Quantum Monte Carlo calculations of lepton-nucleus interactions

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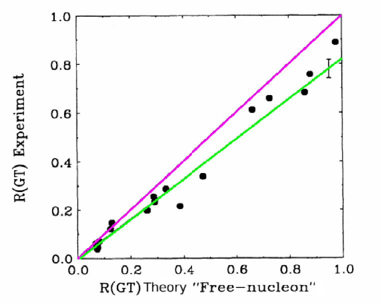
Topical Collaboration Meeting, UNC Chapel Hill, September 2019

Washington University in St. Louis
Arts & Sciences

with
Carlson & Gandolfi (LANL) & Schiavilla (ODU+JLab)
Piarulli (WashU) & Baroni (USC) & Pieper & Wiringa (ANL)
Girlanda (Salento U.) & Marcucci & Viviani & Kievsky (Pisa U/INFN)
Mereghetti & Dekens & Cirigliano & Graesser (LANL)
de Vries (Nikhef) & van Kolck (AU+CNRS/IN2P3)

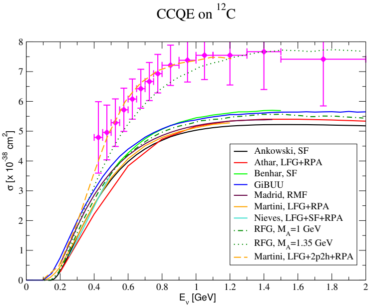
Beta Decay Rate



in 3≤ A≤ 18 $\longrightarrow g_A^{\text{eff}} \simeq 0.80\, g_A$

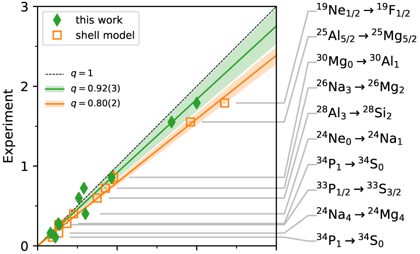
Chou *et al.* PRC47(1993)163

Neutrino-Nucleus Scattering



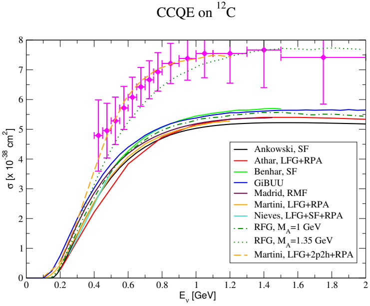
Alvarez-Ruso arXiv:1012.3871

Beta Decay Rate



Gysbers *et al.* Nature Phys.15(2019)

Neutrino-Nucleus Scattering

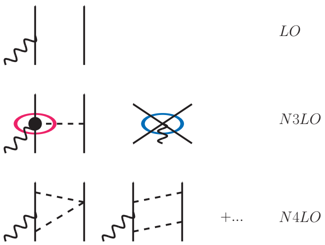


Alvarez-Ruso arXiv:1012.3871

Nuclear Interactions and Axial Currents

$$H = T + V = \sum_{i=1}^{A} t_i + \sum_{i<j} v_{ij} + \sum_{i<j<k} V_{ijk} + \dots$$

so far results are available with AV18+IL7 ($A \leq 10$)
and SNPA or chiral currents (*a.k.a.* hybrid calculations)



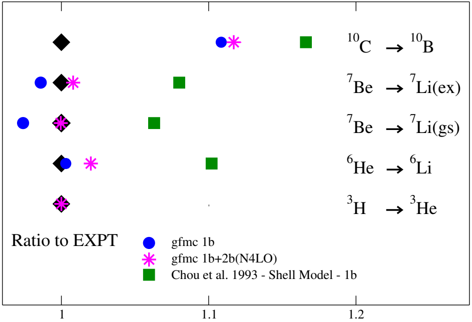
A. Baroni *et al.* PRC93(2016)015501

H. Krebs *et al.* Ann.Phy.378(2017)

- c_3 and c_4 are taken them from Entem and Machleidt PRC68(2003)041001 & Phys.Rep.503(2011)1
- c_D fitted to GT m.e. of tritium Baroni *et al.* PRC94(2016)024003
- cutoffs $\Lambda = 500$ and 600 MeV
- include also N4LO 3b currents (tiny)

* derived by Park *et al.* in the ′90
used at tree-level in many calculations (Song-Ho, Kubodera, Gazit,Marcucci, Lazauskas, Navratil ...)
* pion-pole at tree-level derived
by Klos, Hoferichter *et al.* PLB(2015)B746

Single Beta Decay Matrix Elements in $A = 6\text{–}10$



gfmc (1b) and gfmc (1b+2b); shell model (1b)

SP *et al.* PRC97(2018)022501

A. Baroni *et al.* PRC93(2016)015501 & PRC94(2016)024003

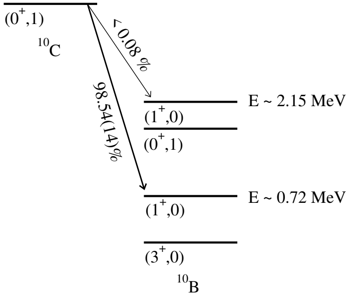
Based on $g_A \sim 1.27$ no quenching factor

GT in ^{3}H is fitted to expt - 2b give a 2% additive contribution to 1b prediction

* similar results were obtained with MEC currents

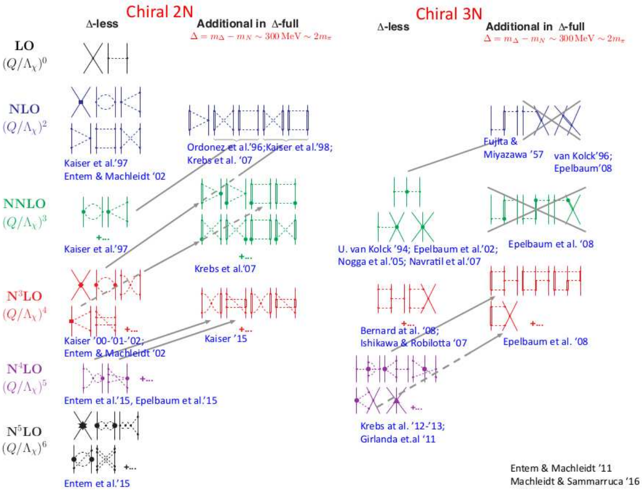
$*$ data from TUNL, Suzuki *et al.* PRC67(2003)044302, Chou *et al.* PRC47(1993)163

^{10}B



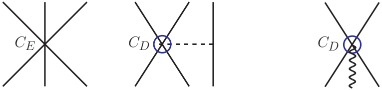
* In ^{10}B, ΔE with same quantum numbers ~ 1.5 MeV
* In $A = 7$, ΔE with same quantum numbers $\gtrsim 10$ MeV

Chiral calculations of beta decay m.e.'s: Nuclear Interaction



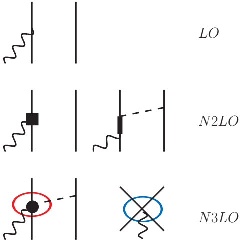
courtesy of M. Piarulli

* Chiral interactions and axial currents



we now use

1. chiral 2– and 3–body interactions with πN and Δ's developed by Piarulli *et al.* and
2. axial currents with Δ's up to N3LO (tree-level) A. Baroni *et al.* arXiv:1806.10245 (2018)



* c_3 and c_4 are taken them from Krebs *et al.* Eur.Phys.J.(2007)A32

* (c_D,c_E) fitted to

a. trinucleon B.E. and *nd* doublet scattering length **NV models**

or

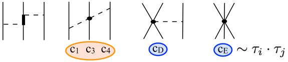
b. trinucleon B.E. and GT m.e. of tritium **NV* models**

A. Baroni *et al.* arXiv:1806.10245 (2018)

Fitting Strategies for (c_D, c_E)

Local chiral 3N potential with Δ's

Inclusion of 3N forces at N2LO:



1) Fit to:

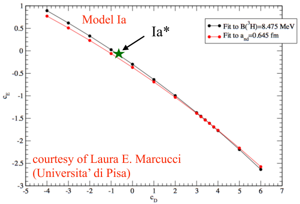
- $E_0(^3\mathrm{H}) = -8.482$ MeV
- $^2a_{nd} = (0.645 \pm 0.010)$ fm

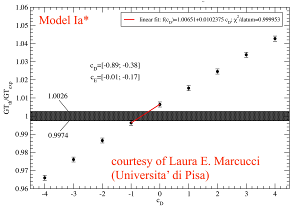
Model	c_D	c_E
Ia	3.666	−1.638
Ib	−2.061	−0.982
IIa	1.278	−1.029
IIb	−4.480	−0.412

2) Fit to:

- $E_0(^3\mathrm{H}) = -8.482$ MeV
- GT m.e. in ^{3}H β-decay

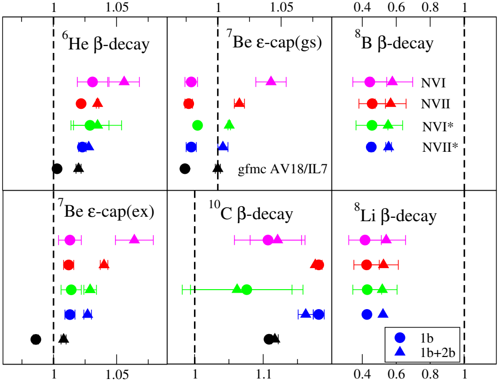
Model	c_D	c_E
Ia*	−0.635(255)	−0.09(8)
Ib*	−4.705(285)	0.550(150)
IIa*	−0.610(280)	−0.350(100)
IIb*	−5.250(310)	0.05(180)





Courtesy of M. Piarulli

Single Beta Decay Matrix Elements in $A = 6$–10 in chiEFT



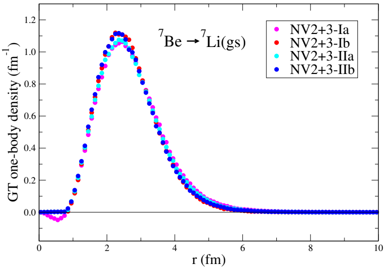
NVI - database fitted up to 125 MeV - c_D, c_E fitted to B.E. and *nd*-scattering length (VMC calculations)
NVII - database fitted up to 200 MeV - c_D, c_E fitted to B.E. and *nd*-scattering length (VMC calculations)
NVI* - database fitted up to 125 MeV - c_D, c_E fitted to B.E. and GT triton (VMC calculations)
NVII* - database fitted up to 200 MeV - c_D, c_E fitted to B.E. and GT triton (VMC calculations)

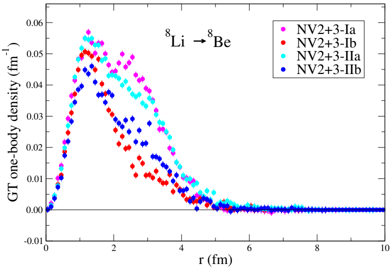
PRELIMINARY VMC CALCULATIONS

AV18+IL7 - database fitted up to 350 MeV - c_D fitted to GT triton (GFMC calculations)

in collaboration with Piarulli *et al.*

Single Beta Decay Matrix Element Densities in chiEFT



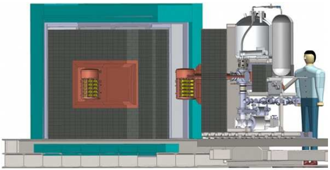


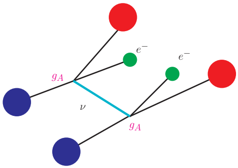
in collaboration with Piarulli *et al.*

based on chiral axial currents from A. Baroni *et al.* PRC93(2016)015501 & arXiv:1806.10245 (2018)

PRELIMINARY

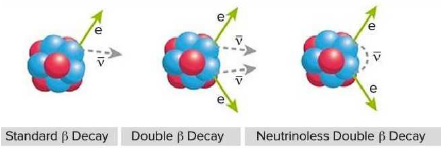
Neutrinoless Double Beta Decay





"The average momentum is about 100 MeV, a scale set by the average distance between the two decaying neutrons" cit. Engel&Menéndez

* Decay rate $\propto$ (nuclear matrix elements) $^2 \times \langle m_{\beta\beta}\rangle^2$ *



Double beta-decay Potentials

$$
\begin{aligned}
v_\nu &\sim L_\nu\, \tau_{1,+}\, \tau_{2,+} \frac{\boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2}{m_\pi\, \mathbf{q}^2} + \cdots + v_\nu{}^{\mathrm{N2LO-loop}\,*} \\
v_{\pi\pi} &\sim L_{\pi\pi}\, \tau_{1,+}\, \tau_{2,+} \frac{\boldsymbol{\sigma}_1 \cdot \mathbf{q}\, \boldsymbol{\sigma}_2 \cdot \mathbf{q}}{m_\pi\, (\mathbf{q}^2 + m_\pi^2)^2} \\
v_\pi &\sim L_\pi\, \tau_{1,+}\, \tau_{2,+} \frac{\boldsymbol{\sigma}_1 \cdot \mathbf{q}\, \boldsymbol{\sigma}_2 \cdot \mathbf{q}}{m_\pi^3\, (\mathbf{q}^2 + m_\pi^2)} \\
v_{\mathrm{NN}} &\sim L_{\mathrm{NN}}\, \tau_{1,+}\, \tau_{2,+} \frac{\boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2}{m_\pi^3}
\end{aligned}
$$

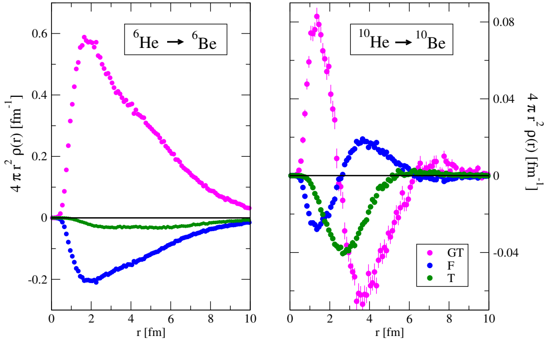
$L_{\pi\pi}$, L_π, L_{NN} encode hadronic and model dependent particle physics

* Cirigliano & Dekens & Mereghetti & Walker-Loud in arXiv:1710.01729

IN COLLABORATION WITH
Emanuele Mereghetti & Wouter Dekens & Cirigliano & Carlson & Wiringa

PRC97(2018)014606

F, GT, and T Transition Densities



* $\Delta T = 0$

^{6}He(1)→^{6}Be(1)
^{8}He(2)→^{8}Be*(2)
^{10}Be(1)→^{10}C(1)

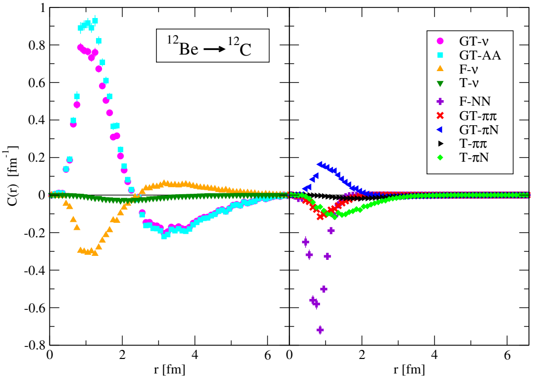
* $\Delta T = 2$

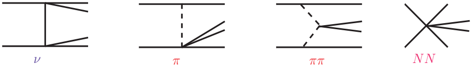
^{8}He(1)→^{8}Be(0)
^{10}He(3)→^{10}Be(1)
^{12}Be(2)→^{12}C(0)

$$F = \tau_{1,+}\,\tau_{2,+} \;\; ; GT = \tau_{1,+}\,\tau_{2,+}\,\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2 \;\; ; T = \tau_{1,+}\,\tau_{2,+}\,S_{12}$$

SP *et al.* PRC97(2018)014606

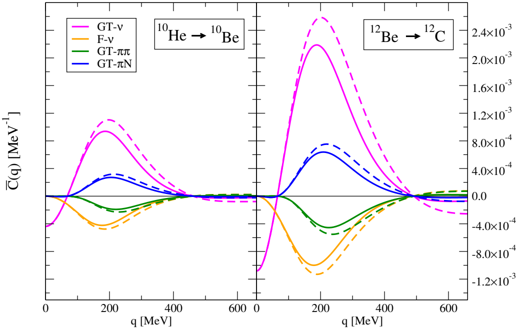
Double beta-decay Matrix Elements





SP *et al.* PRC97(2018)014606

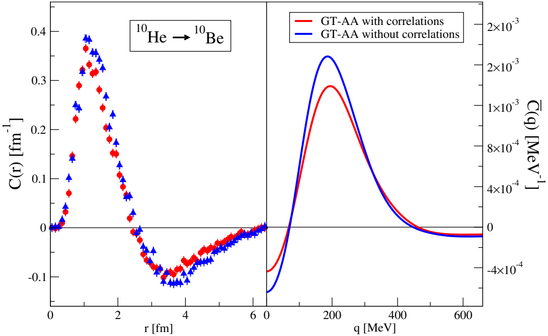
Momentum Dependence and Sensitivity to N2LO effects
i.e., 'dipole' nucleonic form factors and $\upsilon_\nu^{\mathrm{N2LO-loop}}$



- Peaks at ~ 200 MeV
- Form factors on/off $\rightarrow \sim 10\%$ variation same size as $\upsilon_\nu^{\mathrm{N2LO-loop}}$ from Cirigliano *et al.* arXiv:1710.01729
- $A = 10$ highly suppressed w.r.t. $A = 12$ (clusterization matter?)
- $A = 12$ 'most similar' to experimental cases

SP *et al.* PRC97(2018)014606

Sensitivity to ‘pion-exchange-like’ correlations

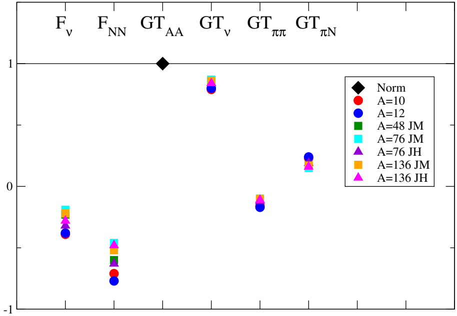


- $*$ no ‘pion-exchange-like’ correlation operators U_{ij}
- $*$ yes ‘pion-exchange-like’ correlation operators U_{ij}
- $*$ $\sim 10\%$ increase in the matrix elements corresponds to a ‘g_A-quenching’ of ~ 0.95
- $*$ as opposed to ~ 0.83 found in $A = 10$ single beta decay

$*$ Correlations reduce the m.e.’s (also true for μ’s and GT’s) $*$

SP *et al.* PRC97(2018)014606

Comparison with Calculations of Larger Nuclei



JM = Javier Menendez private communication
JH = Hyvärien *et al.* PRC91(2015)024613

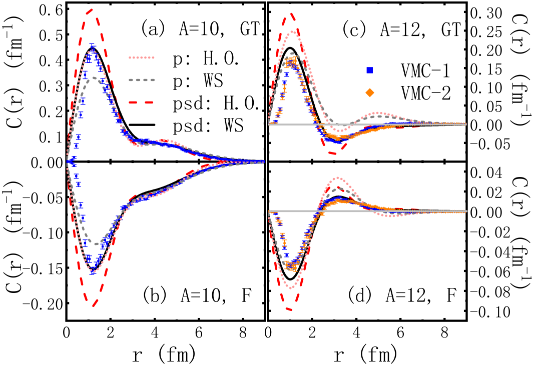
* Relative size of the matrix elements is approximately the same in all nuclei
* Short-range terms approximately the same in all nuclei

SP *et al.* - PRC97(1018)014606

Benchmark with Shell Model

Model dependence

size's space *p* vs *psd* & H.O. vs W.S. wave functions



X. Wang *et al.* arXiv:1906.06662 (2019)

Summary and Outlook

Two-nucleon correlations and two-body electroweak currents are crucial to explain available experimental data of both static (ground state properties) and dynamical (cross sections and rates) nuclear observables

* We validate the computational framework vs electromagnetic data
* Two-body currents can give $\sim 30-40\%$ contributions and improve on theory/EXPT agreement
* Calculations of $\beta-$ and $\beta\beta-$decay m.e.'s in $A \leq 12$ indicate two-body physics (currents and correlations) is required
* Short-Time-Approximation to evaluate ν-A scattering in $A > 12$ nuclei is in excellent agreement with exact calculations and data

* We are developing a coherent picture for lepton-nucleus interactions *

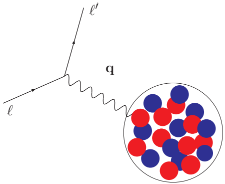
Inclusive (e,ν) scattering

* inclusive xsecs *

$$\frac{d^2\sigma}{dE' d\Omega_{e'}} = \sigma_M \left[v_L R_L(q,\omega) + v_T R_T(q,\omega) \right]$$

$$R_\alpha(q,\omega) = \sum_f \delta\left(\omega + E_0 - E_f\right) |\langle f | O_\alpha(\mathbf{q}) | 0 \rangle|^2$$

Longitudinal response induced by $O_L = \rho$
Transverse response induced by $O_T = \mathbf{j}$
... 5 nuclear responses in ν-scattering...



* Sum Rules *

Exploit integral properties of the response functions + closure to avoid explicit calculation of the final states

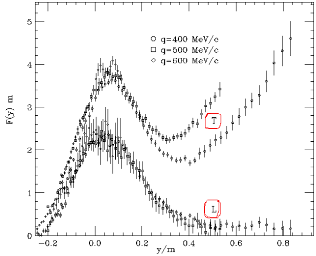
$$S(q,\tau) = \int_0^\infty d\omega\, K(\tau,\omega) R_\alpha(q,\omega)$$

* Coulomb Sum Rules *

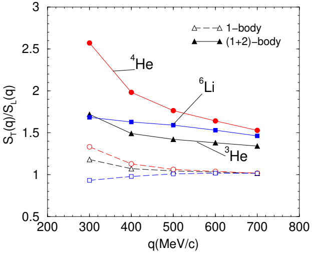
$$S_\alpha(q) = \int_0^\infty d\omega\, R_\alpha(q,\omega) \propto \langle 0 | O_\alpha^\dagger(\mathbf{q}) O_\alpha(\mathbf{q}) | 0 \rangle$$

Lessons learned from exact calculations and electromagnetic data

Longitudinal and transverse responses of ^{12}C

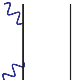


Benhar, Day, Sick Rev.Mod.Phys.80(2008)198

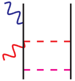


Carlson et al. PRC65(2002)024002

Fermi Gas prediction $F_L = F_T$



$\langle \mathbf{j}^{\dagger}_{1b}\ \mathbf{j}_{1b} \rangle > 0$



$\langle \mathbf{j}^{\dagger}_{1b}\ \mathbf{j}_{2b}\ v_{\pi} \rangle \propto \langle v^2_{\pi} \rangle > 0$

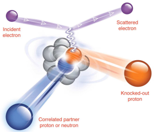
$$S_T(q) \quad \propto \quad \langle 0|\mathbf{j}^{\dagger}\ \mathbf{j}|0\rangle \propto \langle 0|\mathbf{j}_{1b}{}^{\dagger}\ \mathbf{j}_{1b}|0\rangle + \langle 0|\mathbf{j}_{1b}{}^{\dagger}\ \mathbf{j}_{2b}|0\rangle + \dots$$

- $\mathbf{j} = \mathbf{j}_{1b} + \mathbf{j}_{2b}$

The enhancement of the transverse response is due to interference between 1b and 2b currents AND presence of two-nucleon correlations

• 2-body physics essential to explain data •

QMC Calculations of Lepton-Nucleus Scattering: Challenges and Opportunities



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1. How to describe electroweak-scattering off $A > 12$ without losing two-body physics (*i.e.*, two-body correlations and currents)?

2. How to incorporate (more) exclusive processes?

3. How to incorporate relativistic effects?

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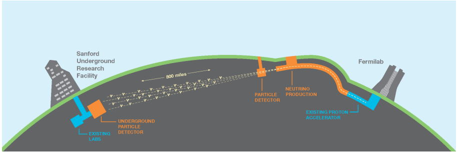


Factorization

$$R(q,\omega) = \sum_f \delta\left(\omega + E_0 - E_f\right) \langle 0|O^\dagger(\mathbf{q})|f\rangle\langle f|O(\mathbf{q})|0\rangle$$

$$R(q,\omega) = \int dt \langle 0|O^\dagger(\mathbf{q})\, e^{i(H-\omega)t}\, O(\mathbf{q})|0\rangle$$

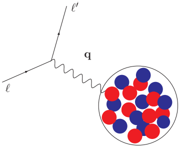
At short time, expand $P(t) = e^{i(H-\omega)t}$ and keep up to 2b-terms

$$H \sim \sum_i t_i + \sum_{i<j} v_{ij}$$

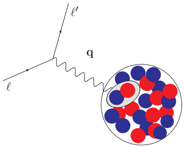
and

$$O_i^\dagger P(t) O_i + O_i^\dagger P(t) O_j + O_i^\dagger P(t) O_{ij} + O_{ij}^\dagger P(t) O_{ij}$$

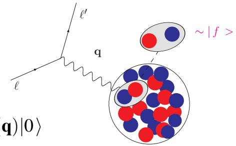
1b



2b



Factorization up to two-body operators: The Short-Time Approximation (STA)



Response functions are given by the scattering off pairs of fully interacting nucleons that propagate into a correlated pair of nucleons

$$R(q,\omega) = \sum_f \delta\left(\omega + E_0 - E_f\right) \langle 0|O^\dagger(\mathbf{q})|f\rangle\langle f|O(\mathbf{q})|0\rangle$$

$$O(\mathbf{q}) = O^{(1)}(\mathbf{q}) + O^{(2)}(\mathbf{q}) = 1\mathrm{b} + 2\mathrm{b}$$

$$|f\rangle \sim |\psi_{p',P',J,M,L,S,T,M_T}(r,R)\rangle = \text{correlated two-nucleon w.f.}$$

* We retain two-body physics consistently in the nuclear interactions and electroweak currents
* $R_\alpha(q,\omega)$ requires only direct calculation of g.s. $|0\rangle$ w.f.'s *
* STA can describe pion-production induced by e and ν

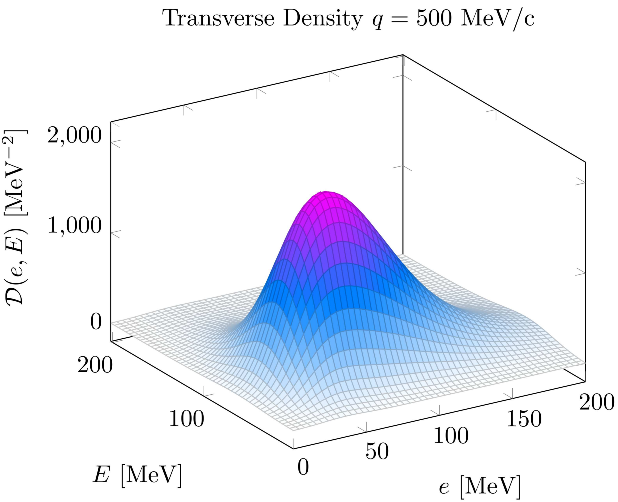
* Definition: Response Density $\mathscr{D}$ *

$$R(q,\omega) \sim \int \delta\left(\omega + E_0 - E_f\right) d\Omega_{P'}\, d\Omega_{p'}\, dP'\, dp' \left[p^{2\prime} P^{2\prime} \langle 0|O^\dagger(\mathbf{q})|\mathbf{p}',\mathbf{P}'\rangle\langle \mathbf{p}',\mathbf{P}'|O(\mathbf{q})|0\rangle\right]$$

$$\sim \int \delta\left(\omega + E_0 - E_f\right) dP'\, dp'\, \mathscr{D}(p',P';q)$$

has info on the nucleus soon after the probe interacts with the pair of nucleons;
provides more "exclusive" info in terms of nucleon-pair kinematics;
correctly accounts for interference terms

Short-Time Approximation: Response Densities



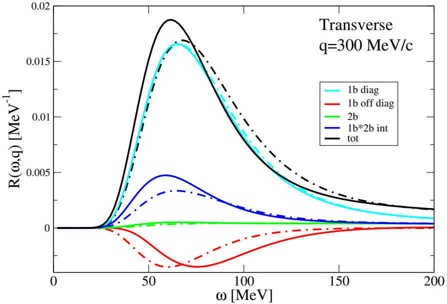
Transverse "response-density" 1b + 2b for ^4He

$\mathscr{D}(p', P'; q)$

* Preliminary results *

Short-Time Approximation: Propagator

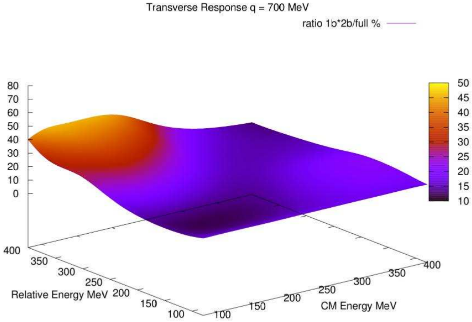
$\dashrightarrow$ Plane Wave Propagator vs $\longrightarrow$ Correlated Propagator



$$R_{\alpha}(q,\omega) \sim \int \delta\left(\omega + E_0 - E_f\right) d\Omega_P \, d\Omega_p \, dP \, dp \left[p^2 P^2 \langle 0|O^{\dagger}_{\alpha}(\mathbf{q})|\mathbf{p},\mathbf{P}\rangle\langle \mathbf{p},\mathbf{P}|O_{\alpha}(\mathbf{q})|0\rangle\right]$$

* Preliminary results *

^{4}He Transverse Response Density $q = 700$ MeV



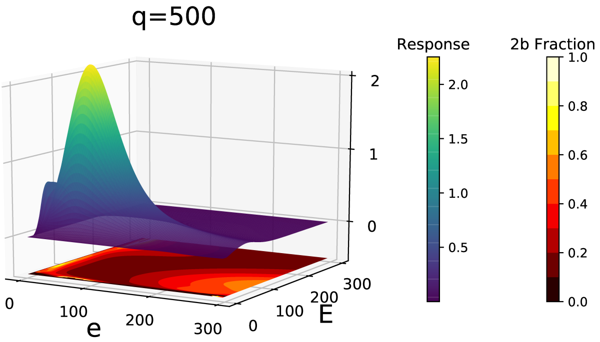
Transverse "response-density" $\mathscr{D}(p', P'; q)$

Ratio (1b×2b+2b) / tot

* Preliminary results *

Short-Time Approximation: Transverse Response Densities

^{4}He Transverse Response Density $q = 500$ MeV

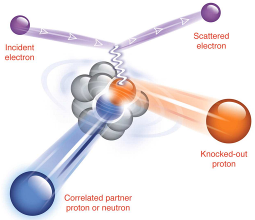


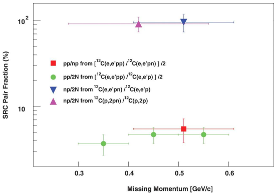
Transverse “response-density” 1b + 2b for ^4He

$\mathscr{D}(p', P'; q)$

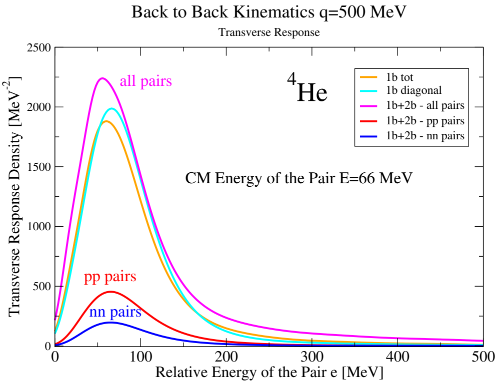
* Preliminary results *

Short-Time Approximation: back to back scattering



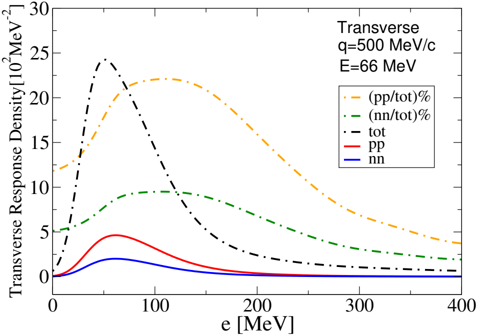


JLab, Subedi et al. Science320(2008)1475



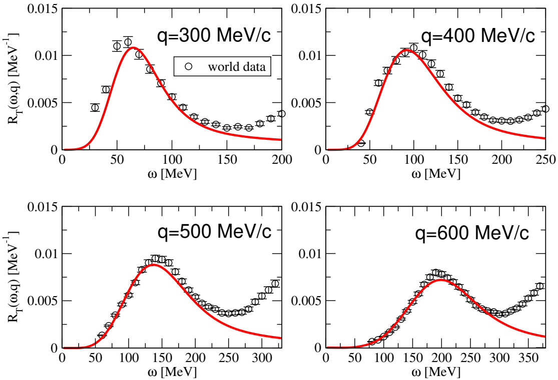
Short-Time Approximation: back to back scattering and particle identity

^{4}He Transverse Responses, $q = 500$



* Preliminary results *

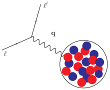
Short-Time Approximation: Comparison with data



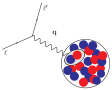
* Preliminary results *

Short-Time Approximation: Summary

1b



2b



What it is

* It is based on factorization at short-time
* Retains two-body operators correlating nucleon-pairs and associated two-body currents
* Describes the scattering of leptons off pairs of fully interacting nucleons
* Lepton-nucleus interaction occurs via 1b and 2b currents and ensuing interference terms
* It provides response functions
* It provides response densities as function of the relative and total energy of a nucleon-pair
* It can accommodate for semi-inclusive processes, pion-production, relativity
* It can be implemented in AFDMC to study $A \sim 40$ systems

Where we are

* Electromagnetic Response Functions and Densities of ^{4}He, ^{3}H and ^{3}He are available for values of $|\mathbf{q}|$ and $E \leq 800$ MeV

Work in progress

* Implementation of axial currents into VMC codes
* Implementation of the STA into VMC codes for ^{12}C

Summary and Outlook

Two-nucleon correlations and two-body electroweak currents are crucial to explain available experimental data of both static (ground state properties) and dynamical (cross sections and rates) nuclear observables

* We validate the computational framework vs electromagnetic data

* Two-body currents can give $\sim 30-40\%$ contributions and improve on theory/EXPT agreement

* Calculations of $\beta-$ and $\beta\beta-$decay m.e.'s in $A \leq 12$ indicate two-body physics (currents and correlations) is required

* Short-Time-Approximation to evaluate υ-A scattering in $A > 12$ nuclei is in excellent agreement with exact calculations and data

* We are developing a coherent picture for lepton-nucleus interactions *