Neutrino properties and double beta decay

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TC-May20

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CMU Deliverables

Peer-reviewed journals:

1. F. Ahmed and M. Horoi, "Interference Effects for 0vββ Decay in the Left-Right Symmetric Model", Phys. Rev. C 101, 035504 (2020), https://doi.org/10.1103/PhysRevC.101.035504.

2. M. Horoi, "On the MSW-like neutrino mixing effects in atomic weak interactions and double beta decays", EPJA 56, 39 (2020), https://doi.org/10.1140/epja/s10050-020-00042-x .

3. C. F. Jiao, M. Horoi, and A. Neacsu, "Neutrinoless double-decay of ¹²⁴Sn, ¹³⁰Te, and ¹³⁶Xe in the Hamiltonian-based generatorcoordinate method", Phys. Rev. C 98, 064324, https://doi.org/10.1103/PhysRevC.98.064324.

4. M. Horoi and A. Neacsu, "Shell model study of using an effective field theory for disentangling several contributions to neutrinoless double-beta decay", Phys. Rev. C 98, 035502 (2018), https://doi.org/10.1103/PhysRevC.98.035502.

5. F. Ahmed, A. Neacsu, and M. Horoi, "Interference between light and heavy neutrinos for 0vββ decay in the left–right symmetric model", Physics Letters B 769, 299–304 (2017), <u>https://doi.org/10.1016/j.physletb.2017.03.066</u>.

6. A. Neacsu and M. Horoi, "Shell Model Studies of Competing Mechanisms to the Neutrinoless Double-Beta Decay in ¹²⁴Sn, ¹³⁰Te, and ¹³⁶Xe", Advances in High Energy Physics 2016, 1903767 (2016), https://doi.org/10.1155/2016/1903767.

Peer-reviewed Proceedings:

7. M. Horoi, "Neutrinoless double beta decay of atomic nuclei", AIP Proceedings **2165**, 020012 (2019).

8. M. Horoi, "Nuclear Structure for Double Beta Decay", in 12th INTERNATIONAL SPRING SEMINAR ON NUCLEAR PHYSICS, Ischia, May 15-19, 2017, volume 966 of J. Phys.: Conf. Series, page 012009, (2018), https://doi.org/10.1088/1742-6596/966/1/012009.

9. M. Horoi, "Towards a complete description of the neutrinoless double beta decay", in Matrix Elements for the Double beta decay EXperiments: MEDEX'17, Prague, June 1-6, 2017, volume 1894 of AIP Conference Proceedings, page 020011, (2017), https://doi.org/ 10.1063/1.5007636.

3. C. F. Jiao, M. Horoi, and A. Neacsu, "Neutrinoless double-decay of ¹²⁴Sn, ¹³⁰Te, and ¹³⁶Xe in the Hamiltonianbased generator-coordinate method", Phys. Rev. C 98, 064324, https://doi.org/10.1103/PhysRevC.98.064324.





FIG. 5. The differences of Gamow-Teller part of NMEs between our GCM and SM calculations against the pair-spin I for ¹²⁴Sn, ¹³⁰Te, and ¹³⁶Xe.

GCM and Shell Model give similar NME if the same model spaces and Hamiltonians are used

4. M. Horoi and A. Neacsu, "Shell model study of using an effective field theory for disentangling several contributions to neutrinoless double-beta decay", Phys. Rev. C 98, 035502 (2018), https://doi.org/10.1103/PhysRevC.98.035502.



and A. Neacsu, "Shell model study of using an effective field theory for disentangling several CENTRAL MICHIGANs to neutrinoless double-beta decay", Phys. Rev. C 98, 035502 (2018), https://doi.org/10.1103/ PhysRevC.98.035502.

PHYSICAL REVIEW D 92, 036005 (2015)



 $\eta_N \propto \frac{I}{m_{W_R}^4 m_N}$



$$\mathcal{L}_D = \frac{g}{\left(\Lambda_D\right)^{D-4}} \mathcal{O}_D$$

$$m_e \bar{\epsilon}_5 = \frac{g^2 (yv)^2}{\Lambda_5}, \qquad \frac{G_F \bar{\epsilon}_7}{\sqrt{2}} = \frac{g^3 (yv)}{2(\Lambda_7)^3}, \\ \frac{G_F^2 \bar{\epsilon}_9}{2m_p} = \frac{g^4}{(\Lambda_9)^5}, \qquad \frac{G_F^2 \bar{\epsilon}_{11}}{2m_p} = \frac{g^6 (yv)^2}{(\Lambda_{11})^7}$$

TABLE VIII. The BSM effective scale (in GeV) for different dimension-D operators at the present ¹³⁶Xe half-life limit (Λ_D^0) and for $T_{1/2} \approx 1.1 \times 10^{28}$ years (Λ_D) .

\mathcal{O}_D	$ar{\epsilon}_D$	$\Lambda_D^0(y=1)$	$\Lambda_D^0(y=y_e)$	$\Lambda_D(y=y_e)$
\mathcal{O}_5	$2.8 \cdot 10^{-7}$	$2.12\cdot 10^{14}$	1904	19044
\mathcal{O}_7	$2.0 \cdot 10^{-7}$	$3.75\cdot 10^4$	541	1165
\mathcal{O}_9	$1.5 \cdot 10^{-7}$	$2.47 \cdot 10^3$	2470	3915
\mathcal{O}_{11}	$1.5 \cdot 10^{-7}$	$1.16 \cdot 10^3$	31	43

 $g \approx 1$ v = 174 GeV $y_e = 3 \times 10^{-6}$ electron mass Yukawa



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gram a We the state on grange component of the $0\nu\beta\beta$ diaexchange a light neutrino. In this case, the Lagrangian can be expressed in terms of effective couplings [15]:

$$T[^{76}Ge]/T[^{A}Z]$$

$$\mathcal{L}_{6} = \frac{G_{F}}{\sqrt{2}} \left[j_{V-A}^{\mu} J_{V-A,\mu}^{\dagger} + \sum_{\alpha,\beta}^{*} \epsilon_{\alpha}^{\beta} j_{\beta} J_{\alpha}^{\dagger} \right], \quad (2)$$

where $J^{\dagger}_{\alpha} = \bar{u} \mathcal{O}_{\alpha} d$ and $j_{\beta} = \bar{e} \mathcal{O}_{\beta} \nu$ are hadronic and and leptonic Lorentz currents, respectively The definitionitions of the $\mathcal{O}_{\alpha,\beta}$ operators are given in Eq. (3) of Ref. [15]. 7 The LNV parameters are $\epsilon_{\alpha}^{\beta} = \{\epsilon_{V-AV-A}^{V+AV+A}, \epsilon_{V+A}^{V+A}, \epsilon_{S\pm P}^{S+P}, \epsilon_{TL}^{TR}, \epsilon_{TR}^{TR}\}$. The symbol indicates that the term with $\alpha = \beta = (V - A)$ is explicitly takehaken out of the sum. $G_F = 1.1663787 \times 10^{-5} \text{ GeV}^{-2}$ denotes the Fermi² coupling constant.

The The $0\nu\beta\beta$ decay amplitude is proportional to the timeordered product of two effective Lagrangians [15]: -

$$T_{J}(\mathcal{L}_{6}^{\langle I \rangle}(\mathcal{L}_{6}^{\langle 1 \rangle}(\mathcal{L}_{6}^{\langle 2 \rangle}))) = \frac{G_{F}^{2}}{22} T \left[j_{V-A} J_{V-A}^{\dagger} j_{V-A} j_{V-A} J_{V-A}^{\dagger} + \epsilon_{\alpha}^{\beta} \epsilon_{\gamma}^{\delta} j_{\beta} J_{\alpha}^{\dagger} j_{\delta} J_{\gamma}^{\dagger} \right].$$
(3)
+ $\epsilon_{\alpha}^{\beta} j_{\beta} J_{\alpha}^{\dagger} j_{V-A} J_{V-A}^{\dagger} + \epsilon_{\alpha}^{\beta} \epsilon_{\gamma}^{\delta} j_{\beta} J_{\alpha}^{\dagger} j_{\delta} J_{\gamma}^{\dagger} \right].$ (3)
- $\eta_{0\nu} \quad \mathcal{E}_{V+A}^{V+A} \quad \mathcal{E}_{V-A}^{V+A} \quad \mathcal{E}_{S\pm P}^{S+P} \quad \mathcal{E}_{TR}^{TR} \quad \eta_{\pi\nu}$

Super-NEMO TC Collab. UNC, Feb 2,2018

ε₁

arxiv:1801.04496

E^{RR}₃

E^{LR}₃

 $\left[T_{1/2}^{0\nu}\right]^{-1} = g_A^4 \left[\sum_i \left|\mathcal{E}_i\right|^2 \mathcal{M}_i^2 + \operatorname{Re}\left[\sum_{i \neq j} \bigotimes \mathcal{M}_{ij}\right]\right].$

E₅

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dominates the process) of the Livy parameter 10.1703/

Here, the \mathcal{E}_i contain the neutrino physics parameters, t_{-} imeters, with $\mathcal{E}_1 = \eta_{0\nu}$ representing the exchange of light left-= tht lefthanded neutrinos corresponding to Fig. 2b, $\mathcal{E}_{2-7} = \{\epsilon_{V-A}^{V+A}, \epsilon_{V+A}^{V+A}, \epsilon_{S\pm P}^{S+P}, \epsilon_{TL}^{TR}, \epsilon_{TR}^{TR}, \eta_{\pi\nu}\}$ at the long-range parameters appearing in Figs. It at the longg- 2-7 =longıd $\mathcal{E}_{8-15} = \{\varepsilon_1, \varepsilon_2, \varepsilon_3^{LLz(RRz)}, \varepsilon_3^{LRz(RLz)}, \varepsilon_4^{130}, \varepsilon_{12}^{\varepsilon_1}, \eta_{2\pi}\}$ 2e, and denote the short-range parameters at the quark level in- n- $\{\eta_{2\pi}, \eta_{2\pi}\}$ volved in the diagram of Fig. 2d, 2f, 2g. Following Refs. s. level in-[13–15, 45], we write \mathcal{M}_i^2 as combinations of NME de- eng Refs. scribed in Eqs. (8, 10, 12, 14, and 16) (see also Eq.(18) 3) ME dein the Appendix for the individual NME) and integrated ed Eq.(18)PSF [44] denoted with $G_{01} - G_{09}$. Our values of the 1e egrated PSF are presented in Table I. In some cases the inter- r_{-} of the ference terms $\mathcal{E}_{\alpha}\mathcal{E}_{\beta}\mathcal{M}_{\alpha\beta}$ are small [48] and can be ne- eie interglected. Considering an on-axis approach when extracting the LNV parameters limits, the interference terms are be nenot taken into account in our analysis. In the following, extractwe extract the on-axis values of these parameters using ims are ıg - the most recent experimental limits of the half-lives, as Howing, \mathbf{as} presented in Table I. rs using

lives, as

2,2010

 $C_{\nu\lambda}$, $C_{\nu\eta}$

The precise definitions are definitions at the Model Studies of Competing Mechanisms to the Neutrinoless Double-Beta Decay in ¹²⁴Sn, $C_1 \langle \nu \rangle_{\nu^2}^2 \subset C_1^{30}$ Te, and $C_2^{136} \times C_2^{"}$, Advances in High Energy Physics 2016, 1903767 (2016), **displicitions** (101156), **displicitions** (10116), **displic**

 $C_4 \langle X \rangle_{\lambda^2}^2 G_{M} C_{\overline{4}} \langle X \rangle_{\eta^2}^2 = \langle C_{\overline{4}} \langle X \rangle_{\eta^2}^2$

 $\begin{array}{l} = 6 \end{array} \\ = 6 \end{array} \\ = 6 \end{array} \\ = 6 \end{array} \\ = 8 \bigg{ = 8 } 0 \\ = 8 \end{array} \\ = 8 \bigg{ = 8 } 0 \\ = 8 \end{array} \\ = 8 \bigg{ = 8 } 0 \\ = 8 \end{array} \\ = 8 \bigg{ = 8 } 0 \\ = 8 \end{array} \\ = 8 \bigg{ = 8 } 0 \\ = 8 \end{array} \\ = 8 \bigg{ = 8 } 0 \\ = 8 \end{array} \\ = 8 \bigg{ = 8 } 0 \\ = 8 \end{array} \\ = 8 \bigg{ = 8 } 0 \\ = 8 \\ = 8 \bigg{ = 8 } 0 \\ = 8 \\$ ctorshare presenteer appearented in Fahr Bant Art be context of $\frac{F d\Omega}{2\pi} \int_{-\frac{T+1}{2\pi}}^{\frac{T+1}{2}} \frac{d\Omega}{B(e_1)} \int_{-\frac{T}{2\pi}}^{\frac{T+1}{2}} \frac{d\Omega}{B(e_1)} \int_{-\frac{T}{2\pi}}^{\frac{T}{2}} \frac{d\Omega}{B(e_1)} \frac{d\Omega}{B(e_1)} \int_{-\frac{T}{2\pi}}^{\frac{T}{2}} \frac{d\Omega}{B(e_1)} \frac{d\Omega}{B(e_1)} \int_{-\frac{T}{2\pi}}^{\frac{T}{2}} \frac{d\Omega}{B(e_1)} \frac{d\Omega}{B($ ighthsylattatisht averent e associatevenessociate the neutrino parameters and termination of the former metering and the second where $d\Omega = \frac{1}{2\pi} \frac{1}{d\cos\theta} = \frac{2\pi d\cos\theta}{2\pi}$ R. Arnold \mathcal{U} al. Provide New Physics Models of Neutrino Rest Double Beta Decay with SuperNEMO 150 $\langle \lambda \rangle = |\eta_{\lambda}|$, (6b) $\langle \lambda \rangle = |\eta_{\lambda}|$ B. Energy distributions (6b)В. (6c)(6c)but we leave them in this generic form for the case that the effective of the constraint of the case that the effective of the energy discontribute. For example, and the effective of the energy of the energy of the two outgo-contribution from a mechanism whose amplitude is proceeded of the energy of the energy of the two outgo-portioned with the energy of the energy of the two outgo-the the hirder of the energy of the two outgo-the the hirder of the energy of the two outgo-portioned with the energy of the two outgo-the the hirder of the energy of the two outgo-the the hirder of the energy of the two outgo-the the hirder of the energy of the two outgo-the the hirder of the energy of the two outgo-the the hirder of the two hirder of the energy of the two outgo-the hirder of the hirder of the energy of the two outgo-the hirder of the hirder of the energy of the two outgo-ear matrix elements and the interference phases. The energy of one electron as: presented in Eque energy of the two energy of the energy of the two energy of the t III. $0\nu\beta\beta$ DECAY ELECTRONS $\frac{20}{2}(0\nu)^{3}$ CGS $\frac{1}{2}(-1)^{2}$ CGS $\frac{1}{2}(-1)^{$ TThe expression of the standes is for a f blewhratter if a The differential decay rates of the 0 prove of the origination of the transmission of the consequence of the origination of th $t = \varepsilon_{e1} - \frac{T}{\varepsilon_{e2}} = \frac{Q_{\beta\beta}}{m_e c^2}. \qquad T = \frac{Q_{\beta\beta}}{m_e c^2}.$



2. M. Horoi, "On the MSW-like neutrino mixing effects in atomic weak interactions and double beta decays", EPJA 56, 39 (2020), https://doi.org/10.1140/epja/s10050-020-00042-x .

MCENTRAL MICHIGANttel, "Effects of Atomic-Scale Electron Density Profile and a Fast and Efficient Iteration Algorithm for Matter Effect of Neutrino Oscillation", Universe 6, 16 (2020).

Atomic nucleus is a high electron density medium:

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2. M. Horoi, "On the MSW-like neutrino mixing effects in atomic weak interactions and double beta decays", 9 (2020), https://doi.org/10.1140/epja/s10050-020-00042-x.

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$$m_{\beta\beta} = \sum_{k=0}^{N} U_{ek}^2 m_k$$

Do we really know U_{ek} ?

DUNE/LBNF

Coupled Dirac equations for neutrino mass-eigenstates:

$$i\frac{d}{dt}\begin{pmatrix}\psi_{1}\\\psi_{2}\\\psi_{3}\end{pmatrix} = \begin{bmatrix} p_{x}\alpha_{x} + m_{1}\beta & 0 & 0\\ 0 & p_{x}\alpha_{x} + m_{2}\beta & 0\\ 0 & 0 & p_{x}\alpha_{x} + m_{3}\beta \end{bmatrix} + U^{\dagger} \begin{pmatrix}V_{e}(x) + V_{N} & 0 & 0\\ 0 & V_{N} & 0\\ 0 & 0 & V_{N} \end{pmatrix} U \begin{bmatrix}\psi_{1}\\\psi_{2}\\\psi_{3}\end{pmatrix}$$

In-matter neutrino optical potential:

$$V_e(eV) = \pm \sqrt{2}G_F N_e \approx \pm 1.26 \times 10^{-37} N_e \ (cm^{-3})$$
$$V_N(eV) \approx \mp G_F N_n / \sqrt{2} \approx \mp 6.3 \times 10^{-38} N_n \ (cm^{-3})$$

Reduction to a "time-dependent" Schroedinger-like equation for amplitudes:

$$i\frac{d}{dx}\begin{pmatrix}\nu_{e}\\\nu_{\mu}\\\nu_{\tau}\end{pmatrix} = \begin{bmatrix}U\begin{pmatrix}0 & 0 & 0\\0 & \Delta m_{21}^{2}/2E & 0\\0 & 0 & \Delta m_{31}^{2}/2E\end{pmatrix}U^{\dagger} + \begin{pmatrix}V_{e}(x) & 0 & 0\\0 & 0 & 0\\0 & 0 & 0\end{bmatrix}\begin{bmatrix}\nu_{e}\\\nu_{\mu}\\\nu_{\tau}\end{pmatrix} \equiv H(x)\begin{pmatrix}\nu_{e}\\\nu_{\mu}\\\nu_{\tau}\end{pmatrix}$$

Condition:

Amplitudes:

Constant electron density: the eigenvalues method

$$HU^{m} = U^{m}M \rightarrow \begin{bmatrix} 0 & 0 & 0 \\ 0 & \Delta m_{21}^{2}/2E & 0 \\ 0 & 0 & \Delta m_{31}^{2}/2E \end{bmatrix} U^{\dagger} + \begin{pmatrix} V_{e} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \end{bmatrix} U^{m} = U^{m} \begin{pmatrix} 0 & 0 & 0 \\ 0 & \Delta M_{21}^{2}/2E & 0 \\ 0 & 0 & \Delta M_{31}^{2}/2E \end{pmatrix}$$

The flavor oscillation probability becomes:

$$P_{\alpha \to \beta} = \delta_{\alpha\beta} - 4\sum_{i>j} Re\left(U^{m^*}_{\ \alpha i}U^{m}_{\ \beta i}U^{m}_{\ \beta j}U^{m^*}_{\ \beta j}\right) \sin^2\left(\frac{\Delta M^2_{ij}L}{4E}\right) + 2\sum_{i>j} Im\left(U^{m^*}_{\ \alpha i}U^{m}_{\ \beta i}U^{m}_{\ \beta j}U^{m^*}_{\ \beta j}\right) \sin^2\left(\frac{\Delta M^2_{ij}L}{2E}\right)$$

Take the case of two flavors: $U = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \rightarrow U^m = \begin{pmatrix} \cos\theta_m & \sin\theta_m \\ -\sin\theta_m & \cos\theta_m \end{pmatrix}$

0

$$\Delta M_{21}^2 = \Delta m_{21}^2 \sqrt{\left(\cos 2\theta - 2V_e E/\Delta m_{21}^2\right)^2 + \sin^2 2\theta} \qquad \sin 2\theta_m = \frac{\Delta m_{21}^2}{\Delta M_{21}^2} \sin 2\theta$$
$$P_{e \to \mu, \tau} = \sin^2 2\theta_m \sin^2 \left(\frac{\Delta M_{21}^2 L}{4E}\right)$$

Constant electron density: the eigenvalues method

$$HU^{m} = U^{m}M \rightarrow \begin{bmatrix} 0 & 0 & 0 \\ 0 & \Delta m_{21}^{2}/2E & 0 \\ 0 & 0 & \Delta m_{31}^{2}/2E \end{bmatrix} U^{\dagger} + \begin{pmatrix} V_{e} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \end{bmatrix} U^{m} = U^{m} \begin{pmatrix} 0 & 0 & 0 \\ 0 & \Delta M_{21}^{2}/2E & 0 \\ 0 & 0 & \Delta M_{31}^{2}/2E \end{pmatrix}$$

3 flavors: no compact solution

TC-May20

Perturbations approach: to get an idea here is one of them

$$P(\nu_{\alpha} \rightarrow \nu_{\beta}) = \delta_{\alpha\beta}$$

$$+ 4 \left[\{A_{+-}^{\alpha\beta}\} s_{\phi}^{2} c_{\phi}^{2} + \epsilon \{B_{+-}^{\alpha\beta}\} (J_{r} \cos \delta) \frac{(\Delta m_{\text{ren}}^{2})^{2} \{(\lambda_{+} - \lambda_{-}) - (\Delta m_{\text{ren}}^{2} - a)\}}{(\lambda_{+} - \lambda_{-})^{2} (\lambda_{+} - \lambda_{0})} \right] \sin^{2} \frac{(\lambda_{+} - \lambda_{-})L}{4E}$$

$$+ 4 \left[\{A_{+0}^{\alpha\beta}\} c_{\phi}^{2} + \epsilon \{B_{+0}^{\alpha\beta}\} (J_{r} \cos \delta/c_{13}^{2}) \frac{\Delta m_{\text{ren}}^{2} \{(\lambda_{+} - \lambda_{-}) - (\Delta m_{\text{ren}}^{2} + a)\}}{(\lambda_{+} - \lambda_{-})(\lambda_{+} - \lambda_{0})} \right] \sin^{2} \frac{(\lambda_{+} - \lambda_{0})L}{4E}$$

$$+ 4 \left[\{A_{-0}^{\alpha\beta}\} s_{\phi}^{2} + \epsilon \{B_{-0}^{\alpha\beta}\} (J_{r} \cos \delta/c_{13}^{2}) \frac{\Delta m_{\text{ren}}^{2} \{(\lambda_{+} - \lambda_{-}) + (\Delta m_{\text{ren}}^{2} + a)\}}{(\lambda_{+} - \lambda_{-})(\lambda_{-} - \lambda_{0})} \right] \sin^{2} \frac{(\lambda_{-} - \lambda_{0})L}{4E}$$

$$+ 8\epsilon J_{r} \frac{(\Delta m_{\text{ren}}^{2})^{3}}{(\lambda_{+} - \lambda_{-})(\lambda_{-} - \lambda_{0})} \sin \frac{(\lambda_{+} - \lambda_{-})L}{4E} \sin \frac{(\lambda_{-} - \lambda_{0})L}{4E}$$

$$\times \left[\{C^{\alpha\beta}\} \cos \delta \cos \frac{(\lambda_{+} - \lambda_{0})L}{4E} + \{S^{\alpha\beta}\} \sin \delta \sin \frac{(\lambda_{+} - \lambda_{0})L}{4E} \right], \quad (3.17)$$

Integration method: rewrite the time-dependent Schroedinger eq. in dimensionless form

$$i\frac{d}{ds}\begin{pmatrix}\nu_{e}\\\nu_{\mu}\\\nu_{\tau}\end{pmatrix} = \begin{bmatrix} U\begin{pmatrix}0 & 0 & 0\\0 & \alpha & 0\\0 & 0 & \gamma \end{bmatrix} U^{\dagger} + \begin{pmatrix}A(s) & 0 & 0\\0 & 0 & 0\\0 & 0 & 0\end{bmatrix} \begin{bmatrix}\nu_{e}\\\nu_{\mu}\\\nu_{\tau}\end{pmatrix} \equiv H(s)\begin{pmatrix}\nu_{e}\\\nu_{\mu}\\\nu_{\tau}\end{pmatrix} \equiv \begin{bmatrix}UD_{1}U^{\dagger} + D_{2}\end{bmatrix}\begin{pmatrix}\nu_{e}\\\nu_{\mu}\\\nu_{\tau}\end{pmatrix}$$

Definition of the dimensionless variables:

 $s = x/x_u$ $x_u = (2E\hbar c)/|\delta m_{31}^2|$ the unit length $\alpha = \delta m_{21}^2 / |\delta m_{31}^2| \qquad \gamma = \delta m_{31}^2 / |\delta m_{31}^2|$ $A(s) = 2EV_e(x) / |\delta m_{31}^2| \propto N_e(x)$

The iterations approach:

$$S(\Delta s_i) = U_A(s_i)UU_fU^{\dagger}$$

$$S(s) = \prod_{i=1}^{N} S(\Delta s_i) \qquad P_{\beta \to \alpha} = \left| \begin{array}{c} S_{\alpha\beta}(s) \right|^2 \\ U_A(s_i) \equiv e^{-i\Delta s_i D_2(s_i)} = \begin{pmatrix} e^{-i\Delta s_i A(s_i)} & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \qquad U_f \equiv e^{-i\Delta s_i D_1} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{-i\Delta s_i \alpha} & 0 \\ 0 & 0 & e^{-i\Delta s_i \gamma} \end{pmatrix}$$

 $N \approx 15$ is enough for good accuracy:

Method works well for the Earth's crust variable density:

Neutrinos in atomic nuclei

Atomic nucleus is a high electron density medium:

Consider 2 electrons in the lowest s-orbital of an Hydrogen-like atom

Electron density near nucleus:

$$N_e(r) \approx \frac{2}{\pi} \left(\frac{Z}{a_B}\right)^3 e^{-2rZ/a_B}$$

Electron density inside nucleus: $N_e(0) \approx \frac{2}{\pi} \left(\frac{Z}{a_B}\right)^3$

 $\rho_{\text{Suncore}} \approx 150 \text{ g}/\text{cm}^3$

² x 1s el. Hydrogen-like density

Matter effects in neutrino oscillations

Electron density unevenly distributed in condensed matter: spikes

DFT calculations of SiO₂ electron density (all atomic units)

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Matter density model

- Different spike shapes produce the same result
- The 3D topology of atoms can be simulated in 1D with random spikes
- Actual density is a mixture: $\rho_{\text{mixed_spikes}} = 0.6 \rho_{\text{spikes}} + 0.4 \rho_{\text{flat}}$
- $\rho_{ave} = \rho_{flat} = 3.8 \text{ g/cm}^3 \text{ (PREM)}$

Are there any effects of the spikes in the electron density?

Apparently yes for very long baseline in neutrino oscillations!

 $E_{\nu_u} = 0.50 \; GeV$

$$i\frac{d}{ds} \begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = \begin{bmatrix} U \begin{pmatrix} 0 & 0 & 0 \\ 0 & \alpha & 0 \\ 0 & 0 & \gamma \end{pmatrix} U^{\dagger} + \begin{pmatrix} A(s) & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \begin{bmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} \equiv H(s) \begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix}$$

TC-May20

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Integration method: rewrite the time- dependent Schroedinger eq. in dimensionless form

$$i\frac{d}{ds}\begin{pmatrix}\nu_{e}\\\nu_{\mu}\\\nu_{\tau}\end{pmatrix} = \begin{bmatrix} U\begin{pmatrix}0 & 0 & 0\\0 & \alpha & 0\\0 & 0 & \gamma \end{bmatrix} U^{\dagger} + \begin{pmatrix}A(s) & 0 & 0\\0 & 0 & 0\\0 & 0 & 0\end{bmatrix} \begin{bmatrix}\nu_{e}\\\nu_{\mu}\\\nu_{\tau}\end{pmatrix} \equiv H(s)\begin{pmatrix}\nu_{e}\\\nu_{\mu}\\\nu_{\tau}\end{pmatrix} \equiv \begin{bmatrix}UD_{1}U^{\dagger} + D_{2}\end{bmatrix}\begin{pmatrix}\nu_{e}\\\nu_{\mu}\\\nu_{\tau}\end{pmatrix}$$

Consider $\Delta s_i \ll 1$ and $A(s) = \Delta s_i \overline{A}(s_i) \delta(s)$ near electron density spikes:

$$i\frac{d\nu_e(s)}{ds} = A(s)\nu_e(s) \implies \nu_e(s_a) = e^{-i\Delta s_i\bar{A}(s_i)}\nu_e(s_b)$$

The contribution to $S(\Delta s_i)$ through the Dirac delta potential:

$$U_A(s_i) \equiv e^{-i\Delta s_i D_2(s_i)} = \begin{pmatrix} e^{-i\Delta s_i \bar{A}(s_i)} & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1 \end{pmatrix}$$

Therefore, the piece-wise S-matrix formula is the same:

$$S(\Delta s_i) = U_A(s_i)UU_fU^{\dagger}$$

 $S(s) = \prod_{i=1}^{N} S(\Delta s_i)$ $P_{\beta \to \alpha} = \left| S_{\alpha\beta}(s) \right|^2$

Conclusions

- We presented a fast and reliable algorithm to calculate the neutrino oscillation probabilities through matter of varying density (more info in Universe 6, 16 (2020)).
 - The algorithm was extended to the case where the sterile neutrinos are present.
- We use this algorithm to show that the electron density spikes near the atomic nuclei can be treated as a local average density.
 - This statement can be extended to the neutron density spikes contributing to V_N (needed if the sterile neutrinos are present).
- **Related:** we showed (EPJA **56**, 39 (2020)) that the large neutrino optical potential due to the electron density spikes in the atomic nuclei does not affect the neutrinoless double-beta probability for the mass mechanism.