s)\begin{pmatrix}\nu_e\\ \nu_\mu\\ \nu_\tau\end{pmatrix}$$

$$A(s) = 2EV_e(x)/|\delta m^2_{31}| \propto N_e(x)$$

$$V_e(x) = \pm 1.26 \times 10^{-37} N_e(x)\ (cm^{-3})$$

$$P_{\nu_\mu \to \nu_e} = \left|\nu_e\right|^2$$



$P_{\nu_\mu \to \nu_e}$ mixed-spikes

$P_{\nu_\mu \to \nu_e}$ flat

-> Gran Sasso or

CERN -> Sanford

Neutrinos traveling in matter:

Integration method: rewrite the time- dependent Schroedinger eq. in dimensionless form

$$i\frac{d}{ds}\begin{pmatrix}\nu_e\\ \nu_\mu\\ \nu_\tau\end{pmatrix} = \left[U\begin{pmatrix}0 & 0 & 0\\ 0 & \alpha & 0\\ 0 & 0 & \gamma\end{pmatrix}U^\dagger + \begin{pmatrix}A(s) & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & 0\end{pmatrix}\right]\begin{pmatrix}\nu_e\\ \nu_\mu\\ \nu_\tau\end{pmatrix} \equiv H(s)\begin{pmatrix}\nu_e\\ \nu_\mu\\ \nu_\tau\end{pmatrix} \equiv \left[UD_1U^\dagger + D_2\right]\begin{pmatrix}\nu_e\\ \nu_\mu\\ \nu_\tau\end{pmatrix}$$

Consider $\Delta s_i \ll 1$ and $A(s) = \Delta s_i \bar{A}(s_i)\delta(s)$ near electron density spikes:

$$i\frac{d\nu_e(s)}{ds} = A(s)\nu_e(s) \qquad \Longrightarrow \qquad \nu_e(s_a) = e^{-i\Delta s_i \bar{A}(s_i)}\nu_e(s_b)$$

The contribution to $S(\Delta s_i)$ through the Dirac delta potential:

$$U_A(s_i) \equiv e^{-i\Delta s_i D_2(s_i)} = \begin{pmatrix}e^{-i\Delta s_i \bar{A}(s_i)} & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{pmatrix}$$

Therefore, the piece-wise S-matrix formula is the same:

$$\boxed{S(\Delta s_i) = U_A(s_i)UU_fU^\dagger}$$

$$S(s) = \prod_{i=1}^{N} S(\Delta s_i)$$

$$P_{\beta\to\alpha} = \left|S_{\alpha\beta}(s)\right|^2$$

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Conclusions

- We presented a fast and reliable algorithm to calculate the neutrino oscillation probabilities through matter of varying density (more info in Universe **6**, 16 (2020)).
  - The algorithm was extended to the case where the sterile neutrinos are present.
- We use this algorithm to show that the electron density spikes near the atomic nuclei can be treated as a local average density.
  - This statement can be extended to the neutron density spikes contributing to V_N (needed if the sterile neutrinos are present).
- **Related:** we showed (EPJA **56**, 39 (2020)) that the large neutrino optical potential due to the electron density spikes in the atomic nuclei does not affect the neutrinoless double-beta probability for the mass mechanism.