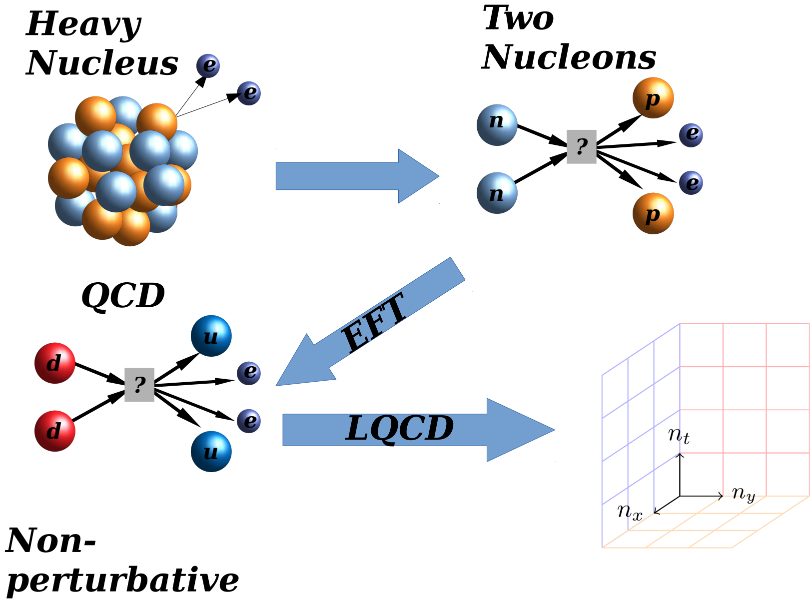
Progress On Lattice Calculations for $\beta\beta$ Decays

Henry Monge-Camacho

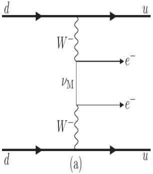
University of North Carolina at Chapel Hill

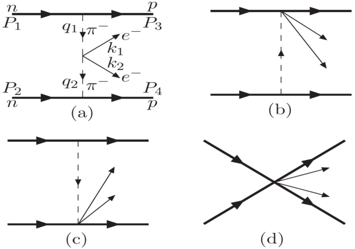
DBD Collaborating Meeting
UNC at Chapel Hill, September 6, 2019

LQCD for ββ Decay

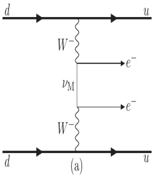


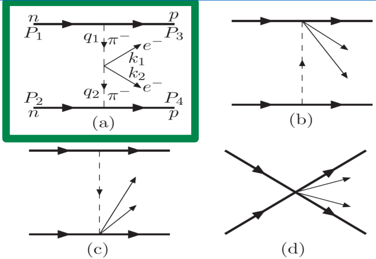
Short-Range Contributions

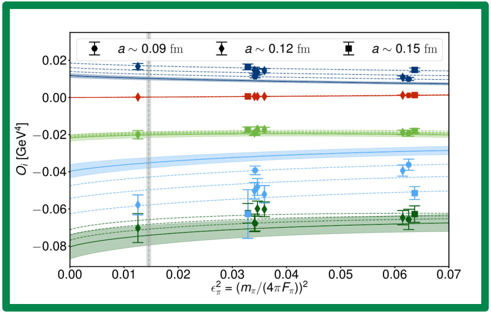




Short-Range Contributions

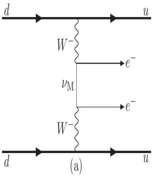


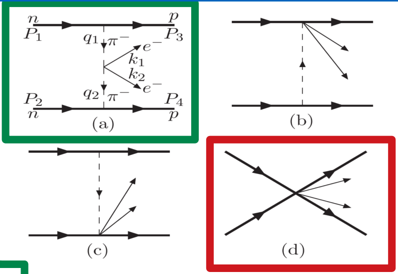


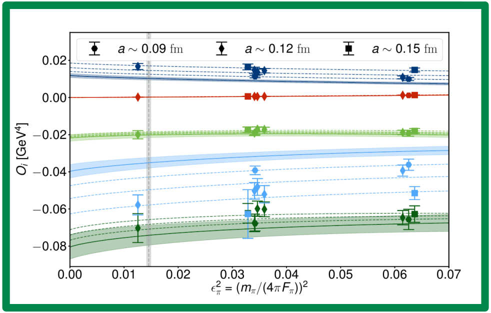


A. Nicholson et al. (2018). In: *Phys. Rev. Lett.* 121.17, p. 172501. DOI: 10.1103/PhysRevLett.121.172501. arXiv: 1805.02634 [nucl-th]

Short-Range Contributions



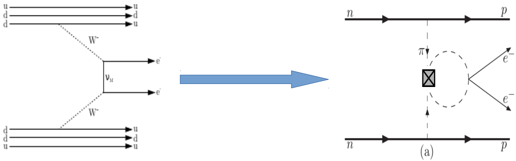




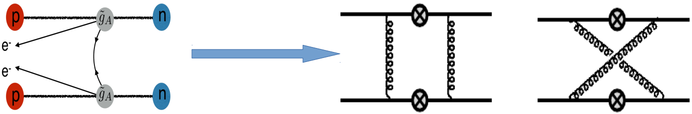
A. Nicholson et al. (2018). In: *Phys. Rev. Lett.* 121.17, p. 172501. DOI: 10.1103/PhysRevLett.121.172501. arXiv: 1805.02634 [nucl-th]

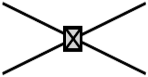
$\otimes = \bar{q}\Gamma^1 q$ $\boxtimes = \bar{q}\Gamma^1 q\bar{q}\Gamma^2 q$

Short-range contributions



Long-range contributions





B. C. Tiburzi, M. L. Wagman, F. Winter, E. Chang, Z. Davoudi, W. Detmold, K. Orginos, M. J. Savage, and P. E. Shanahan (2017). In: *Phys. Rev.* D96.5, p. 054505. arXiv: 1702.02929 [hep-lat]

$$\mathcal{O}_{1+}^{++} = (\bar{q}_L \tau^+ \gamma^\mu q_L)(\bar{q}_R \tau^+ \gamma^\mu q_R)$$
$$\mathcal{O}_{2\pm}^{++} = (\bar{q}_R \tau^+ q_L)\,(\bar{q}_R \tau^+ q_L) + (\bar{q}_L \tau^+ q_R)(\bar{q}_L \tau^+ q_R)$$
$$\mathcal{O}_{3\pm}^{++} = (\bar{q}_L \tau^+ q_L)(\bar{q}_L \tau^+ q_L) + (\bar{q}_R \tau^+ q_R)(\bar{q}_R \tau^+ q_R)$$

Mix under renormalization

$$\mathcal{O}_{4\pm}^{++} = (\bar{q}_L \tau^+ \gamma^\mu q_L \mp \bar{q}_R \tau^+ \gamma^\mu q_R)(\bar{q}_L \tau^+ q_R - \bar{q}_R \tau^+ q_L)$$
$$\mathcal{O}_{5\pm}^{++} = (\bar{q}_L \tau^+ \gamma^\mu q_L \pm \bar{q}_R \tau^+ \gamma^\mu q_R)(\bar{q}_L \tau^+ q_R + \bar{q}_R \tau^+ q_L)$$

Suppresed by m_e

Operator Basis

$$\mathcal{O}_{1+}^{++} = (\bar{q}_L^a \tau^+ \gamma^\mu q_L^a)(\bar{q}_R^b \tau^+ \gamma^\mu q_R^b)$$
$$\mathcal{O}_{2+}^{++} = (\bar{q}_R^a \tau^+ q_L^a)\,(\bar{q}_R^b \tau^+ q_L^b) + (\bar{q}_L^a \tau^+ q_R^a)(\bar{q}_L^b \tau^+ q_R^b)$$
$$\mathcal{O}_{3+}^{++} = (\bar{q}_L^a \tau^+ q_L^a)(\bar{q}_L^a \tau^+ q_L^a) + (\bar{q}_R^a \tau^+ q_R^a)(\bar{q}_R^a \tau^+ q_R^a)$$
$$\mathcal{O}_{1+}^{\prime\,++} = (\bar{q}_L^a \tau^+ \gamma^\mu q_L^b)(\bar{q}_R^b \tau^+ \gamma^\mu q_R^a)$$
$$\mathcal{O}_{2+}^{\prime\,++} = (\bar{q}_R^a \tau^+ q_L^b)\,(\bar{q}_R^b \tau^+ q_L^a) + (\bar{q}_L^a \tau^+ q_R^b)(\bar{q}_L^b \tau^+ q_R^a)$$

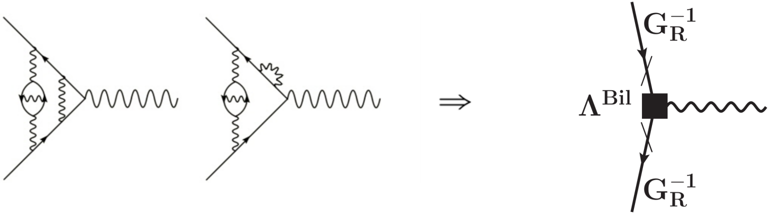
V. Cirigliano, W. Dekens, J. De Vries, M. L. Graesser, E. Mereghetti, S. Pastore, and U. Van Kolck (2018). In: *Phys. Rev. Lett.* 120.20, p. 202001. arXiv: 1802.10097 [hep-ph]

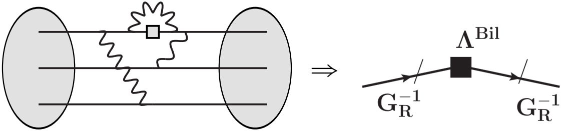
G. Prezeau, M. Ramsey-Musolf, and P. Vogel (2003). In: *Phys. Rev.* D68, p. 034016. arXiv: hep-ph/0303205 [hep-ph]

Non-perturbative Renormalization on the Lattice

In the lattice: $\mathcal{O}^R_{Latt}(a) = Z(\mu, a) \cdot \mathcal{O}^B_{Latt}(a)$

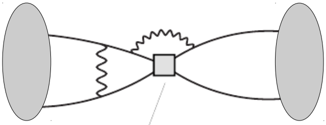
In the continuum: $\mathcal{O}^R_{cont} = \lim_{a\to 0} Z(\mu, a)\mathcal{O}^B_{Latt}(a)$



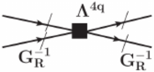


$$= \frac{Z_V(\mu,a)}{Z_q(\mu,a)} \cdot \Lambda_V(\mu, a)$$

$$\Lambda_V(\mu, a, m_q) = \left(1 + \sum_{n=-1,1,2} b_n(am_q)^n\right)\left(\sum_m c_m(a\mu)^m\right) + \sum_k d_k(a\mu)^k$$



$$
\begin{pmatrix}
\mathbf{Z_{11}} & 0 & 0 & 0 & 0 \\
0 & \mathbf{Z_{22}} & Z_{23} & 0 & 0 \\
0 & Z_{32} & \mathbf{Z_{33}} & 0 & 0 \\
0 & 0 & 0 & \mathbf{Z_{44}} & Z_{45} \\
0 & 0 & 0 & Z_{54} & \mathbf{Z_{55}}
\end{pmatrix}
$$

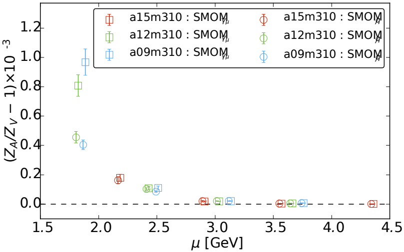


$$= \frac{Z_{4q}(\mu,a)}{Z_V^2(\mu,a)} \cdot \frac{\Lambda_{4q}(\mu,a)}{\Lambda_V(\mu,a)}$$

$$\Lambda_\Gamma(\mu, a, m_q) = \left(1 + \sum_{n=-1,1,2} b_n (a m_q)^n\right)\left(\sum_m c_m (a\mu)^m\right) + \sum_k d_k (a\mu)^k$$

LQCD for $0\nu\beta\beta$

A. Nicholson et al. (2018). In: arXiv: 1805.02634 [nucl-th]
C. C. Chang et al. (2018). In: Nature 558.7708, pp. 91–94. arXiv:1805.12130 [hep-lat]



$$Z^{\mathrm{a09}}(\mu = 3\ \mathrm{GeV}):$$

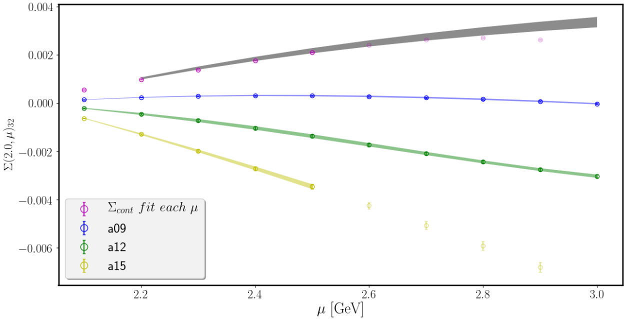
$$\begin{pmatrix} 0.9483(44) & -0.0269(17) & 0 & 0 & 0 \\ -0.0236(29) & 0.9369(54) & 0 & 0 & 0 \\ 0 & 0 & 0.9209(91) & -0.0224(49) & 0 \\ 0 & 0 & -0.0230(28) & 0.9332(47) & 0 \\ 0 & 0 & 0 & 0 & 0.9017(39) \end{pmatrix}$$

Method RI-SMOM[1]:

- Three Lattice spacings: 0.09, 0.12, 0.15fm
- Projectors γ and $\not{q}$ show agreement after $\overline{\mathrm{MS}}$ conversion
- Step scaling functions are raise the renormalization scale (0.15)

[1] C. Sturm, Y. Aoki, N. H. Christ, T. Izubuchi, C. T. C. Sachrajda, and A. Soni (2009). In: Phys. Rev. D80, p. 014501. arXiv:0901.2599 [hep-ph]

Renormalization Constants Running



Renormalization Group ⇒ cont. running $\Sigma(\mu_1, \mu_2) = Z(\mu_1)Z(\mu_2)^{-1}$

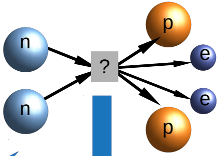
In the Lattice: $\Sigma(\mu_1, \mu_2, a) = \Sigma(\mu_1, \mu_2)_{cont} + \Delta a^2$

Fit assuming smooth μ dependence to obtain $\Sigma(\mu_1, \mu_2)_{cont}$

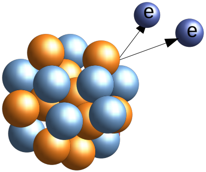
R. Arthur and P. A. Boyle (2011). In: *Phys. Rev.* D83, p. 114511. arXiv: 1006.0422 [hep-lat]

TABLE II. Resulting matrix elements extrapolated to the physical point, renormalized in RI/SMOM and $\overline{\text{MS}}$, both at $\mu = 3$ GeV.

$O_i[\text{GeV}]^4$	RI/SMOM $\mu = 3$ GeV	$\overline{\text{MS}}$ $\mu = 3$ GeV
O_1	$-1.96(14) \times 10^{-2}$	$-1.94(14) \times 10^{-2}$
O_1'	$-7.21(53) \times 10^{-2}$	$-7.81(57) \times 10^{-2}$
O_2	$-3.60(30) \times 10^{-2}$	$-3.69(31) \times 10^{-2}$
O_2'	$1.05(09) \times 10^{-2}$	$1.12(10) \times 10^{-2}$
O_3	$1.89(09) \times 10^{-4}$	$1.90(09) \times 10^{-4}$



$$\left\langle {}^{76}\mathbf{Se} \right| \mathbf{V_{nn\to pp}} \left| {}^{76}\mathbf{Ge} \right\rangle$$



$$V_i^{nn\to pp}(|\mathbf{q}|) = -O_i \frac{g_A^2}{4F_\pi^2} \tau_1^+ \tau_2^+ \frac{\sigma_1 \cdot \mathbf{q} \sigma_2 \cdot \mathbf{q}}{(|\mathbf{q}|^2 + m_\pi^2)^2}$$

$nn \longrightarrow pp$

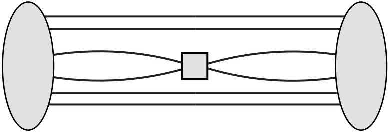
Methods are well known for current insertions between meson states:

$$\langle A|\, \bar{q}\Gamma^1 q\bar{q}\Gamma^2 q\, |B\rangle \qquad \langle A|\, \bar{q}\Gamma^1 q\, |B\rangle$$

Bilinear current insertions between nucleon are known but more complex:

$$\langle NN|\, \bar{q}\Gamma^1 q\, |NN\rangle$$

Four quark current insertions between nucleons even more challenging:



$$\sum_{x,y} \underbrace{\langle NN(y)|\, \bar{q}(x)\Gamma^1 q(x)\bar{q}(x)\Gamma^2 q(x)\, |NN(0)\rangle}_{\text{Momentum projection}}$$

all x-to-all y propagators required

Four-quark Feynman-Hellman Method: $\pi^- \to \pi^+$

Analog of method implemented for baryons and bilinear currents [2]

$\partial_\lambda E_\lambda = \langle n| H_\lambda |n\rangle \qquad S_\lambda = \lambda \int d^4x \bar{\psi}\Gamma^1\psi\bar{\psi}\Gamma^2\psi = \lambda \int d^4x \mathcal{J}(x)$

$\partial_\lambda E_\lambda$

For a meson effective mass:

$$\left.\frac{\partial m_{eff}}{\partial_\lambda}\right|_{\lambda=0} = -\frac{\partial_\lambda C(t+\tau) + \partial_\lambda C(t-\tau) - 2cosh(m_{eff}\tau)\partial_\lambda C(t)}{2\tau C(t) sinh(m_{eff}\tau)}$$

For long enough t $\left.\frac{\partial m_{eff}}{\partial_\lambda}\right|_{\lambda=0} \approx \frac{\mathcal{J}_{00}}{2E_0^2}$

$\partial_\lambda C(t)$

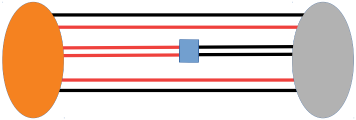
Matrix element is pulled down with ∂_λ

$$N(t) = \int d^4x \left\langle \Omega | T\mathcal{O}(t)\mathcal{J}(x)\mathcal{O}^\dagger(0)|\Omega \right\rangle$$

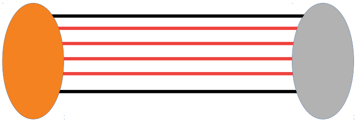
[2] C. Bouchard, C. C. Chang, T. Kurth, K. Orginos, and A. Walker-Loud (2017). In: *Phys. Rev.* D96.1, p. 014504. arXiv: 1612.06963 [hep-lat]

$$\frac{\partial m_{eff}}{\partial_\lambda}\bigg|_{\lambda=0} = -\frac{\partial_\lambda C(t+\tau) + \partial_\lambda C(t-\tau) - 2cosh(m_{eff}\tau)\partial_\lambda C(t)}{2\tau C(t) sinh(m_{eff}\tau)}$$

$\partial_\lambda C_\lambda(t)$ →



$C(t)$ →

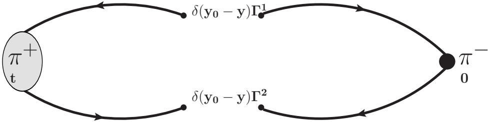


→ $\int dx^4 \mathcal{J}(x)$

C. Bouchard, C. C. Chang, T. Kurth, K. Orginos, and A. Walker-Loud (2017). In: *Phys. Rev.* D96.1, p. 014504. arXiv: 1612.06963 [hep-lat]

Brute force calculation on small Lattice:

$$\int \mathbf{d^4x} \left\langle \boldsymbol{\Omega} | \mathbf{T}\mathcal{O}(\mathbf{t})\mathcal{J}(\mathbf{x})\mathcal{O}^{\dagger}(\mathbf{0}) | \boldsymbol{\Omega} \right\rangle = \sum_{\mathbf{y_0} \in \mathbf{V}}$$

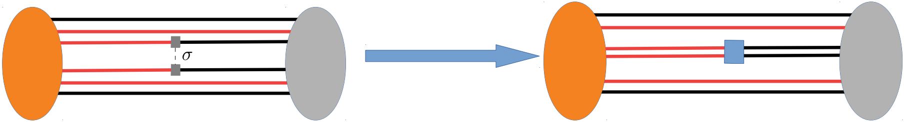


Hubbard-Stratanovich Transformation:

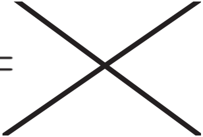
$$e^{-\lambda^2 \int d^4x (\overline{\psi}\Gamma\psi)^2} = \alpha \int_{-\infty}^{\infty} d\sigma e^{-\int d^4x \{ \frac{\sigma^2}{4} + \lambda i \sigma (\overline{\psi}\Gamma\psi) \}}$$

D. J. Gross and A. Neveu (1974). In: *Phys. Rev.* D10, p. 3235
R. L. Stratonovich (1957). In: *Doklady Akad. Nauk S.S.S.R.* 115, p. 1097,J. Hubbard (1959). In: *Phys. Rev. Lett.* 3, pp. 77–80

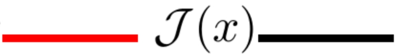
LQCD for $0\nu\beta\beta$



$$\int \mathbf{d}\sigma =$$



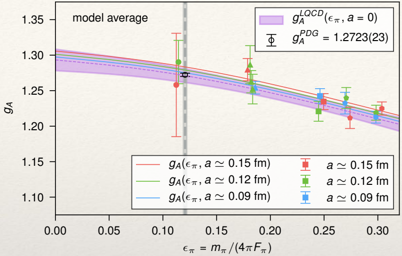
$$= \int d^4x$$



Nucleon Form Factors

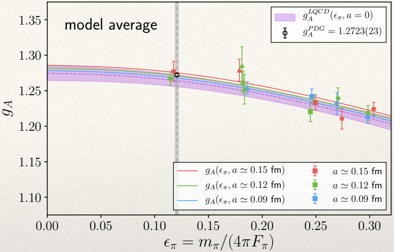
Nature 558 (2018) no. 7708, 91-94
Chang et al. [arXiv:1805.12130]

1 year on Titan (ORNL) + 2 years on GPU machines at LLNL



California Lattice Collaboration

Sierra Early Science PRELIMINARY



- The a12m130 (48^3 x 64 x 20) with 3 sources cost as much as all other ensembles combined
  - 2.5 weekends on Sierra → 16 srcs
  - Now, 32 srcs (un-constrained, 3-state fit)
- We generated a new a15m135XL (48^3 x 64) ensemble (old a15m130 is 32^3 x 48)
  - $M\pi L = 4.93$ (old $M\pi L = 3.2$)
  - $L_5 = 24$, $N_{src} = 16$

$$g_A = 1.2711(125) \rightarrow 1.2641(93) \quad [0.74\%]$$

- We are running $g_A(Q^2)$ on Summit this year (DOE INCITE)
  - We anticipate improving g_A to ~0.5%

Slides from A. Walker-Loud, Lattice 2019

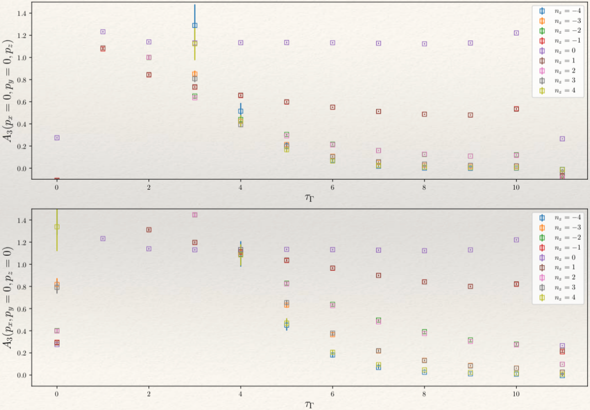
Nucleon Form Factors

Nucleon Axial FormFactor

PRELIMINARY

a09m310

non-zero momentum, t_{sep} = 11



|n|=1, Q=0.196 GeV

|n|=2, Q=0.393 GeV

|n|=3, Q=0.589 GeV

|n|=4, Q=0.785 GeV

24

Slides from A. Walker-Loud, Lattice 2019

Summary:

Non-perturbative renormalization of four-quark operators is finished

A new method is proposed to compute contributions from four-quark operators.

Next steps:

- Reproduce $\pi^- \to \pi^+$ calculation with the new method
- Implement calculation using the Hubbard-Stratanovich transformation
- Apply method to $nn \to pp$ calculation

Nucleon Form Factors: Lots of data to analize

Lattice QCD People

RIKEN-iTHEMS: Chia Cheng Chang

LBNL:André Walker-Loud, Chris Koerber, Ben Hörz, Ken McElvain

LLNL: David Brantley, Pavlos Vranas, Arjun Gambhir

RIKEL-BNL: Enrico Rinaldi

FZJ: Evan Berkowitz

JLab: Bálint Jóo,

W&M: Kostas Orginos, Chris Monahan

Liverpool Univ.: Nicolas Garron

Glasgow: Chris Bouchard

NERSC: Thorsten Kurth

UNC: Amy Nicholson, Henry Monge-Camacho

nVidia: Kate Clark

Thanks!