TRIUMF

Updates on $0\nu\beta\beta$ with the NCSM and SRG Evolution

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Double-Beta Decay Topical Collaboration
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Discovery, accelerated

Outline

- Background
  - recap: SRG
- Two-body operators
  - recap: NCSM vs CC benchmarks in light nuclei
  - progress: more nuclei, improved agreement
- Three-body operators
  - progress: debugging matrix elements / transition densities

Background

- SRG evolution is a unitary transformation which improves convergence

$$U_\alpha \hat{O} U_\alpha^\dagger = O_\alpha^{(1)} + O_\alpha^{(2)} + O_\alpha^{(3)} + \dots$$

- Introduces higher-body terms, $O_\alpha^{(a)}$, determined in the appropriate a-body system ($a \leq A$)
- E.g. if $O = O^{(2)}$:

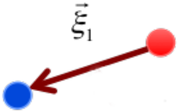
$$O_\alpha^{(2)} = U_\alpha^{(2)} O^{(2)} U_\alpha^{\dagger(2)} \qquad U_\alpha^{(2)} = \sum |\psi_{\alpha,a=2}\rangle \langle\psi_{a=2}|$$

$$O_\alpha^{(3)} = U_\alpha^{(3)} \left\langle O^{(2)} \right\rangle^{(3)} U_\alpha^{\dagger(3)} - \left\langle O_\alpha^{(2)} \right\rangle^{(3)} \qquad U_\alpha^{(3)} = \sum |\psi_{\alpha,a=3}\rangle \langle\psi_{a=3}|$$

Two-body Implementation

For $|\psi_k\rangle = |kj^\pi tt_z\rangle = \sum c^k_{n\ell s} |n\ell s j^\pi tt_z\rangle$, U_α is constructed in blocks:

$$U_\alpha^{j^\pi tt_z} = \sum_k |kj^\pi t, \alpha\rangle \langle kj^\pi t|$$



Non-scalar operators may connect states with $j^\pi tt_z$, e.g.

$$\langle f, j_f| O_\alpha |i, j_i\rangle = \langle f, j_f| U_\alpha^{j_f} O U_\alpha^{j_i \dagger} |j_i\rangle$$

Converting to single-particle basis:

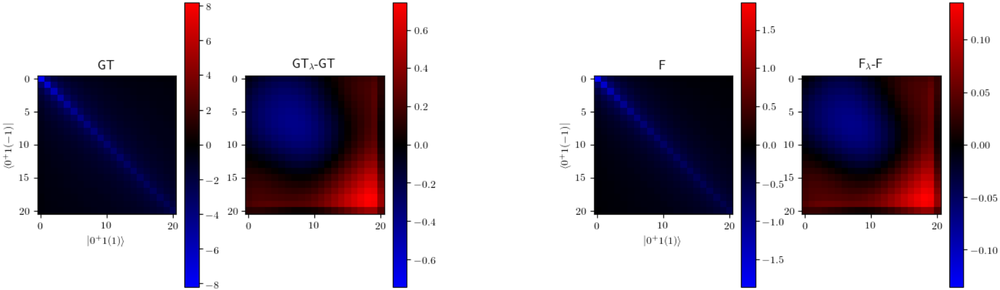
$$\langle a'b' J_{a'b'}| O_\alpha |abJ_{ab}\rangle \qquad a = n_a, \ell_a, j_a$$

$$= \sum_{if} c^{if}_{a'b'ab} \langle f, j_f| O_\alpha |i, j_i\rangle$$

Application to $0\nu\beta\beta$

$$\hat{O}_{0\nu\beta\beta} = \hat{O}_{GT} + \hat{O}_F + \hat{O}_T$$ from J. Engel

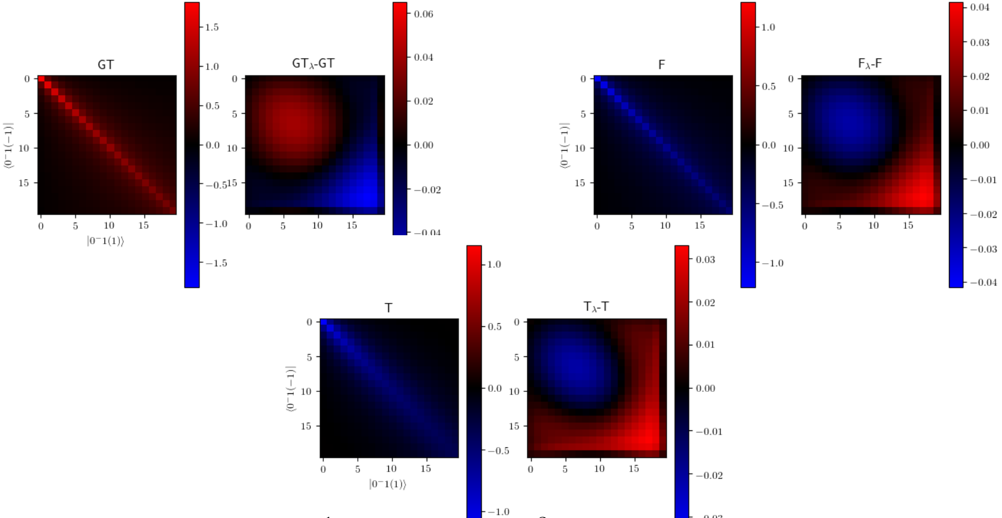
${}^1S_0$:



Two-body SRG: $\lambda = 2$ fm^{-1}, chiral NN @ N^{3}LO

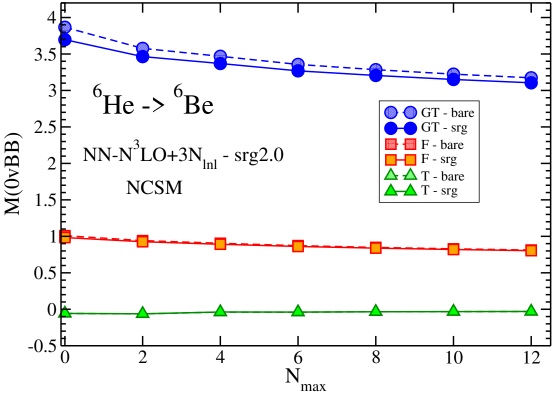
Application to $0\nu\beta\beta$

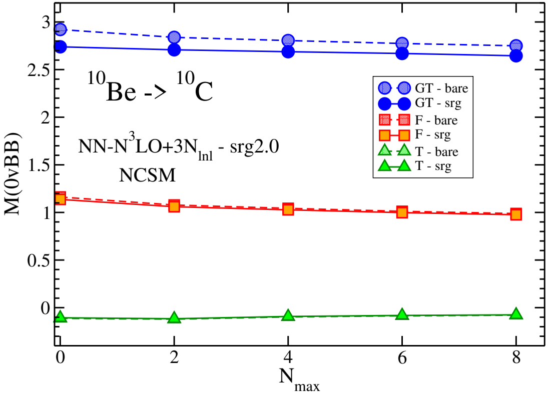
$^3P_0$:



Two-body SRG: $\lambda = 2$ fm^{-1}, chiral NN @ N^{3}LO

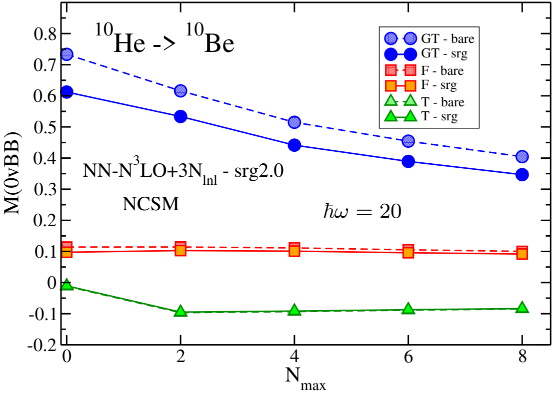
Results for Light Nuclei

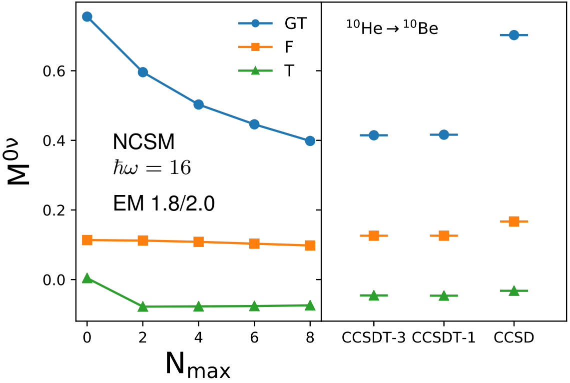




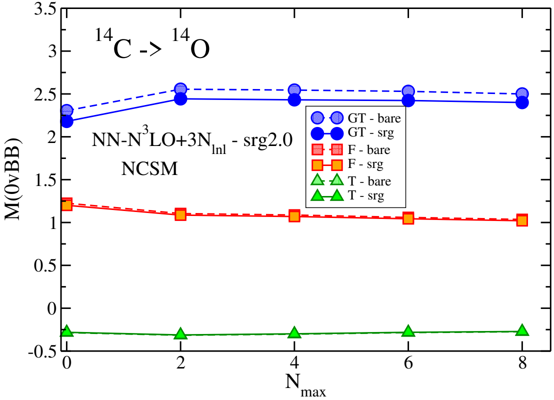
Benchmarks with Coupled Cluster

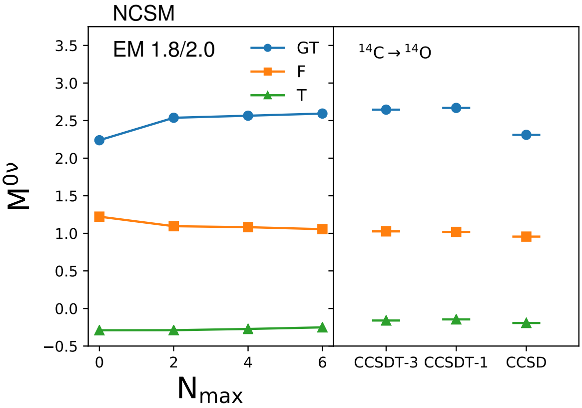
(CC: Sam Novario)





Benchmarks with Coupled Cluster





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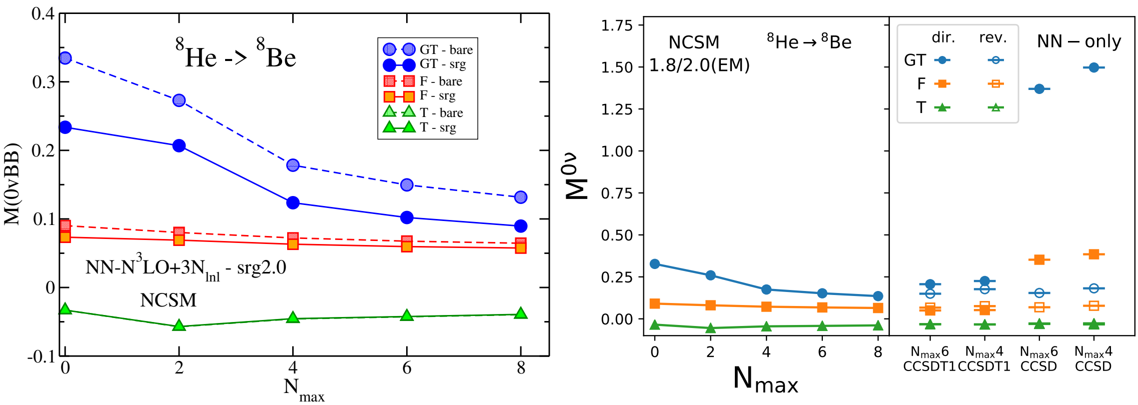

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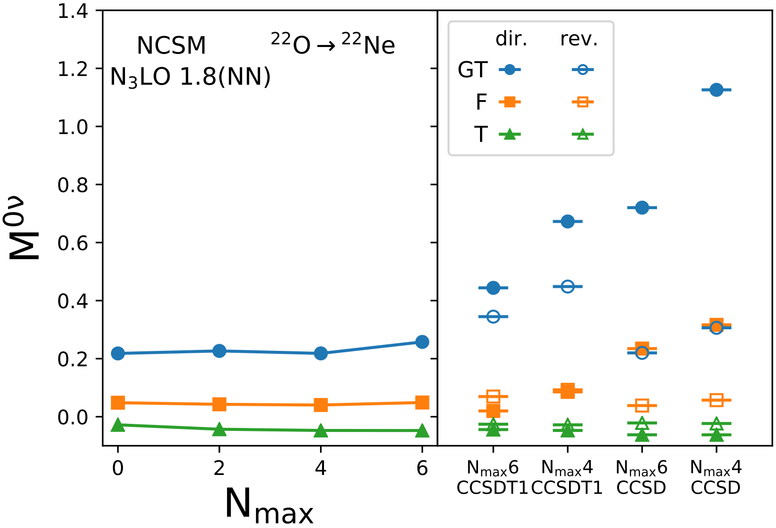
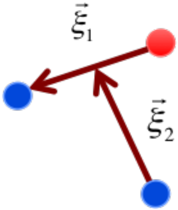


Three-body Implementation

For $|\psi_k\rangle = |kJ^\rho T\rangle = \sum c^k_{Ni} |NiJ^\rho T\rangle$
$|NiJ^\rho T\rangle = \sum \mathcal{C}^{NiJT}_{n\ell sjt;\mathcal{N}\mathcal{L}\mathcal{J}} |(n\ell sjt;\mathcal{N}\mathcal{L}\mathcal{J})JT\rangle \quad (N = 2n + \ell + 2\mathcal{N} + \mathcal{L})$

$$U^{J^\rho T}_\alpha = \sum_k |kJ^\rho T,\alpha\rangle \langle kJ^\rho T|$$



Non-scalar operators may connect states with $J^\rho T(T_z)$, e.g.

$$\langle f, J_f| O_\alpha |i, J_i\rangle = \langle J_f| U^{J_f}_\alpha O U^{J_i\dagger}_\alpha |i, J_i\rangle$$

Converting to single-particle basis:

$$\langle a'b'J_{a'b'}c'J_{a'b'c'}| O_\alpha |abJ_{ab}cJ_{abc}\rangle \qquad a = n_a, \ell_a, j_a$$
$$= \sum_{if} c^{if}_{a'b'c'abc} \langle f, J_f| O_\alpha |i, J_i\rangle$$

Three-body Implementation (cont.)

- Generalized code to calculate:

$$\langle f, J_f || \hat{O}^{(3)} || i, J_i \rangle = \frac{1}{36} \sum \langle \alpha\beta\gamma | \hat{O} | \delta\epsilon\omega \rangle \langle f, J_f || a^\dagger_\alpha a^\dagger_\beta a^\dagger_\gamma a_\omega a_\epsilon a_\delta || i, J_i \rangle$$

- Decouple $\langle abJ_{ab}cJ_{abc}| \hat{O} |deJ_{de}fJ_{def}\rangle \rightarrow \langle abc| \hat{O} |def\rangle$ on the fly
- Scalar operators have been tested: T_{kin}, V_{NN}, V_{3N}, radius
- Matrix elements with $T_{z,i} \neq T_{z,f}$ (and therefore $0\nu\beta\beta$) are currently broken
- Currently comparing 2B vs 3B (without SRG) for test operators:
  - $\tau^+\tau^+$, $T_{kin} \otimes \tau^+\tau^+$, $V_{NN} \otimes \tau^+\tau^+$, etc.

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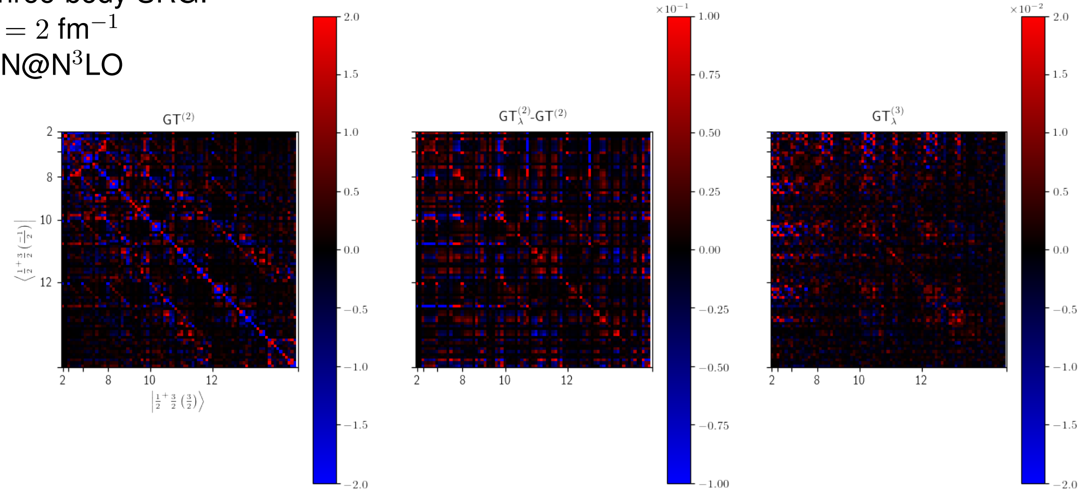
Preliminary Results: $0\nu\beta\beta_{\lambda,3b}$

Three-body SRG:

$\lambda = 2$ fm^{-1}

NN@N^{3}LO



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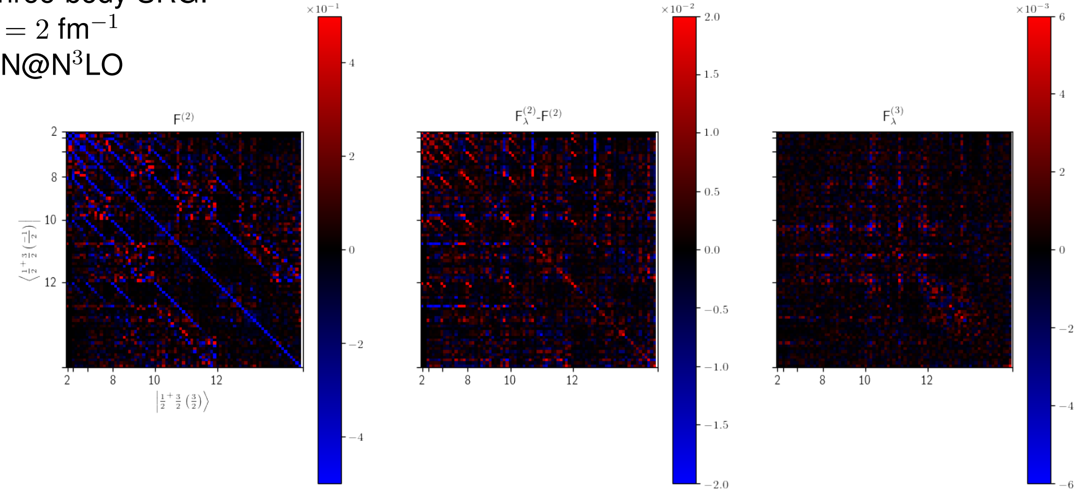

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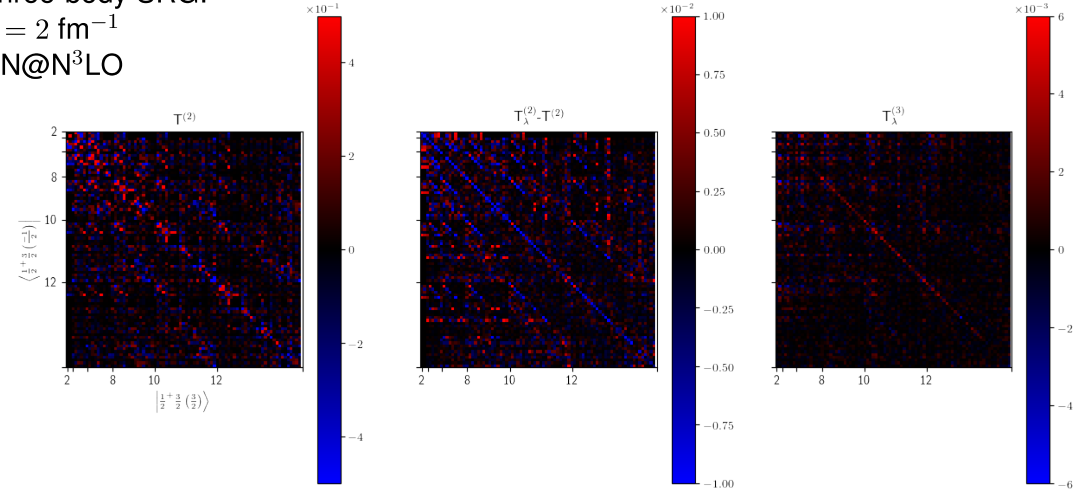


Summary and Outlook

- Operators must be SRG evolved to converge to the correct result
- Method implemented in 2B and 3B for arbitrary operators
- So far: $\sigma\tau$, axial MEC, $0\nu\beta\beta$, radius, E2
- Available in single-particle coordinates
- Results for $0\nu\beta\beta_{\lambda,2b}$: ^{8}H$\rightarrow$^{8}Be, ^{14}C$\rightarrow$^{14}O, etc agree with coupled-cluster
- In progress:
  - Application of $0\nu\beta\beta_{\lambda,3b}$ matrix elements in many-body methods
  - Quantification of 2- and 3-body evolution effects

Extra Slide: $0\nu\beta\beta$ Operators

$$\hat{O}_{0\nu\beta\beta} = \hat{O}_{GT} + \hat{O}_F + \hat{O}_T$$

$$O_\gamma = H_\gamma y_\gamma \tau_1^+ \tau_2^+$$

$$H_\gamma(r_{12}) = \frac{2R}{\pi} \int_0^\infty \mathrm{d}q \frac{q \cdot f_\gamma(q \cdot r_{12}) h_\gamma(q^2)}{q + E_0^{\text{cl}}}$$

$$y_\gamma = \begin{cases} 1 & \gamma = F \\ \sigma_1 \cdot \sigma_2 & \gamma = GT \\ \sqrt{\frac{24\pi}{5}} Y_2(\hat{r_{12}}) \left(3(\sigma_1 \cdot r_{12})(\sigma_2 \cdot r_{12}) - \sigma_1 \cdot \sigma_2\right) & \gamma = T \end{cases}$$

Ab Initio Nuclear Theory

Goal: solve the nuclear eigenvalue problem

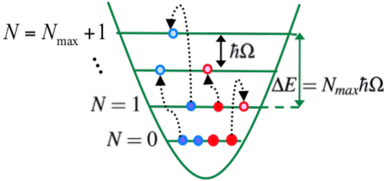
$$H\,|\Psi_k\rangle = E_k\,|\Psi_k\rangle \text{, where } H = \sum_i^A T_i + \sum_{i<j} V_{ij} + \sum_{i<j<f} V_{ijf} + \cdots$$

with nucleons as the degrees of freedom

The No-core Shell Model

Expand in anti-symmetrized products of harmonic oscillator single-particle states

$$|\Psi_k\rangle = \sum_{N=0}^{N_{max}} \sum_j c_{Nj}^k\,|\Phi_{Nj}\rangle$$



Calculations should converge to the exact value as $N_{max} \to \infty$

Motivation

- Problem: Huge model-space size required to accommodate short-range physics
- Solution: use renormalized potentials in smaller model-space
- Caveat: need renormalized operators

Similarity Renormalization Group (SRG)

Unitary transformation that decouples high and low momentum physics

$$H_\alpha = U_\alpha H U_\alpha^\dagger \text{ where } U_\alpha U_\alpha^\dagger = 1$$

$$H_{\alpha=0} = H, \; U_{\alpha=0} = 1$$

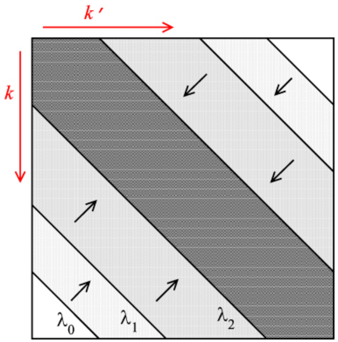
$$\frac{\mathrm{d}H_\alpha}{\mathrm{d}\alpha} = [\eta_\alpha, H_\alpha]$$

$$\eta_\alpha = \frac{\mathrm{d}U_\alpha}{\mathrm{d}\alpha} U_\alpha^\dagger = -\eta_\alpha^\dagger$$

Choose a generator, e.g. $\eta_\alpha = [T, H_\alpha]$

$$\lambda = \alpha^{-1/4}$$

$$H_{\lambda=\infty} = H, \; U_{\lambda=\infty} = 1$$



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General Operators

$$H\,|\Psi_k\rangle = E_k\,|\Psi_k\rangle \;\rightarrow\; H_\alpha\,|\Psi_{k,\alpha}\rangle = E_k\,|\Psi_{k,\alpha}\rangle$$

General operators must also be transformed:

$$\langle\Psi_f|\,\hat{O}\,|\Psi_i\rangle = \langle\Psi_{f,\alpha}|\,\hat{O}_\alpha\,|\Psi_{i,\alpha}\rangle \text{ where } \hat{O}_\alpha = U_\alpha \hat{O} U_\alpha^\dagger$$

$$U_\alpha = \sum_k |\Psi_{k,\alpha}\rangle\,\langle\Psi_k|$$

Induced many-body terms

SRG transformations introduce higher-body terms in operators:

$$U_\alpha \hat{O} U_\alpha^\dagger = \hat{O}_\alpha^{(1)} + \hat{O}_\alpha^{(2)} + \hat{O}_\alpha^{(3)} + \dots$$

Each term, $\hat{O}_\alpha^{(a)}$, must be determined in the appropriate a-body system ($a \leq A$).
E.g. if $O = O^{(2)}$:

$$O_\alpha^{(2)} = U_\alpha^{(2)} O^{(2)} U_\alpha^{\dagger(2)} \qquad U_\alpha^{(2)} = \sum \left|\psi_{\alpha,a=2}\right\rangle \left\langle\psi_{a=2}\right|$$

$$O_\alpha^{(3)} = U_\alpha^{(3)} \left\langle O^{(2)} \right\rangle^{(3)} U_\alpha^{\dagger(3)} - \left\langle O_\alpha^{(2)} \right\rangle^{(3)} \qquad U_\alpha^{(3)} = \sum \left|\psi_{\alpha,a=3}\right\rangle \left\langle\psi_{a=3}\right|$$