

# EFT approach to $0\nu\beta\beta$ : an update

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Los Alamos National Laboratory



# Outline

- Introduction: EFT for LNV and  $0\nu\beta\beta$
- $0\nu\beta\beta$  from light Majorana  $\nu$  exchange in chiral EFT
  - Summary of 2017-18 results
  - 2019 updates

# Credits and references

- Chiral EFT approach for TeV-scale LNV (dim 7, dim 9)

Prezeau, Ramsey-Musolf, Vogel hep-ph/0303205

V. C. , W. Dekens, M. Graesser, E. Mereghetti, J. de Vries,  
1806.02780, JHEP 1812 (2018) 097

- Chiral EFT approach for light Majorana neutrino (dim 5)

V. C., W. Dekens, E. Mereghetti, A. Walker-Loud,  
1710.01729, Phys.Rev. C97 (2018) no.6, 065501

V.C., W. Dekens, J. de Vries, M. Graesser, E. Mereghetti, S. Pastore, U. van Kolck,  
1802.10097, Phys.Rev.Lett. 120 (2018) no.20, 202001

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1907.11254

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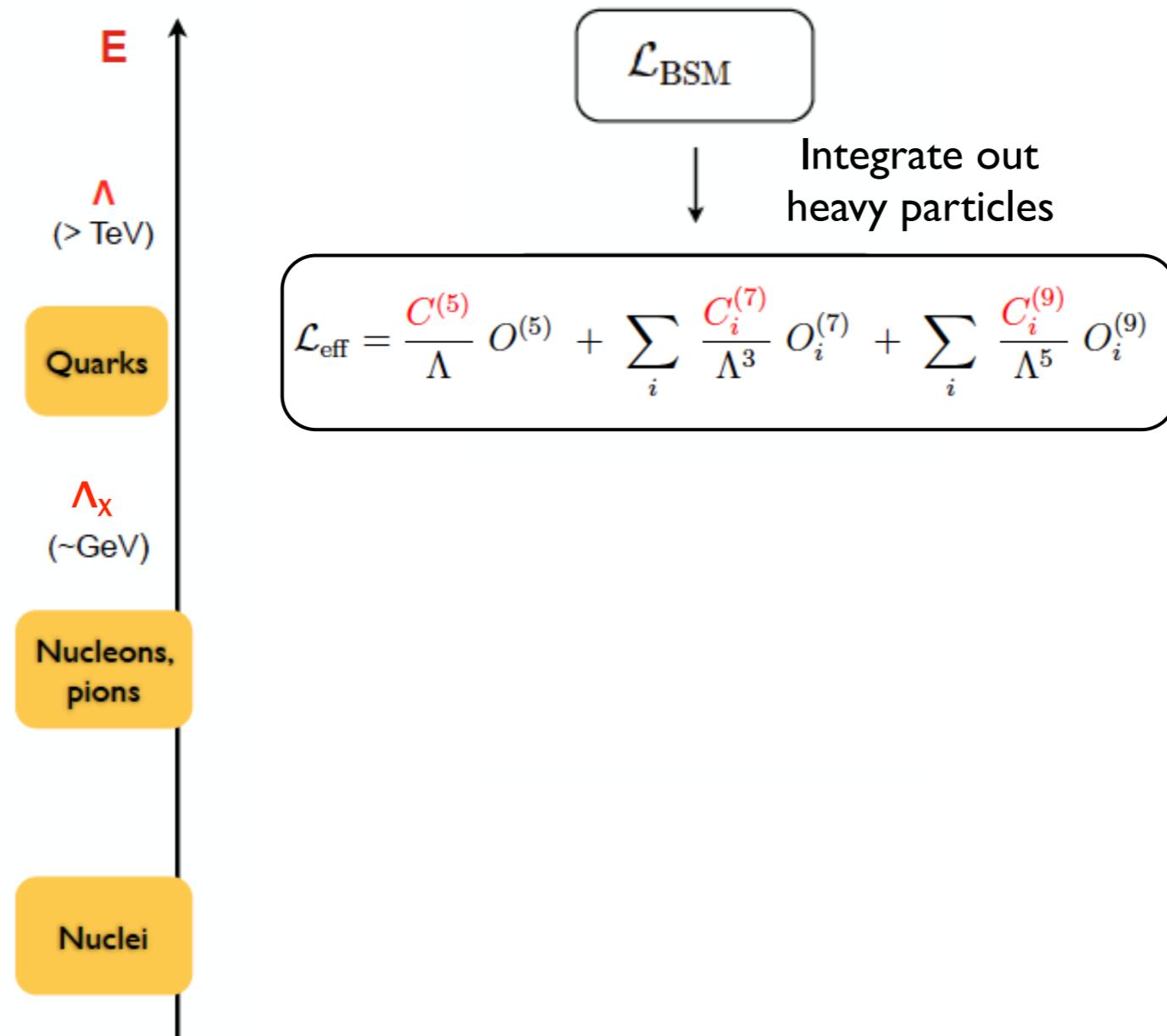
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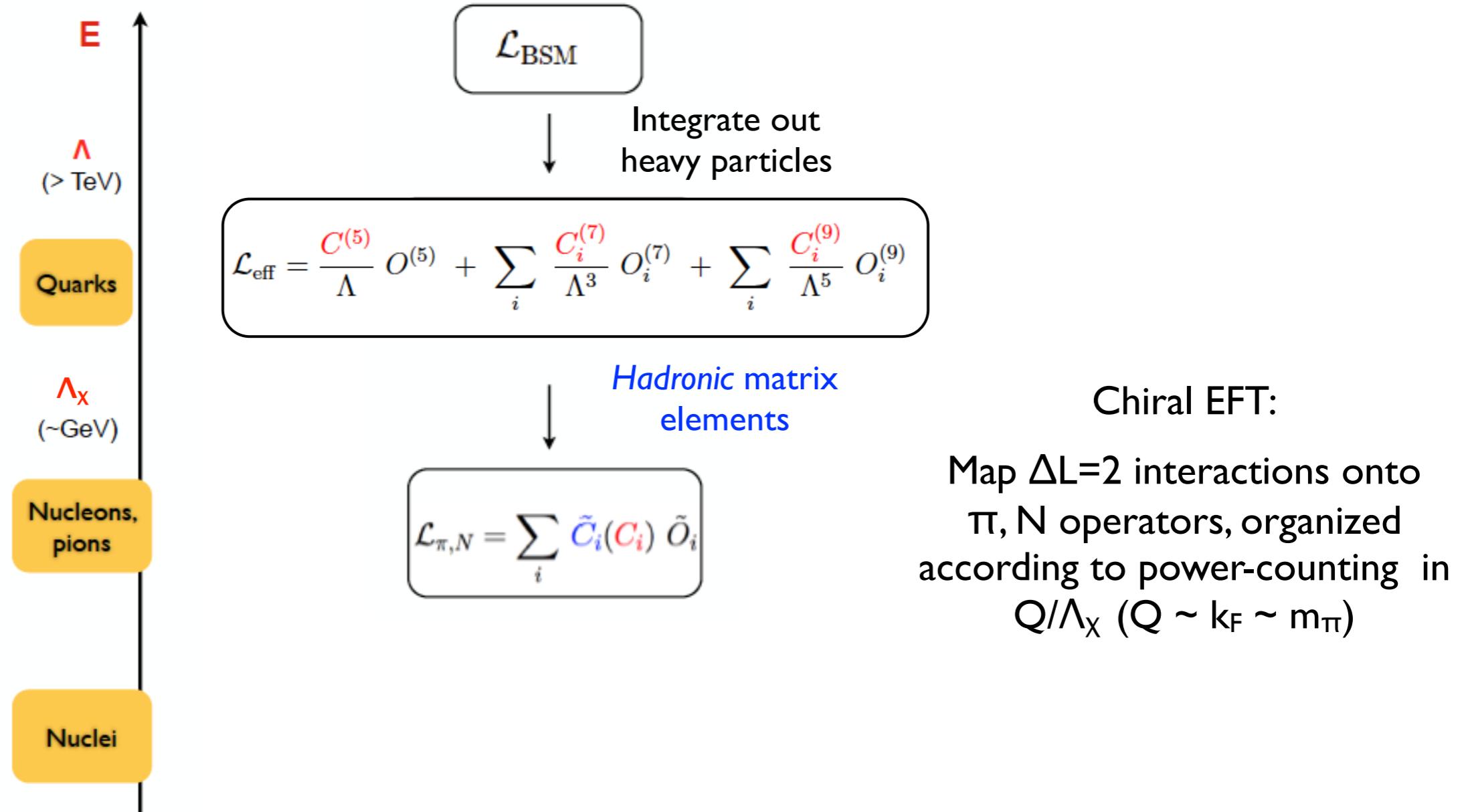
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# EFT framework for $0\nu\beta\beta$

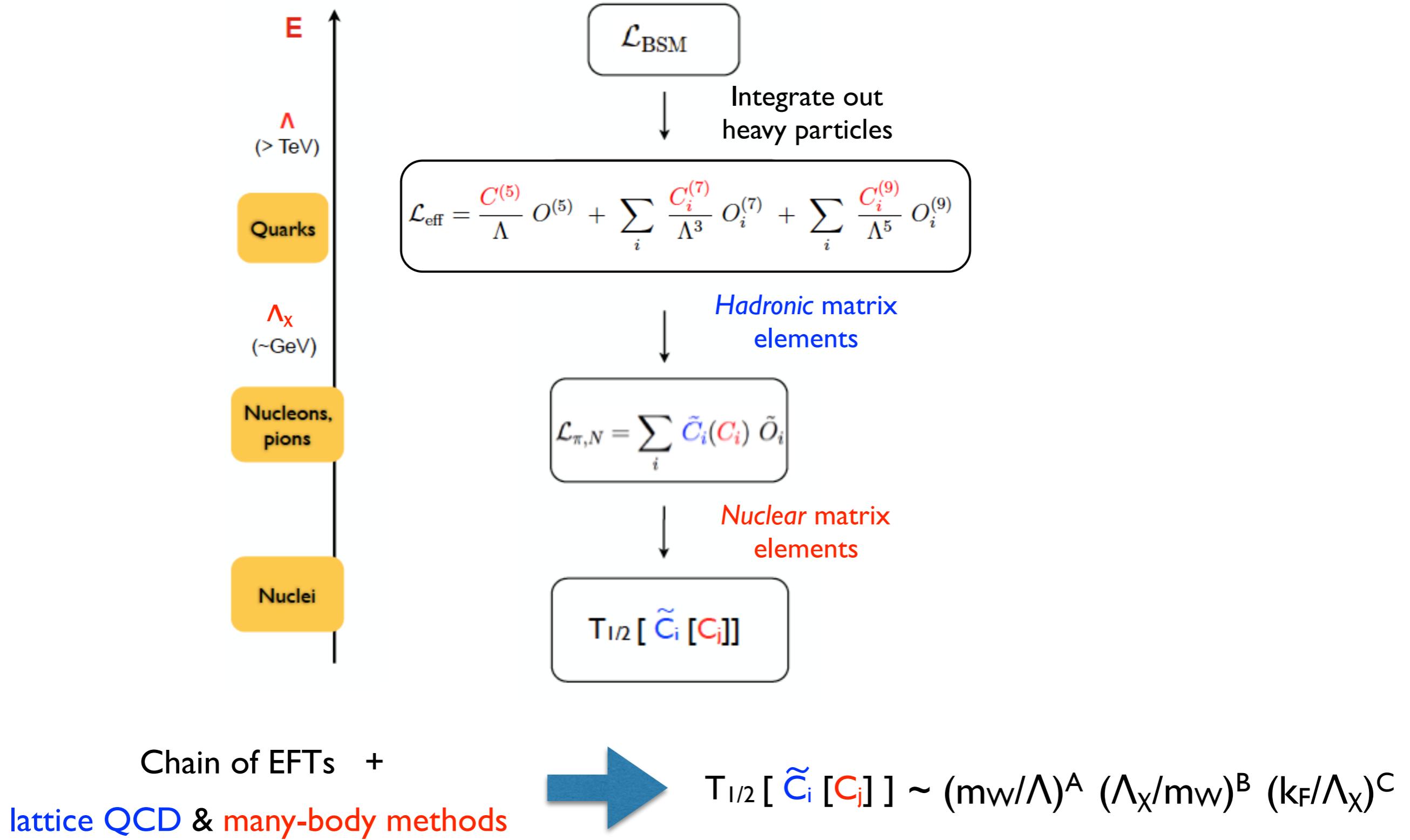


LNV in the  
“Standard Model EFT”:  
only dim=5,7,9,...

# EFT framework for $0\nu\beta\beta$



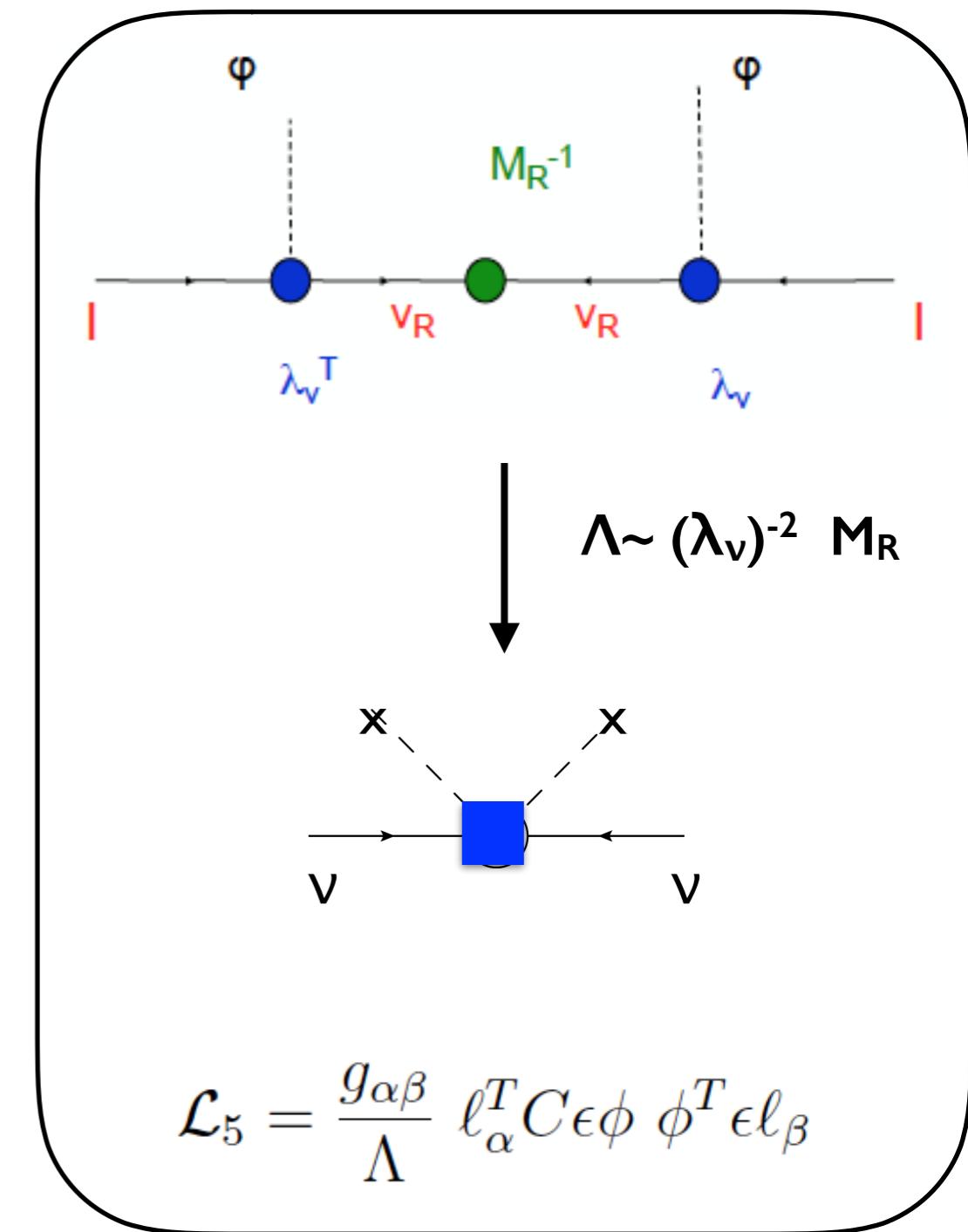
# EFT framework for $0\nu\beta\beta$



# $0\nu\beta\beta$ from light Majorana neutrino (dim-5 operator)

# Underlying mechanism

- Assume that Lepton Number Violation originates at very high scale (e.g. GUT see-saw with heavy  $\nu_R$ )
- Dominant low-energy footprint is the Weinberg operator (= Majorana mass for neutrinos written in  $SU(2)_W$ -invariant way)
- $0\nu\beta\beta$  mediated by light  $\nu_M$  exchange
- Light  $\nu_R$  states can be included ( $> 3$  light Majorana eigenstates)



# GeV-scale effective Lagrangian

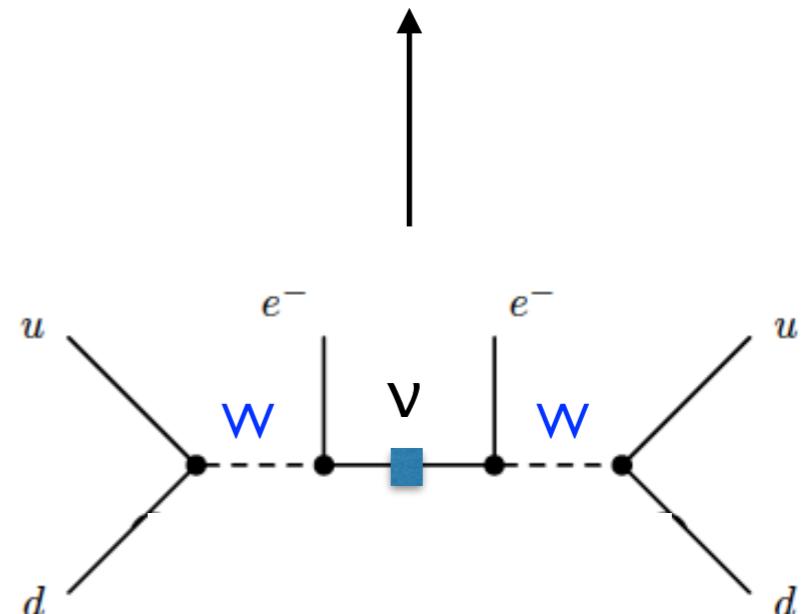
V. C., W. Dekens, E. Mereghetti, A. Walker-Loud, 1710.01729

- QCD + Fermi theory + Majorana mass + local operator

$$\mathcal{L}_{\text{eff}} = \mathcal{L}_{\text{QCD}} - \left\{ 2\sqrt{2}G_F V_{ud} \bar{u}_L \gamma^\mu d_L \bar{e}_L \gamma_\mu \nu_{eL} + \frac{1}{2}m_{\beta\beta} \nu_{eL}^T C \nu_{eL} - C_L O_L + \text{h.c.} \right\}$$

$$O_L = \bar{e}_L e_L^c \bar{u}_L \gamma_\mu d_L \bar{u}_L \gamma^\mu d_L \quad C_L = (8V_{ud}^2 G_F^2 m_{\beta\beta}) / M_W^2 \times (1 + \mathcal{O}(\alpha_s/\pi))$$

- Effect of local operator  
highly suppressed at nuclear  
level  $\sim \mathcal{O}((k_F/M_W)^2)$



# GeV-scale effective Lagrangian

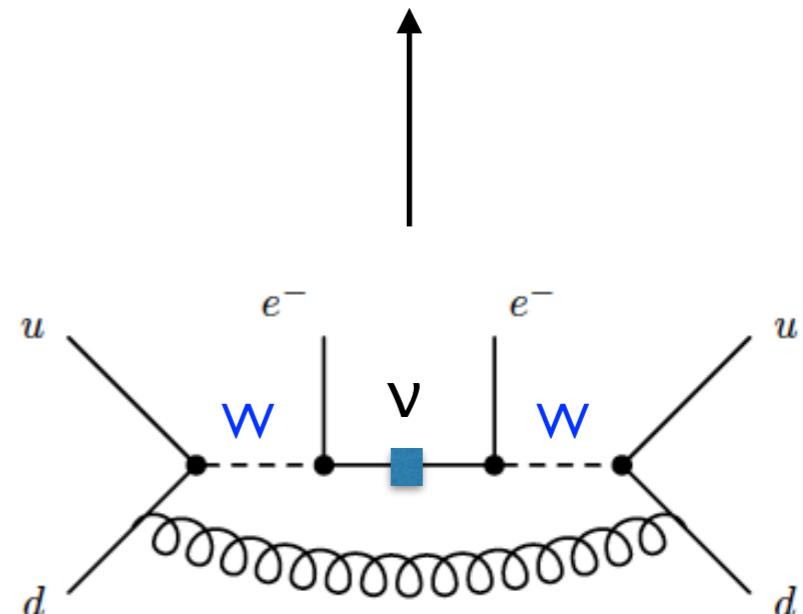
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# $\Delta L=2$ amplitudes

V. C., W. Dekens, E. Mereghetti, A. Walker-Loud, 1710.01729

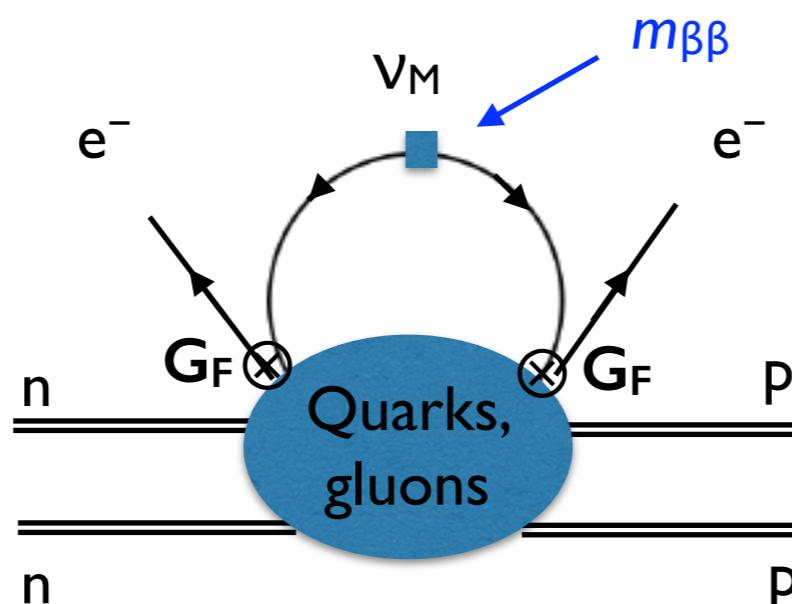
- Determined by neutrino-less non-local effective action

$$S_{\text{eff}}^{\Delta L=2} = \frac{8G_F^2 V_{ud}^2 m_{\beta\beta}}{2!} \int d^4x d^4y S(x-y) \times \bar{e}_L(x) \gamma^\mu \gamma^\nu e_L^c(y) \times T(\bar{u}_L \gamma_\mu u_L(x) \bar{u}_L \gamma_\nu u_L(y))$$

$|p_1 - p_2|/k_F \ll 1.$

Scalar massless propagator

$g^{\mu\nu} \bar{e}_L(x) e_L^c(x) + \dots$



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V. C., W. Dekens, E. Mereghetti, A. Walker-Loud, 1710.01729

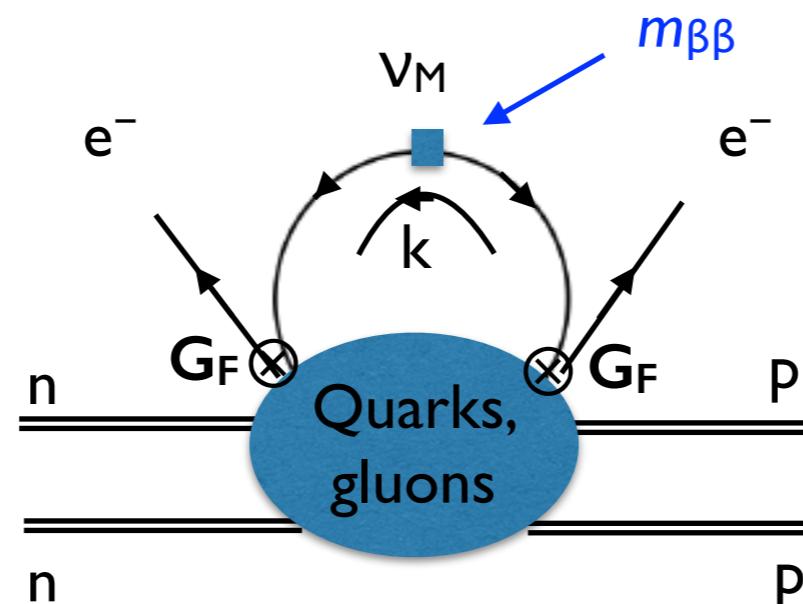
- Determined by neutrino-less non-local effective action

$$\langle e_1 e_2 h_f | S_{\text{eff}}^{\Delta L=2} | h_i \rangle = \frac{8G_F^2 V_{ud}^2 m_{\beta\beta}}{2!} \int d^4x \langle e_1 e_2 | \bar{e}_L(x) e_L^c(x) | 0 \rangle \int \frac{d^4k}{(2\pi)^4} \frac{g^{\mu\nu} \hat{\Pi}_{\mu\nu}^{++}(k, x)}{k^2 + i\epsilon},$$

$$\hat{\Pi}_{\mu\nu}^{++}(k, x) = \int d^4r e^{ik \cdot r} \langle h_f | T(\bar{u}_L \gamma_\mu d_L(x + r/2) \bar{u}_L \gamma_\mu d_L(x - r/2)) | h_i \rangle.$$

Momentum space representation

LNV hadronic amplitudes  
such as  $nn \rightarrow ppee$   
receive contributions from  
all neutrino virtual  
momenta ( $k$ )



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V. C., W. Dekens, E. Mereghetti, A. Walker-Loud, 1710.01729

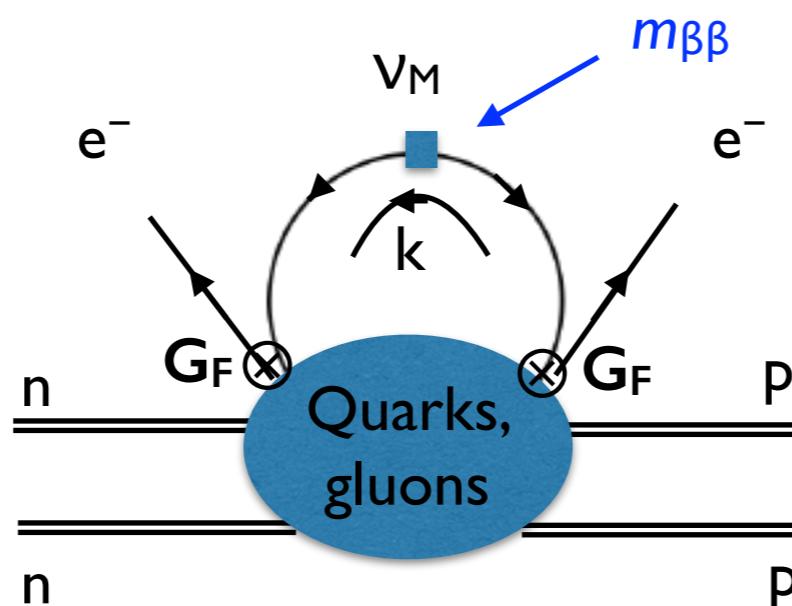
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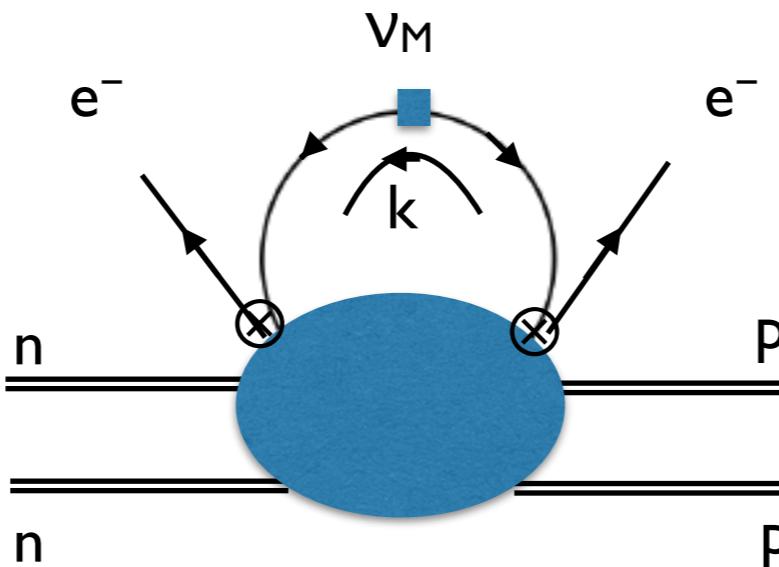
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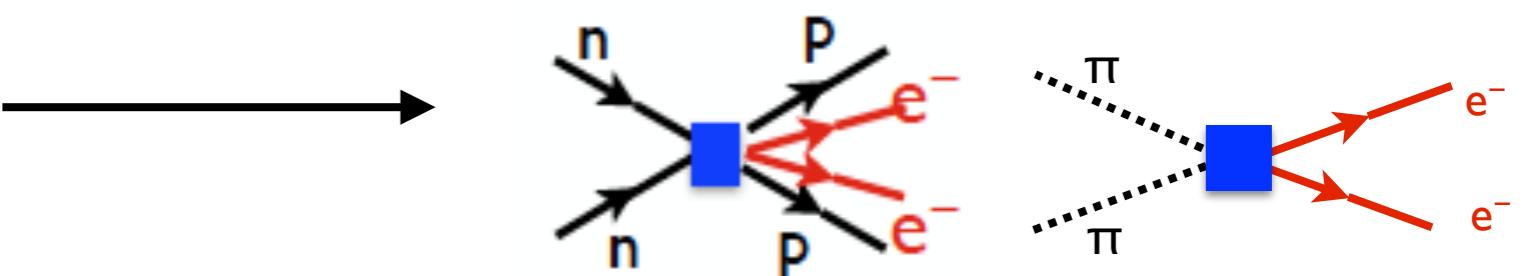
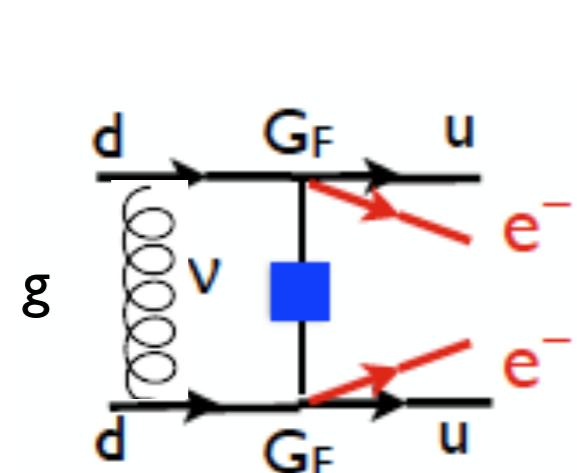
Chiral EFT captures  
contributions from all  
relevant momentum regions

# $\Delta L=2$ amplitudes in EFT



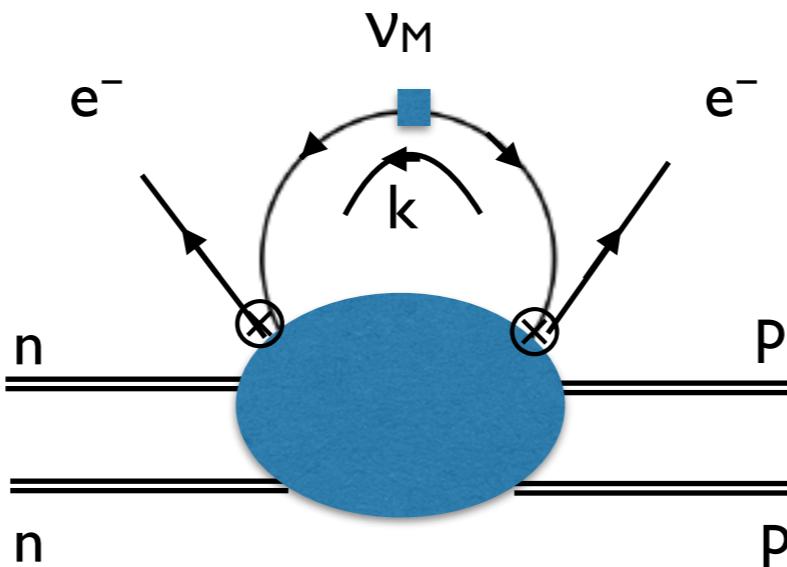
“Hard neutrinos”:  
 $E, |k| > \Lambda_X \sim m_N \sim \text{GeV}$

Short-range  $\Delta L=2$  operators at  
 the hadronic level,  
*still proportional to  $m_{\beta\beta}$*



Short- and pion-range contributions to  
 “Neutrino potential” mediating  $nn \rightarrow pp$

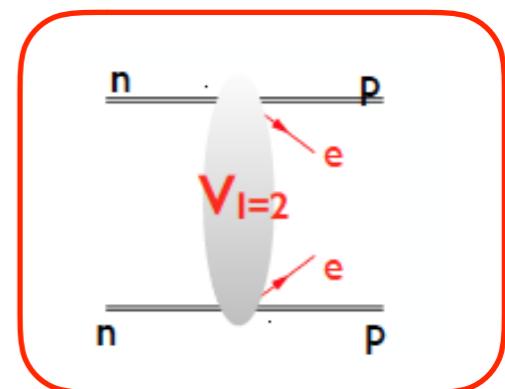
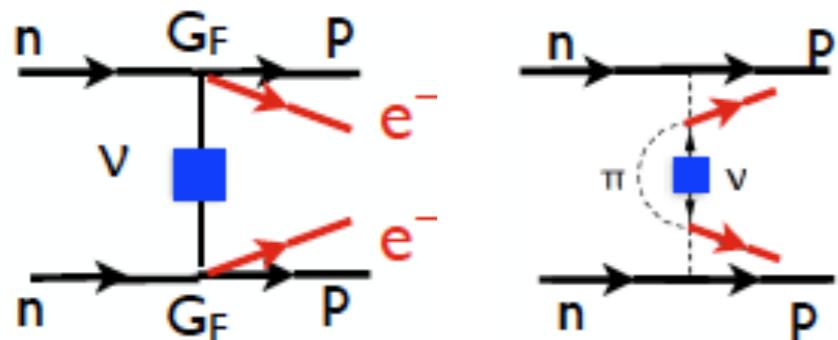
# $\Delta L=2$ amplitudes in EFT



“Soft” & “Potential” neutrinos:

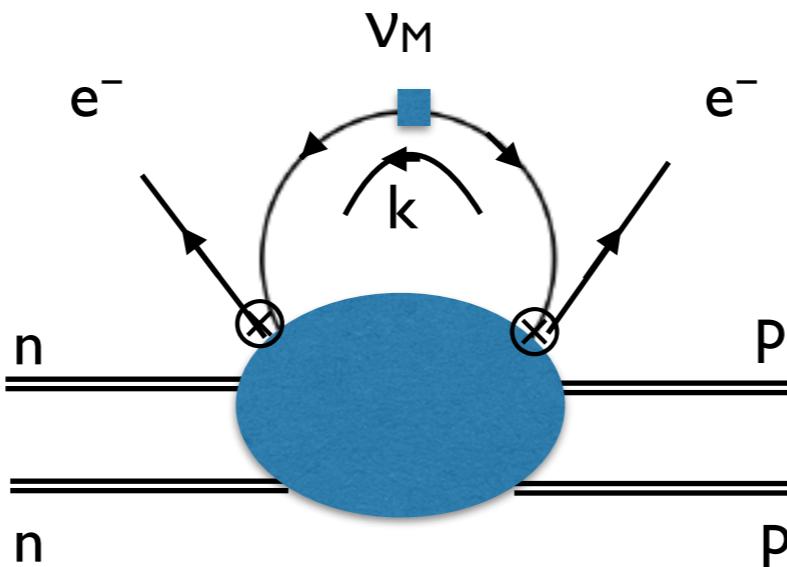
$$(E, |k|) \sim Q \sim k_F \sim m_\pi$$

$$(E, |k|) \sim (Q^2/m_N, Q)$$



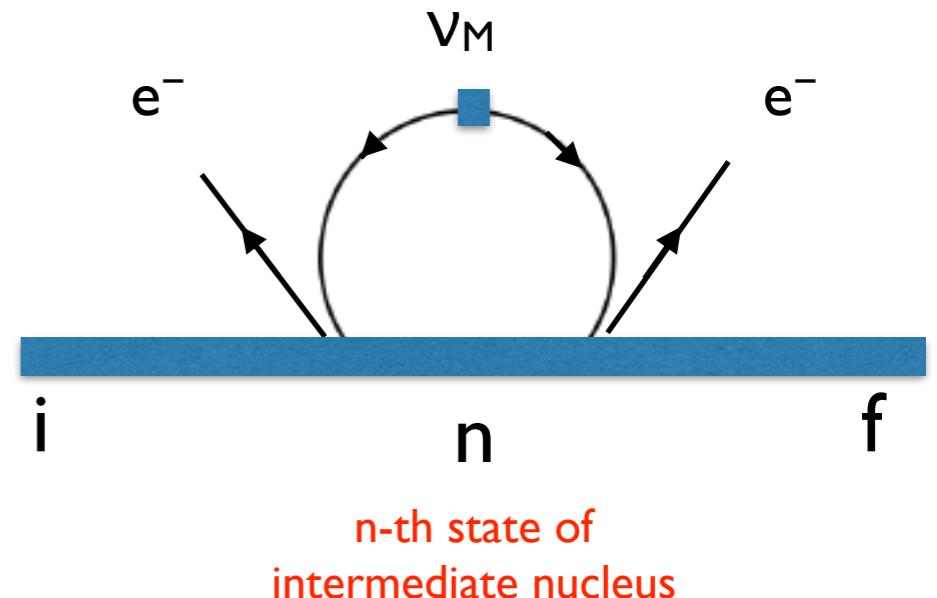
Calculable long- and pion-range contributions to “Neutrino potential” mediating  $nn \rightarrow pp$

# $\Delta L=2$ amplitudes in EFT



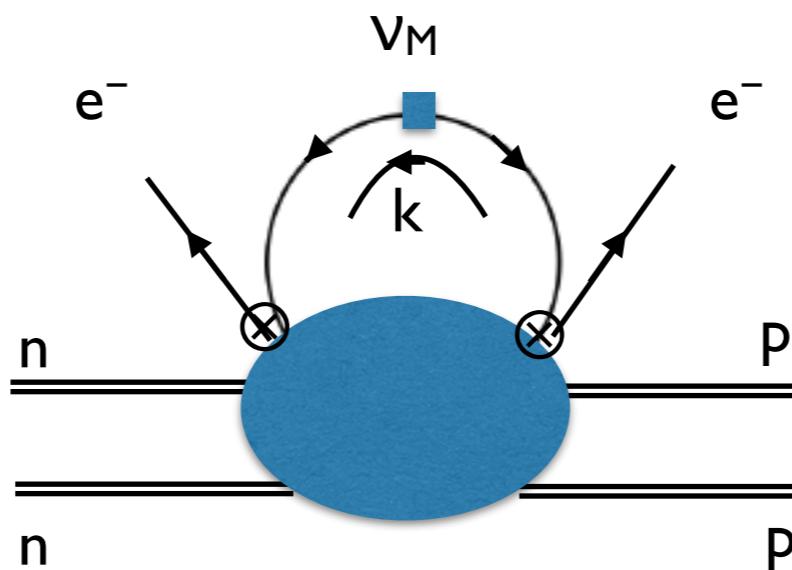
“UltraSoft” neutrinos:

$$(E, |k|) \ll k_F$$



Double insertions of the  
weak current at the  
hadronic / nuclear level

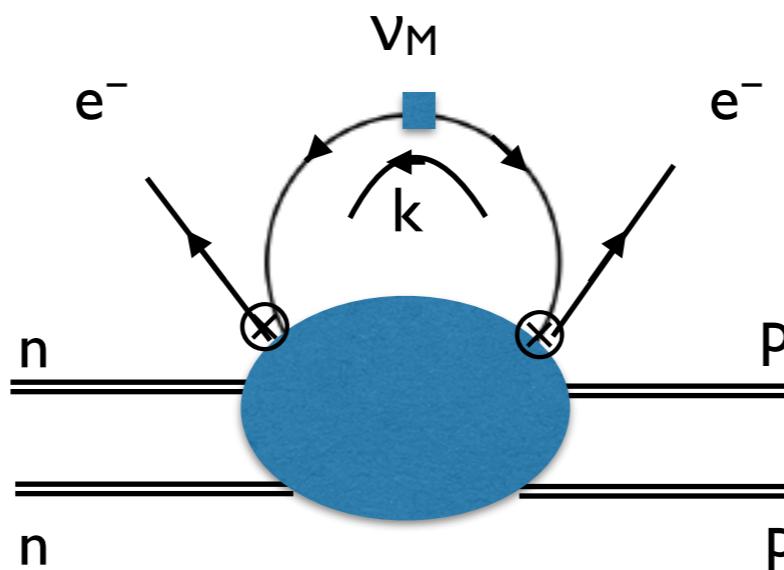
# Nuclear scale effective Hamiltonian



Kinetic terms and strong NN potential

$$H_{\text{Nucl}} = H_0 + \sqrt{2}G_F V_{ud} \bar{N} (g_V \delta^{\mu 0} - g_A \delta^{\mu i} \sigma^i) \tau^+ N \bar{e}_L \gamma_\mu \nu_L + 2G_F^2 V_{ud}^2 m_{\beta\beta} \bar{e}_L e_L^c V_{I=2}$$

# Nuclear scale effective Hamiltonian

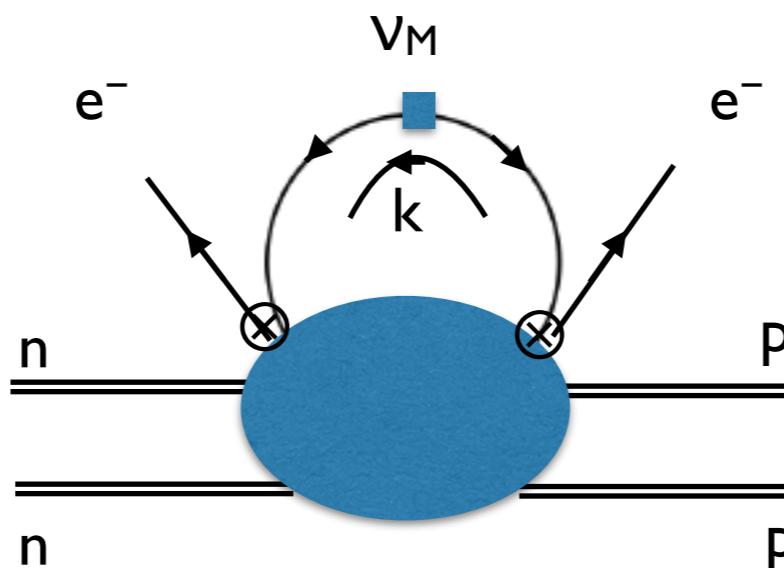


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“Ultra-soft” ( $e, \nu$ ) with  $|p|, E \ll k_F$   
cannot be integrated out

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“Ultra-soft” ( $e, \nu$ ) with  $|p|, E \ll k_F$   
cannot be integrated out

“Isotensor”  $0\nu\beta\beta$  potential mediates  $nn \rightarrow pp$ .  
It can be identified to a given order in  $Q/\Lambda_X$  by  
computing 2-nucleon amplitudes

# Anatomy of $0\nu\beta\beta$ amplitude

Figure adapted from Primakoff-Rosen 1969



Hard, soft, and potential  $\nu$

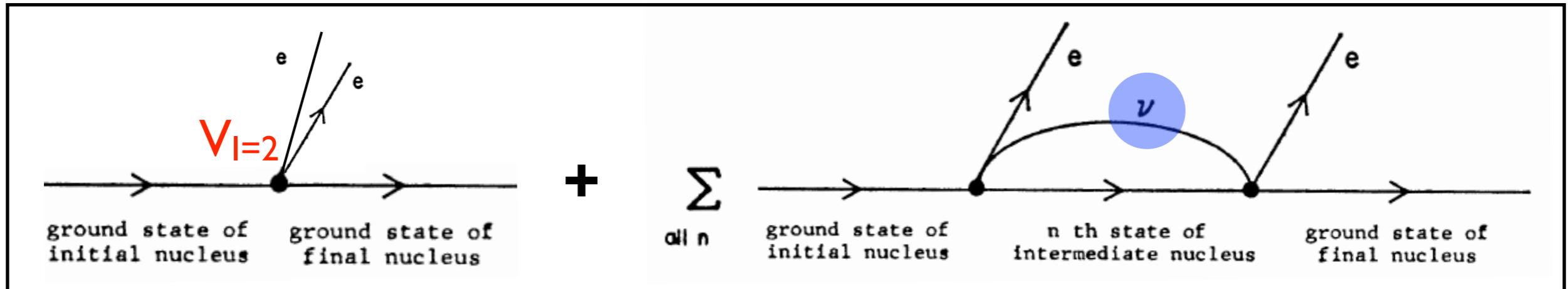
$$V_{I=2} = \sum_{a \neq b} \left( V_{\nu,0}^{(a,b)} + V_{\nu,2}^{(a,b)} + \dots \right)$$

$$V_\nu \sim 1/Q^2, 1/(\Lambda_X)^2, \dots$$

↑      ↑  
LO      N<sup>2</sup>LO

# Anatomy of $0\nu\beta\beta$ amplitude

Figure adapted from Primakoff-Rosen 1969



Hard, soft, and potential  $\nu$

Ultrasoft  $\nu$

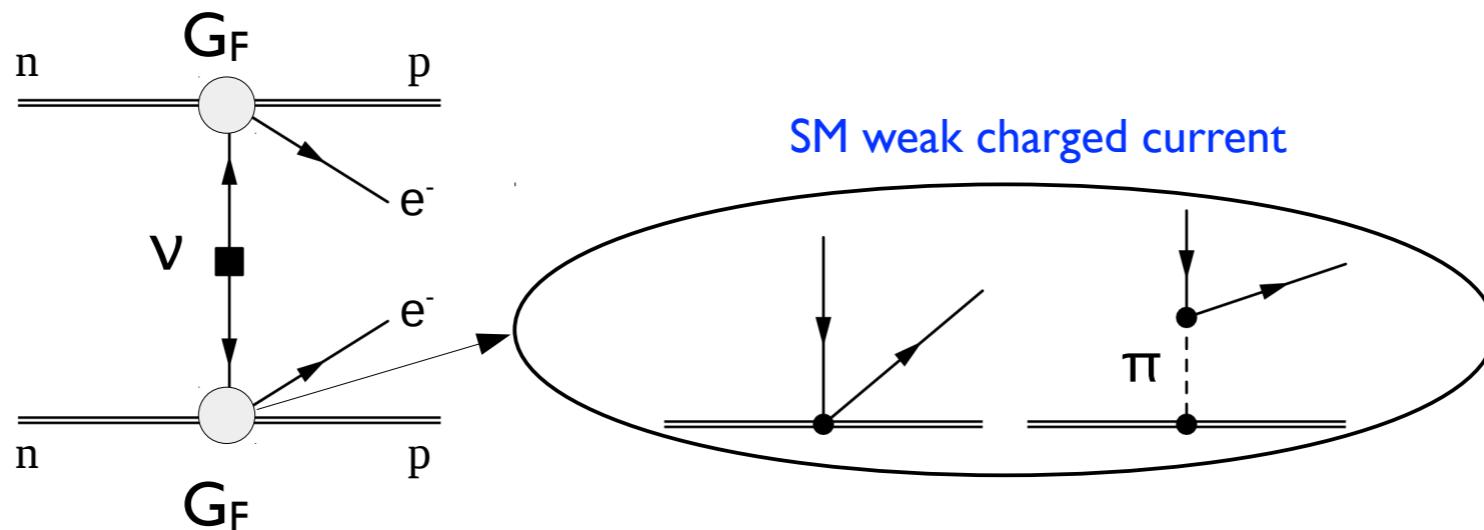
$$V_{I=2} = \sum_{a \neq b} \left( V_{\nu,0}^{(a,b)} + V_{\nu,2}^{(a,b)} + \dots \right)$$

Loop calculable in terms of  $E_n - E_i$  and  $\langle f | J_\mu | n \rangle \langle n | J^\mu | i \rangle$ , that also control  $2\nu\beta\beta$ .  
Contributes to the amplitude at  $N^2LO$

$$V_\nu \sim 1/Q^2, 1/(\Lambda_X)^2, \dots$$

↑      ↑  
LO      N<sup>2</sup>LO

# Leading order $0\nu\beta\beta$ potential

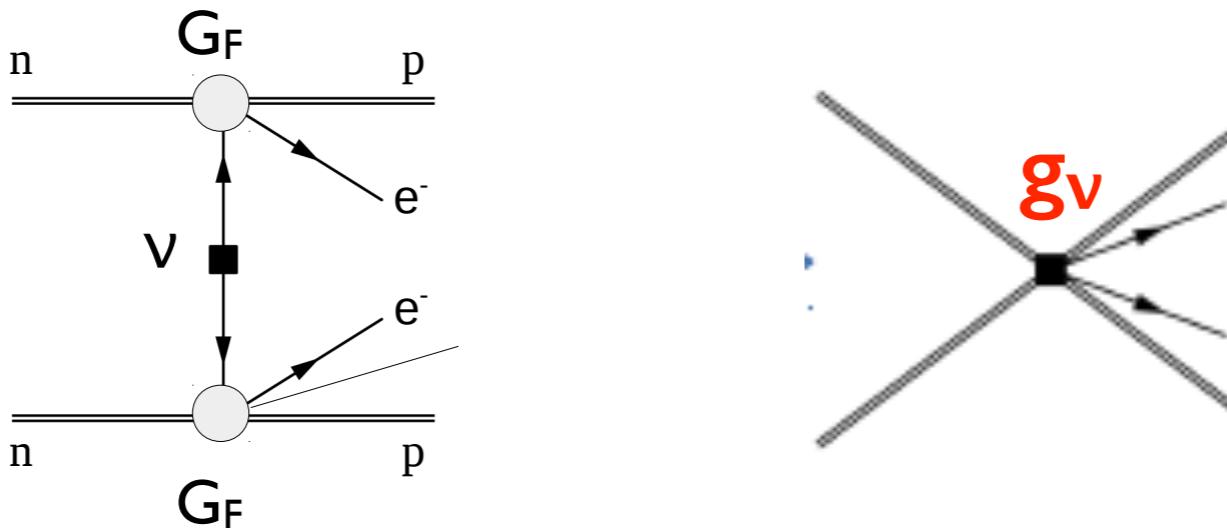


- Tree-level  $\nu_M$  exchange

$$V_{\nu,0}^{(a,b)} = \tau^{(a)} + \tau^{(b)} + \frac{1}{q^2} \left\{ 1 - g_A^2 \left[ \sigma^{(a)} \cdot \sigma^{(b)} - \sigma^{(a)} \cdot q \sigma^{(b)} \cdot q \frac{2m_\pi^2 + q^2}{(q^2 + m_\pi^2)^2} \right] \right\}$$

Hadronic  
input:  $g_A$

# Leading order $0\nu\beta\beta$ potential



- Tree-level  $\nu_M$  exchange

$$V_{\nu,0}^{(a,b)} = \tau^{(a)+} \tau^{(b)+} \frac{1}{q^2} \left\{ 1 - g_A^2 \left[ \sigma^{(a)} \cdot \sigma^{(b)} - \sigma^{(a)} \cdot q \sigma^{(b)} \cdot q \frac{2m_\pi^2 + q^2}{(q^2 + m_\pi^2)^2} \right] \right\}$$

Hadronic input:  $g_A$

- Short-range coupling  $g_\nu \sim 1/Q^2 \sim 1/k_F^2$  (only in  ${}^1S_0$  channel) required by renormalization of  $nn \rightarrow ppee$  amplitude

$$V_{\nu,CT}^{(a,b)} = -2 g_\nu \tau^{(a)+} \tau^{(b)+}$$

$g_\nu \sim 1/\Lambda^2 \sim 1/(4\pi F_\pi)^2$  in NDA / Weinberg counting

# Scaling of contact term

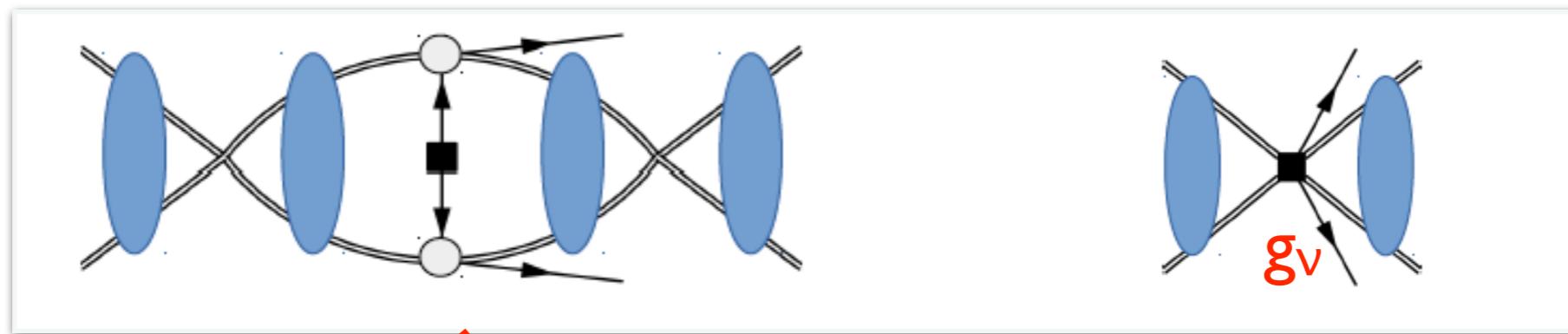
Weinberg 1991, Kaplan-Savage-Wise 1996

- Study  $nn \rightarrow ppee$  amplitude (in  $^1S_0$  channel) with LO strong potential



$$\tilde{C} \sim 4\pi/(m_N Q) \sim l/F_\pi^2 \text{ from fit to } a_{NN}$$

- Renormalization requires contact LNV operator at LO!



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1802.10097,  
Phys.Rev.Lett. 120 (2018)  
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$$\sim \frac{1}{2}(1 + 2g_A^2) \left( \frac{m_N \tilde{C}}{4\pi} \right)^2 \left( \frac{1}{4-d} + \log \mu^2 \right)$$

- The coupling flows to  $g_V \sim l/Q^2 \gg l/(4\pi F_\pi)^2$ , same order as  $l/q^2$  from tree-level neutrino exchange

# Use different regulator and scheme

- Same conclusion obtained by solving the Schroedinger equation
  - Use smeared delta function to regulate short range strong potential:  
 $\tilde{C} \rightarrow \tilde{C}(R_s)$
  - Compute amplitude

$$\delta^{(3)}(\mathbf{r}) \rightarrow \frac{1}{\pi^{3/2} R_s^3} e^{-\frac{r^2}{R_s^2}}$$

$$A_\nu = \int d^3\mathbf{r} \ \psi_{\mathbf{p}'}^-(\mathbf{r}) V_{\nu,0}(\mathbf{r}) \psi_{\mathbf{p}}^+(\mathbf{r})$$

Scattering states “fully correlated” according to the leading order strong potential in the  ${}^1S_0$  channel

# Use different regulator and scheme

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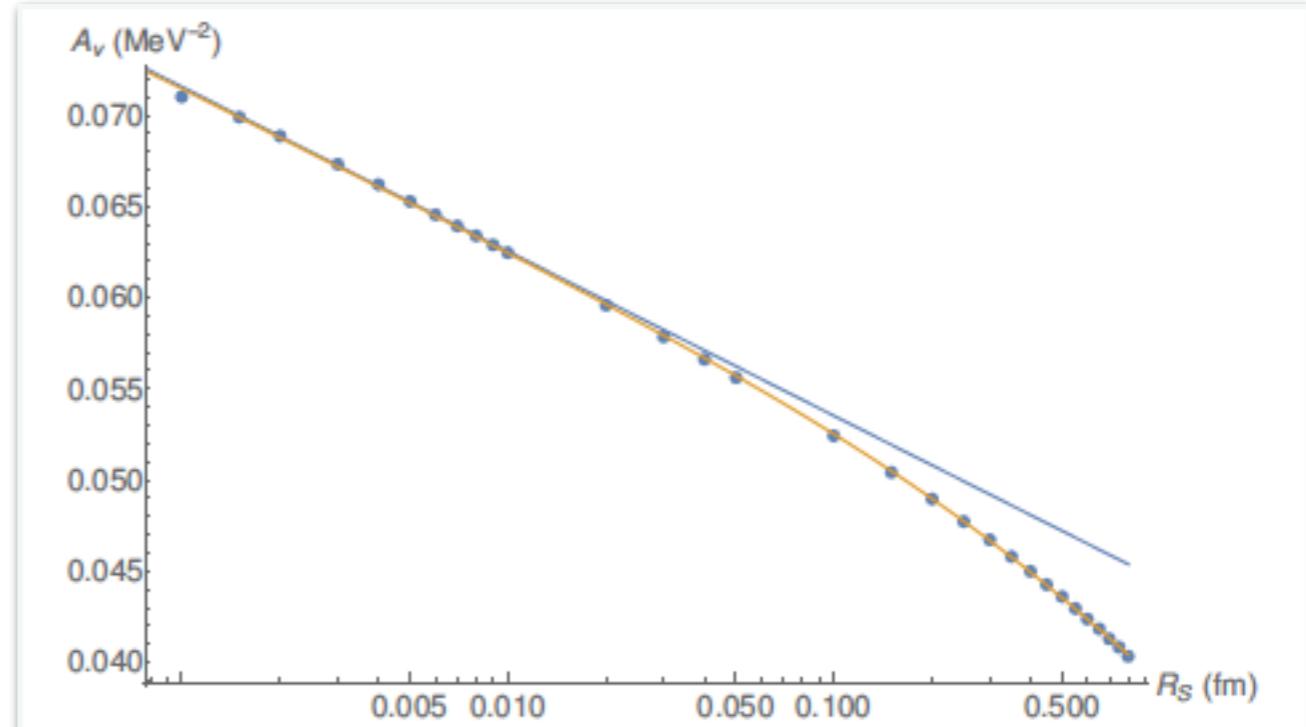
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- Compute amplitude

$$A_\nu = \int d^3\mathbf{r} \ \psi_{\mathbf{p}'}^-(\mathbf{r}) V_{\nu,0}(\mathbf{r}) \psi_{\mathbf{p}}^+(\mathbf{r})$$

- Logarithmic dependence of  $A_\nu$  on  $R_s \Rightarrow$   
need LO counterterm  
to obtain physical,  
regulator-independent  
result



# LO contact in higher waves?

- In  $^3P_0$ , renormalization of pion potential requires a LO contact term

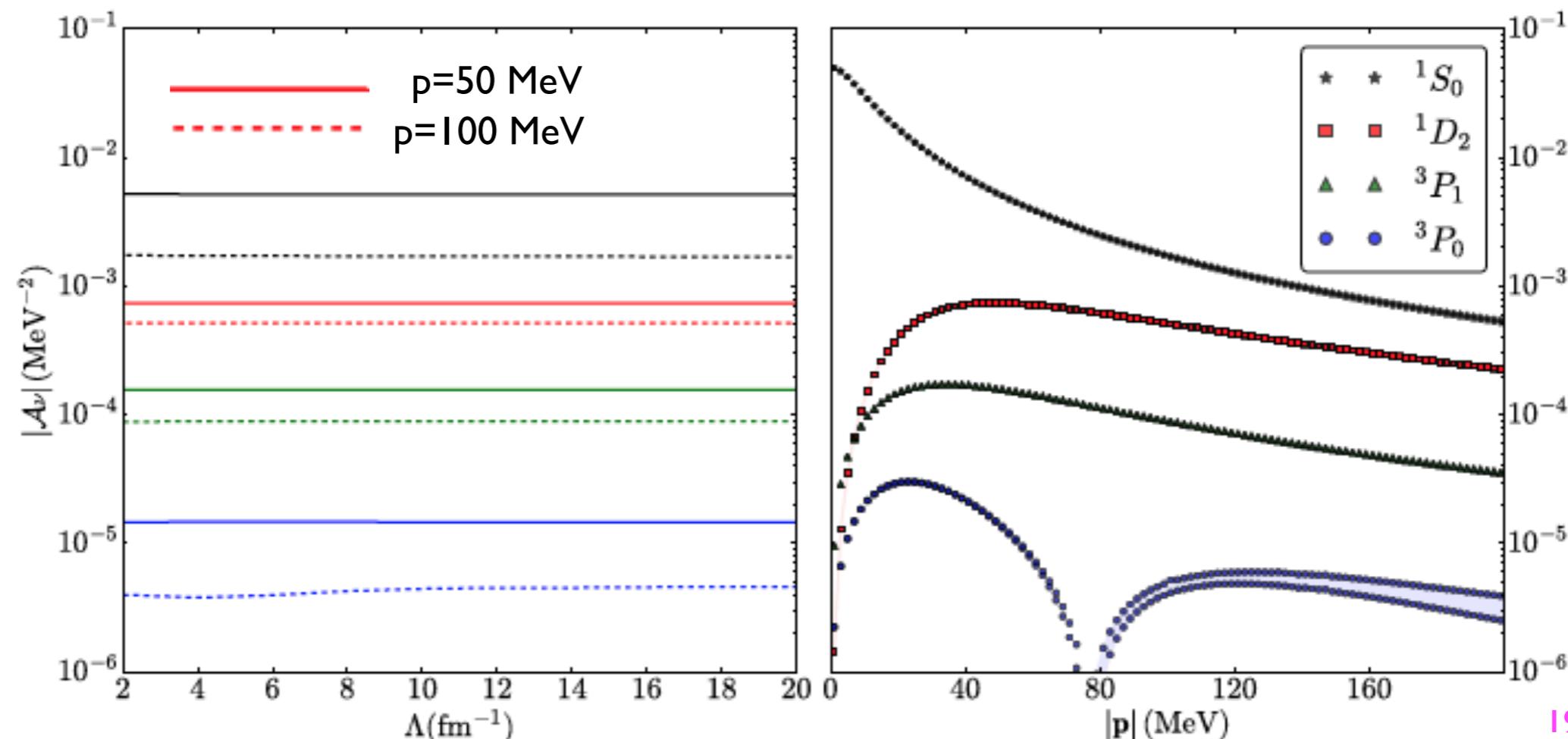
Nogga et al, nucl-th/0506005

$$C_{^3P_0} \left( N^T \vec{P}_{^3P_0} N \right)^\dagger \cdot \left( N^T \vec{P}_{^3P_0} N \right)$$

$$\vec{P}_{^3P_0} = -\frac{i}{\sqrt{8}} \sigma_2 \boldsymbol{\sigma} \cdot \vec{\nabla}_{\tau_2} \vec{\tau}$$

$$C_{^3P_0} = \mathcal{O}(4\pi/(m_N Q^3))$$

- Once strong NN force is renormalized, no cutoff dependence in neutrino-mediated  $nn \rightarrow pp$

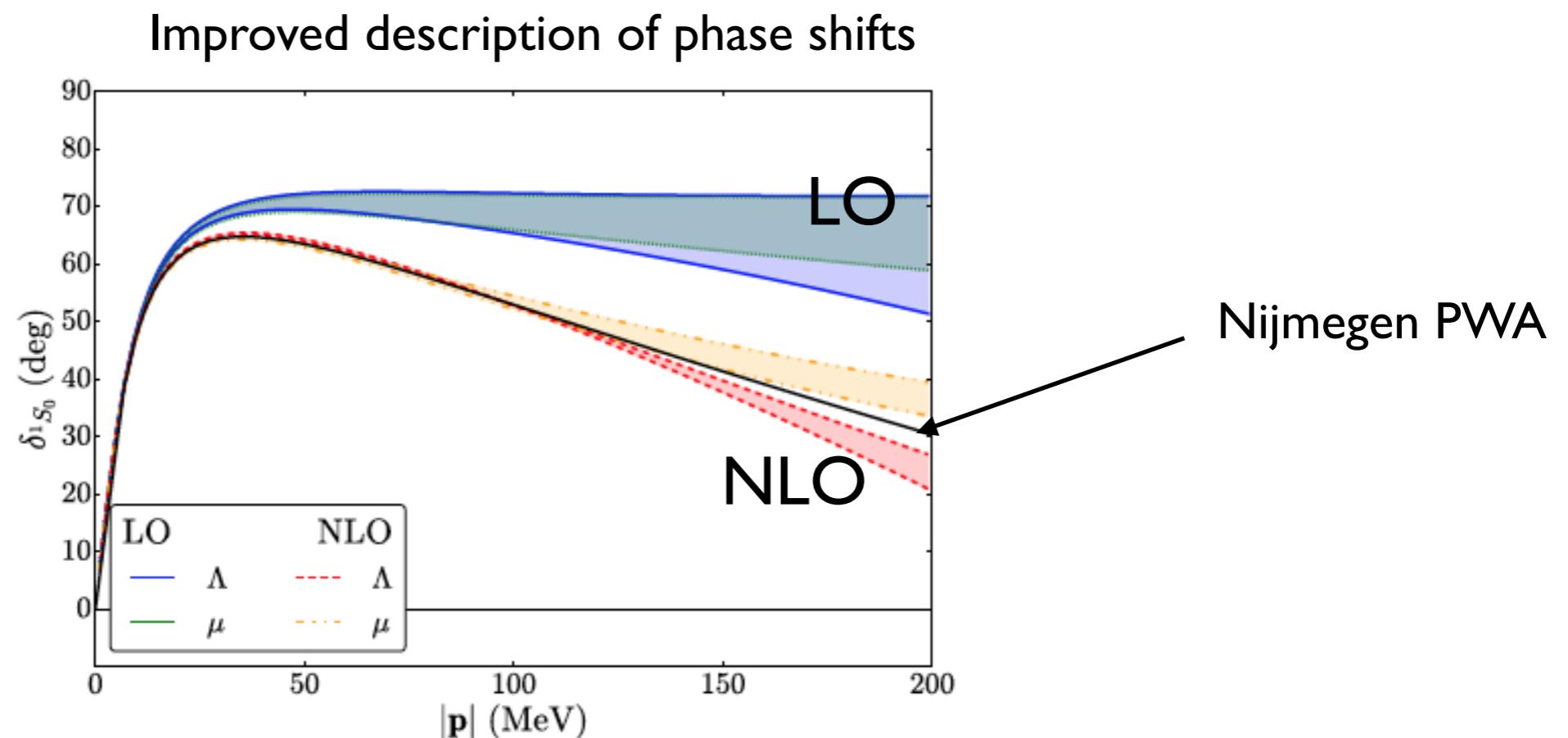


# NLO contact in $^1S_0$ channel?

V.C , W. Dekens, J. de Vries, M. Graesser, E. Mereghetti, S. Pastore, M. Piarulli, U. van Kolck, R. Wiringa, 1907.11254

- Introduce  $V_{\text{Strong}, I} \sim C_2 N D^2 N \bar{N} \bar{N}$  with  $C_2 \sim 4\pi/(MQ^2\Lambda)$

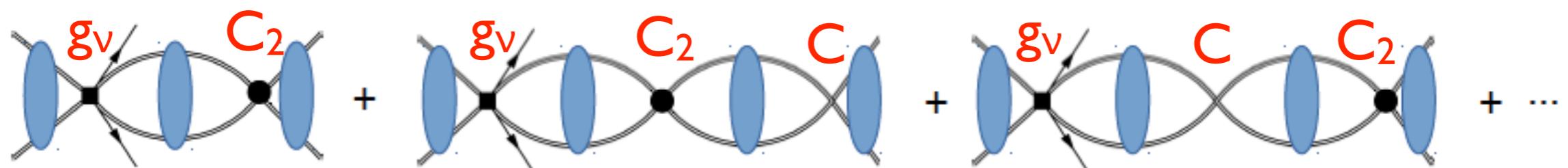
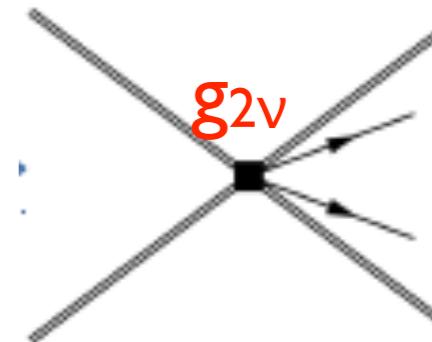
Long-Yang 1202.4053



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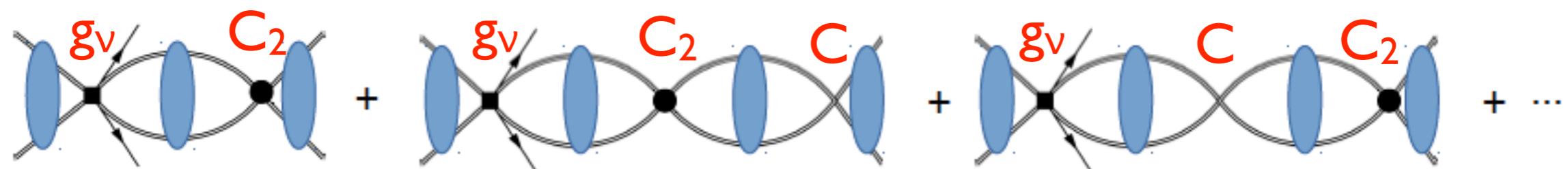
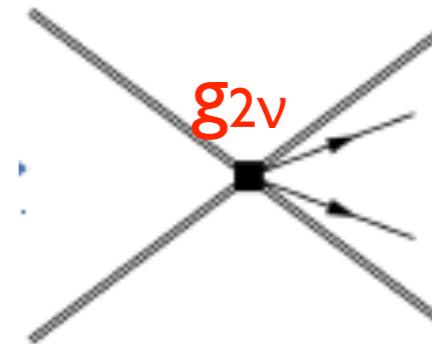
- Introduce  $V_{\text{Strong}, I} \sim C_2 N D^2 N \text{NN}$  with  $C_2 \sim 4\pi/(MQ^2\Lambda)$   
Long-Yang 1202.4053
- Do we need new LNV short range coupling ter at NLO?  
( $V_{v,I} \sim g_{2v} N D^2 N \text{NN}$ )
- RGE imply that  $g_{2v}$  has an “NLO” term  $\sim I/(\Lambda Q^3)$  determined by LO couplings and effective range parameter + unknown N2LO piece



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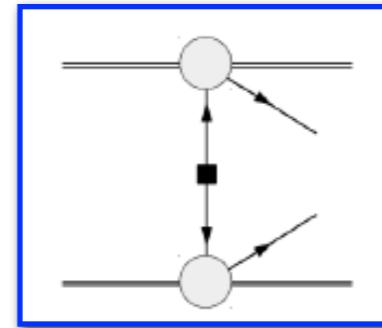
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- RGE imply that  $g_{2v}$  has an “NLO” term  $\sim I/(\Lambda Q^3)$  determined by LO couplings and effective range parameter + unknown N2LO piece



No new parameter needed at NLO

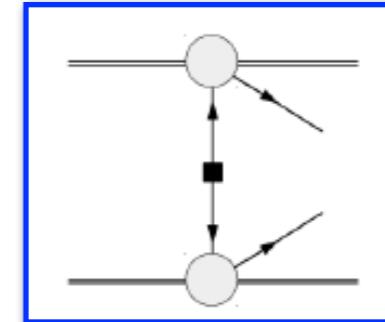
# $N^2LO$ $0\nu\beta\beta$ potential

- Known factorizable corrections  
to 1-body currents (radii, ...)

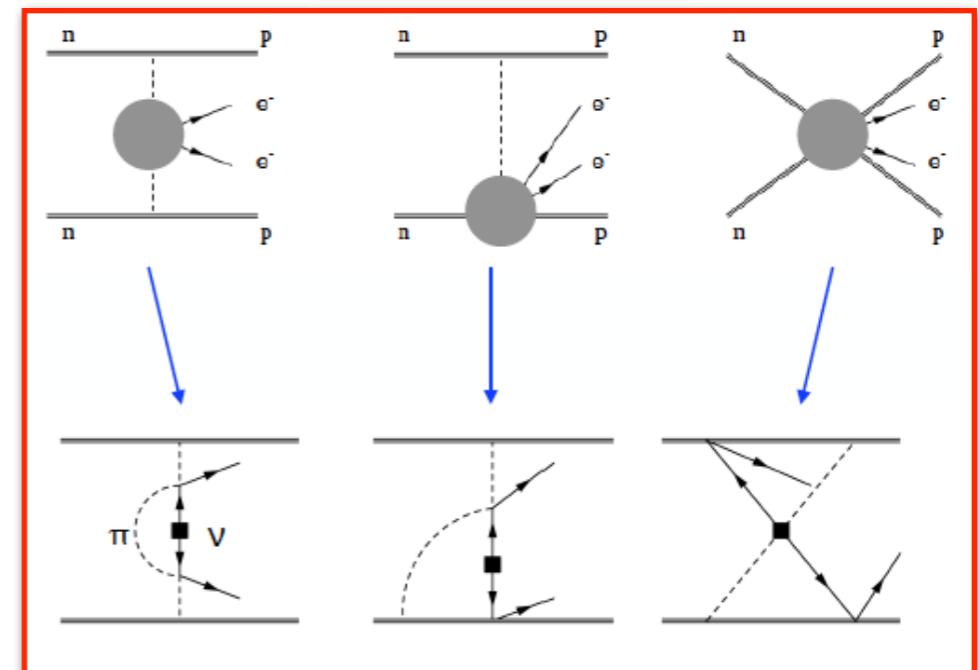


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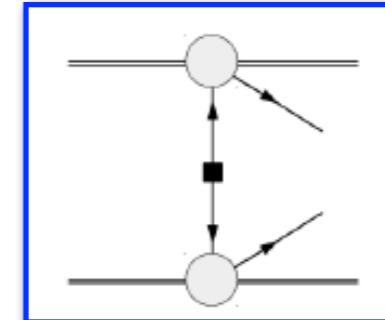
- New non-factorizable contributions to  $V_{v,2} \sim V_{v,0} (k_F/4\pi F_\pi)^2$  [ $\pi$ -N loops and new contact terms]



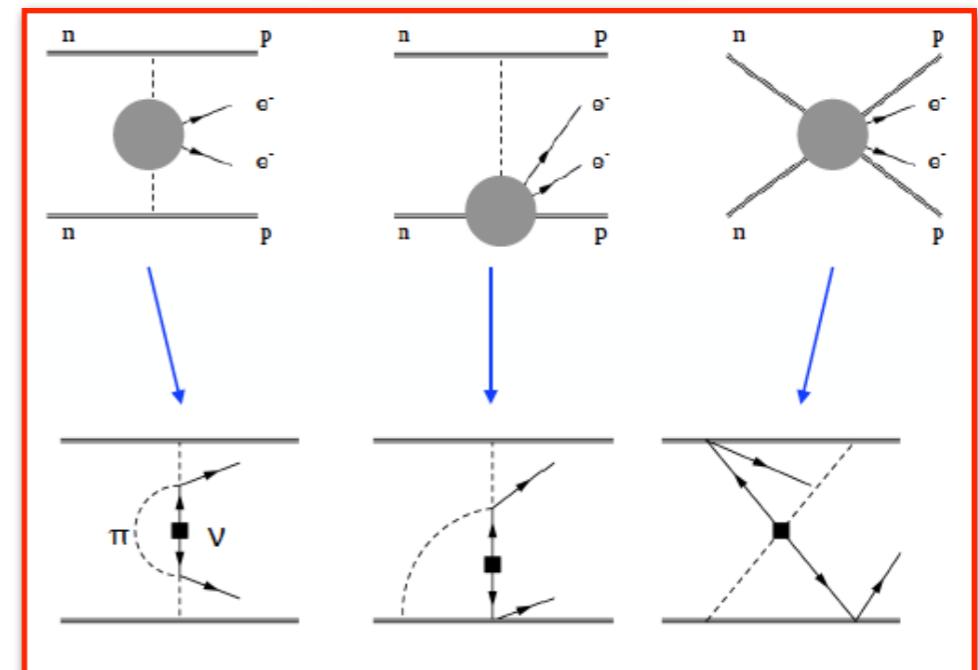
V. C., W. Dekens, E. Mereghetti, A. Walker-Loud, 1710.01729

# N<sup>2</sup>LO 0νββ potential

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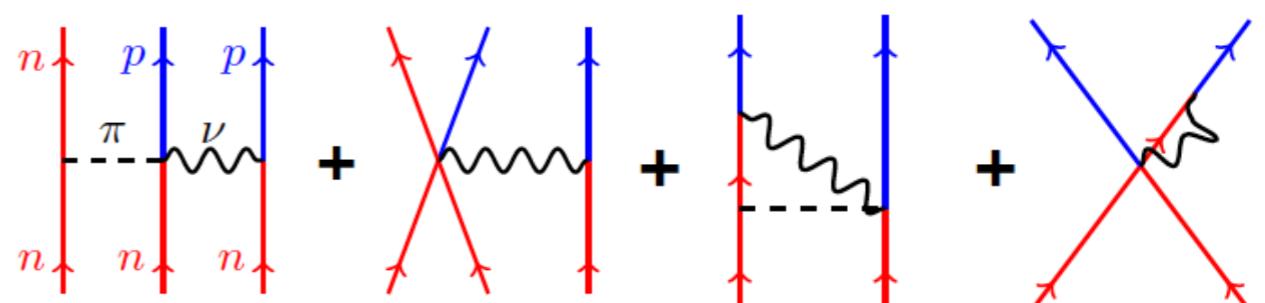


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V. C., W. Dekens, E. Mereghetti, A. Walker-Loud, 1710.01729

- 2-body x 1-body current (and another contact...)



Wang-Engel-Yao 1805.10276

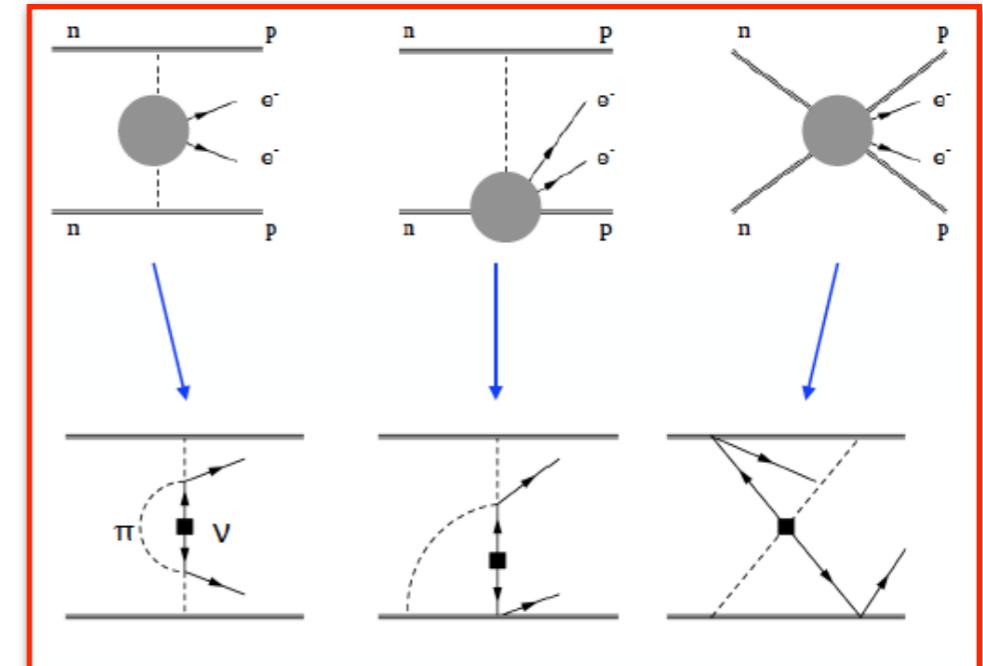
# N<sup>2</sup>LO 0νββ potential

Calculations of these effects in light and heavy nuclei show O(10%) corrections

S. Pastore, J. Carlson, V.C., W. Dekens, E. Mereghetti, R. Wiringa 1710.05026

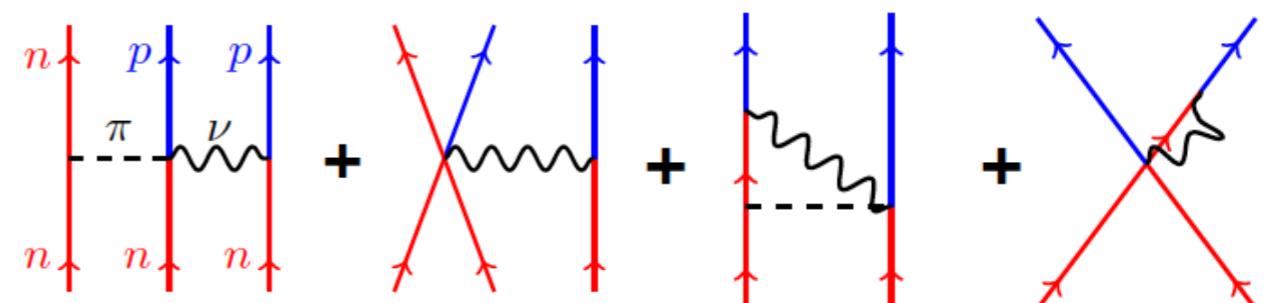
V.C., J. Engel, E. Mereghetti, in preparation

- **New non-factorizable contributions to  $V_{\nu,2} \sim V_{\nu,0} (k_F/4\pi F_\pi)^2$  [ $\pi$ -N loops and new contact terms]**



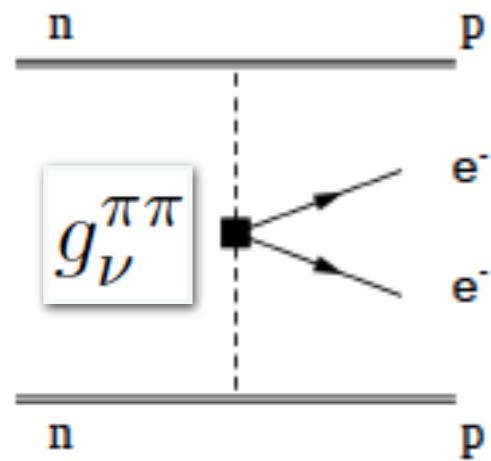
V. C., W. Dekens, E. Mereghetti, A. Walker-Loud, 1710.01729

- **2-body x 1-body current (and another contact...)**

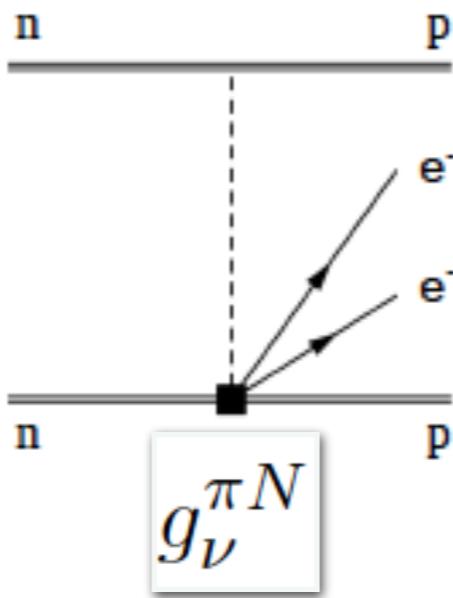


Wang-Engel-Yao 1805.10276

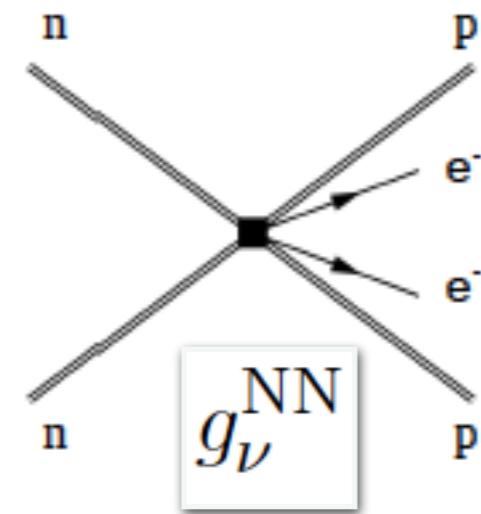
# Estimating the LECs



N2LO



N2LO



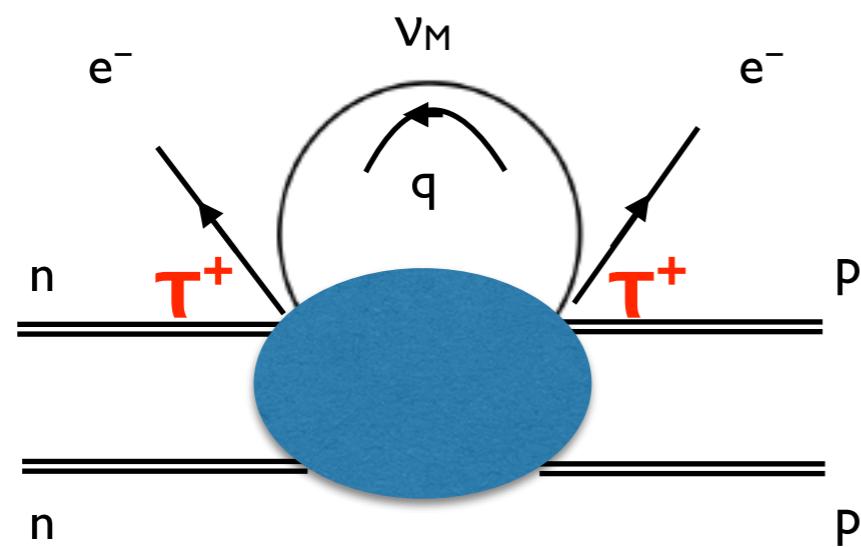
LO

# Estimating the LECs

I. Compute  $\pi^- \rightarrow \pi^+$ ,  $nn \rightarrow pp, \dots$  in lattice QCD and match to EFT

$$S_{\text{eff}}^{\Delta L=2} = i4G_F^2 V_{ud}^2 m_{\beta\beta} \int d^4x \bar{e}_L(x) e_L^c(x) \int d^4y S(x-y) g^{\mu\nu} T(\bar{u}_L \gamma_\mu d_L(x) \bar{u}_L \gamma_\nu d_L(y))$$

Remnant of  $\nu$  propagator



# Estimating the LECs

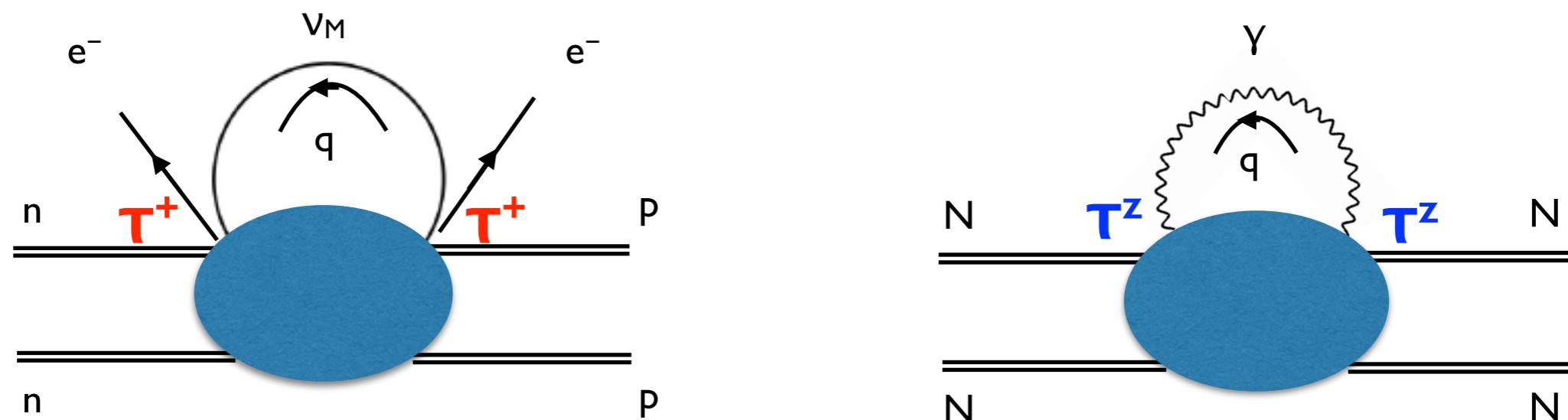
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Remnant of  $\nu$  propagator  
 $\sim \gamma$  propagator in Feynman gauge

$(J_+ \times J_+)$  vs  $(J_{\text{EM}} \times J_{\text{EM}})$   $l=2$

2. Chiral symmetry relates  $(g_\nu)^{AB}$  to one of two  $l=2$  EM LECs (hard  $\gamma$ 's vs  $\nu$ 's)



# Estimating the LECs

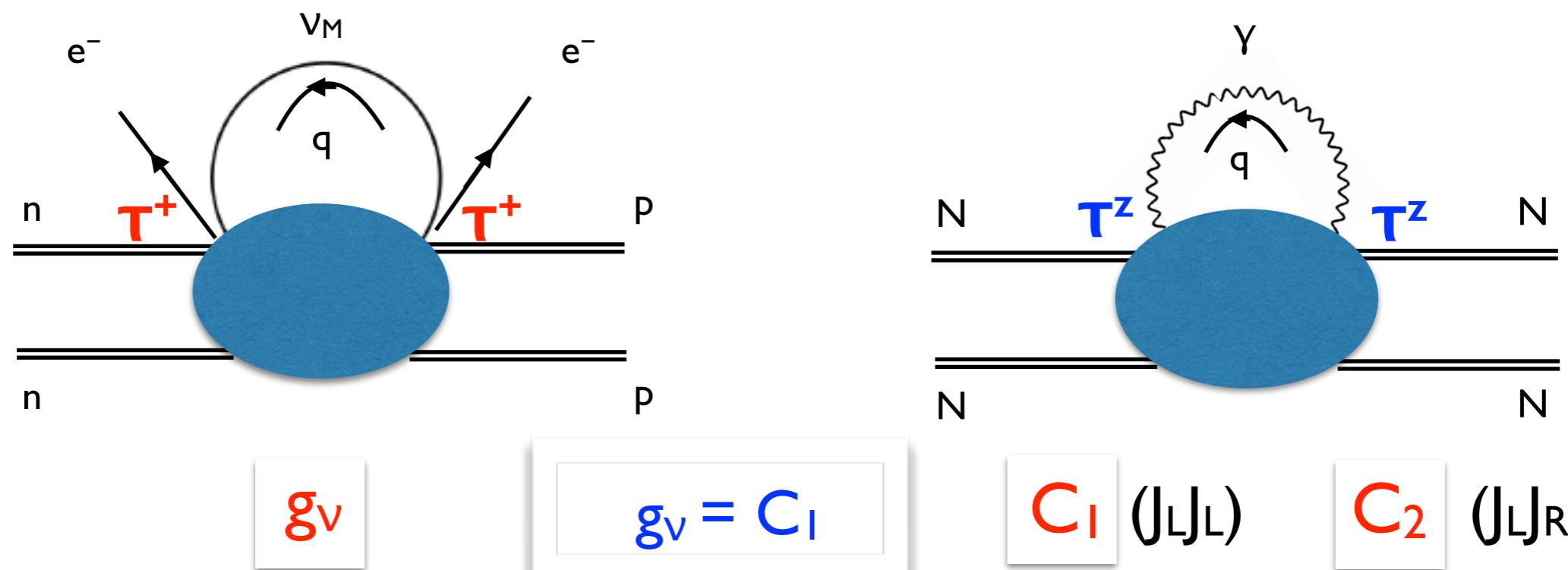
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# $\pi\pi\pi$ contact

- Estimates of  $l=2$  pion coupling in large- $N_c$  inspired resonance approach  
⇒

Ananthanarayan &  
Moussallam  
hep-ph/0405206

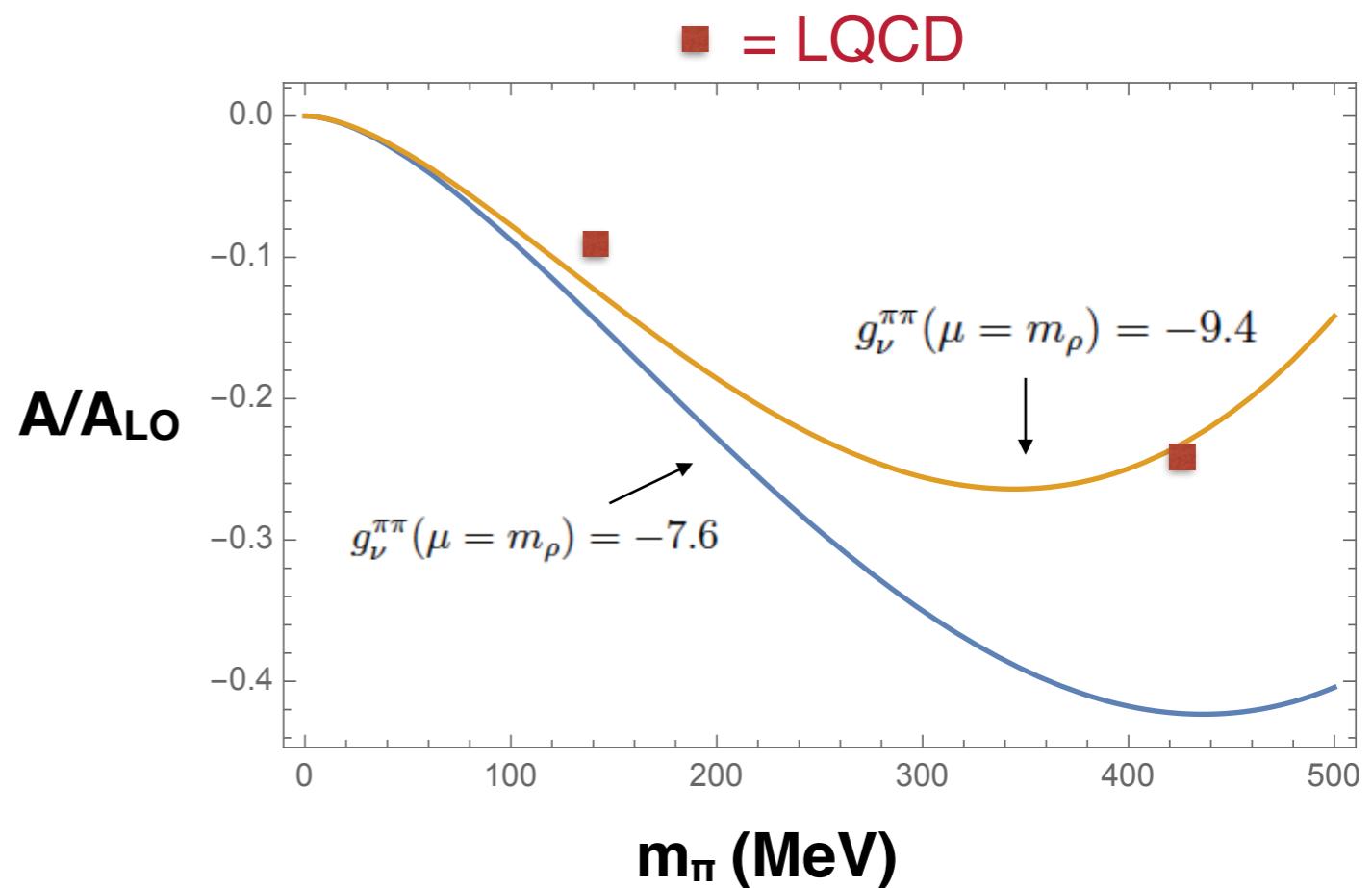
$$g_\nu^{\pi\pi}(\mu = m_\rho) = -7.6$$

~30% uncertainty

- Good agreement with matching 1-loop ChPT  $\pi^-\pi^- \rightarrow e^-e^-$  to LQCD calculation\*

\* Xu Feng et al., 1809.10511

For related work see  
Detmold-Murphy 1811.0554



# NN contact

- Two  $I=2$  operators involving four nucleons

(See also Walzl-Meißner-Epelbaum  
[nucl-th/0010109](https://arxiv.org/abs/nucl-th/0010109))

EM case

$$Q_L = \frac{\tau^z}{2}, Q_R = \frac{\tau^z}{2}$$

$$\frac{e^2}{4} C_1 \left( \bar{N} \mathcal{Q}_L N \bar{N} \mathcal{Q}_L N - \frac{\text{Tr}[\mathcal{Q}_L^2]}{6} \bar{N} \boldsymbol{\tau} N \cdot \bar{N} \boldsymbol{\tau} N + L \rightarrow R \right)$$

$$\frac{e^2}{4} C_2 \left( \bar{N} \mathcal{Q}_L N \bar{N} \mathcal{Q}_R N - \frac{\text{Tr}[\mathcal{Q}_L \mathcal{Q}_R]}{6} \bar{N} \boldsymbol{\tau} N \cdot \bar{N} \boldsymbol{\tau} N + L \rightarrow R \right)$$

$$\mathcal{Q}_L = u^\dagger \mathcal{Q}_L u$$

$$\mathcal{Q}_R = u \mathcal{Q}_R u^\dagger$$

$$u = 1 + \frac{i\boldsymbol{\pi} \cdot \boldsymbol{\tau}}{2F_\pi} + \dots$$

$\Delta L=2$  case

$$Q_L = \tau^+, Q_R = 0$$

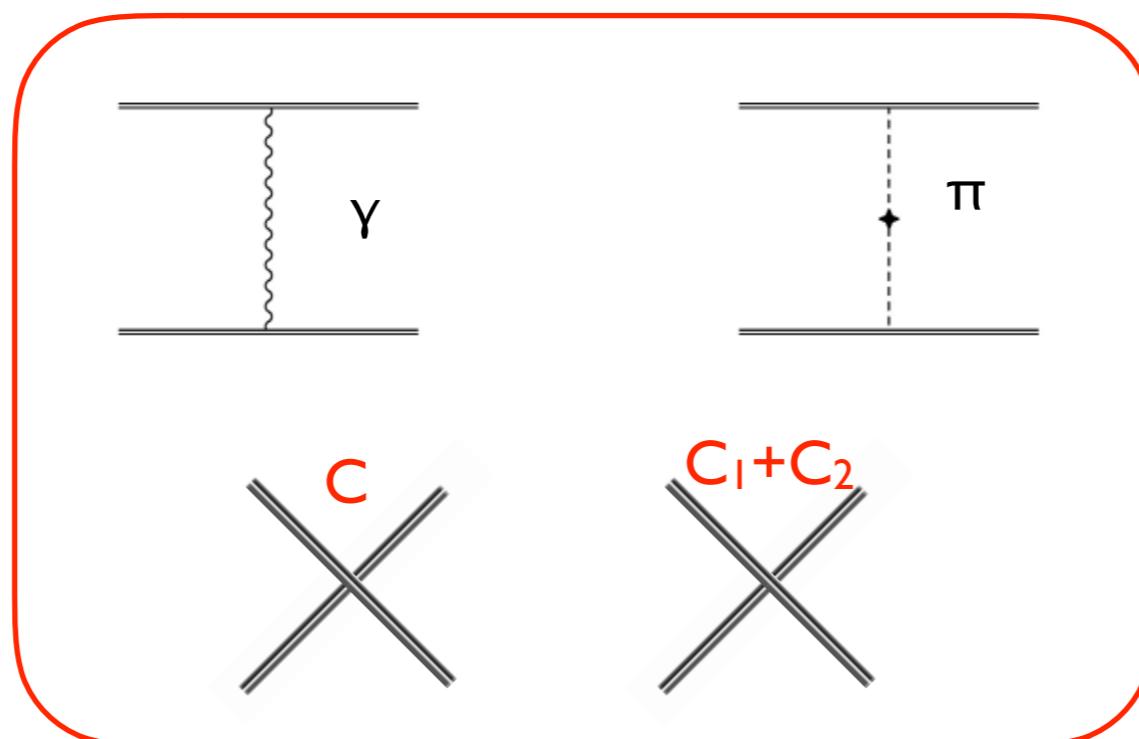
$$8G_F^2 V_{ud}^2 m_{\beta\beta} \bar{e}_L e_L^c \frac{g_\nu}{4} \left( \bar{N} \mathcal{Q}_L N \bar{N} \mathcal{Q}_L N - \frac{\text{Tr}[\mathcal{Q}_L^2]}{6} \bar{N} \boldsymbol{\tau} N \cdot \bar{N} \boldsymbol{\tau} N + L \rightarrow R \right)$$

- Chiral symmetry  $\Rightarrow g_\nu = C_1$
- NN observables *cannot* disentangle  $C_1$  from  $C_2$  (need pions),  
 but provide **data-based estimate of  $C_1+C_2$**

# Connection with data

$$a_{np} = -23.7 \pm 0.02 \text{ fm}, \quad a_{nn} = -18.90 \pm 0.40 \text{ fm}, \quad a_C = -7.804 \pm 0.005 \text{ fm}.$$

- $C_1 + C_2$  controls CIB combination of  $^1S_0$  scattering lengths  $a_{nn} + a_C - 2 a_{np}$

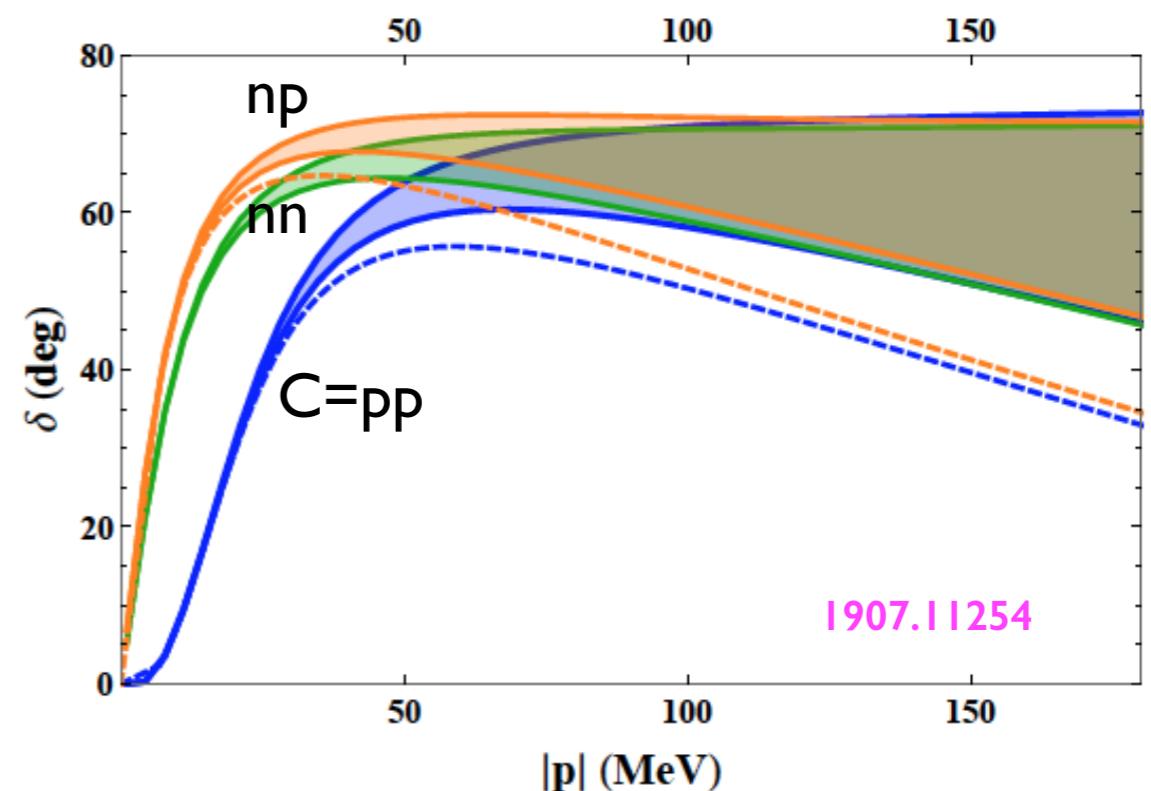


1907.11254

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- $C_1 + C_2$  controls CIB combination of  $^1S_0$  scattering lengths  $a_{nn} + a_C - 2 a_{np}$
- Fit to data, including LO strong, Coulomb potential, pion EM mass splitting, and contact terms confirms the scaling  $C_1 + C_2 \gg 1/(4\pi F_\pi)^2$



$$\frac{C_1 + C_2}{2} \equiv \left( \frac{m_N C}{4\pi} \right)^2 \left( 2.5 - 1.8 \ln(m_\pi/\mu) \right)$$

$$C = -\frac{1}{\tilde{\Lambda}^2}$$

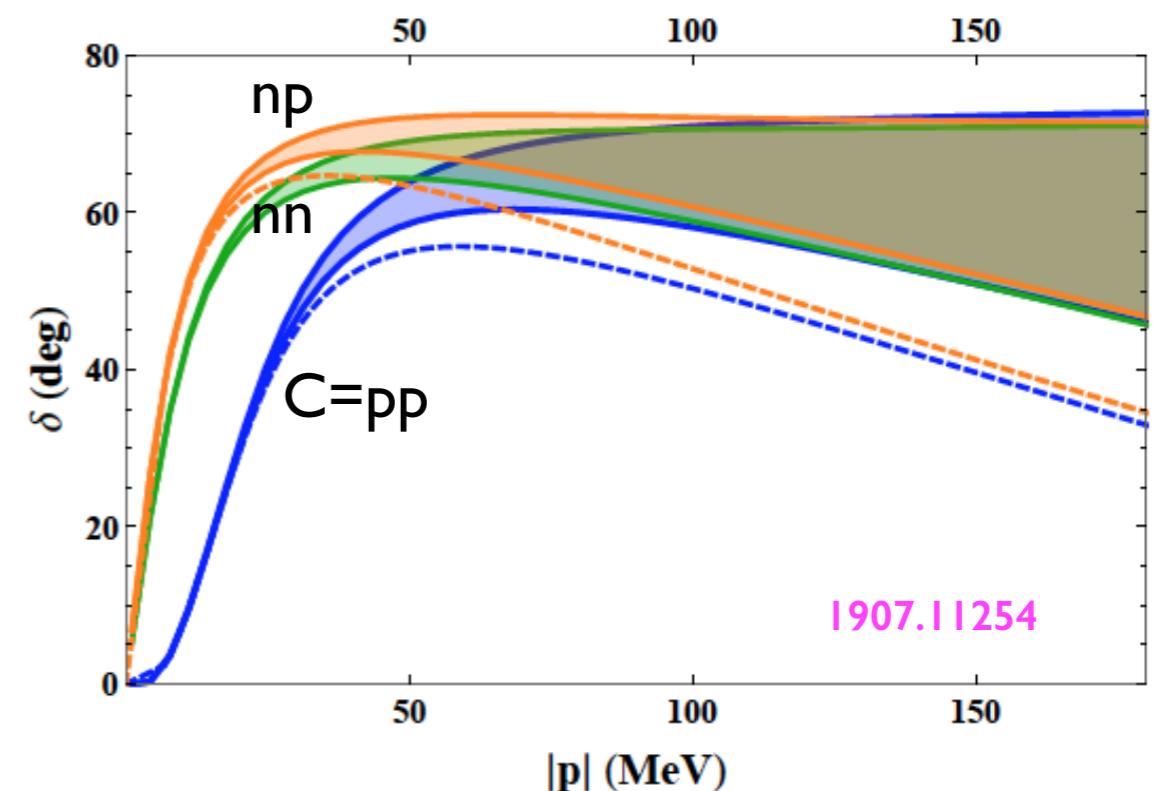
$$\tilde{\Lambda}(\mu = m_\pi) = O(100 \text{ MeV})$$

MS-bar scheme analysis [1907.11254]

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The EFT analysis survives comparison with data

The analog of  $e^2(C_1+C_2)$  is included in all high-quality potentials  
(AV18, CD-Bonn, chiral, ...)

MeV

# $C_1 + C_2$ in “high-quality” NN potentials

1907.111254

$$V_{\text{CIB},S} = -\frac{e^2}{6} \frac{\mathcal{C}_1 + \mathcal{C}_2}{2} T^{(12)} \delta_{R_S}^{(3)}(\mathbf{r})$$

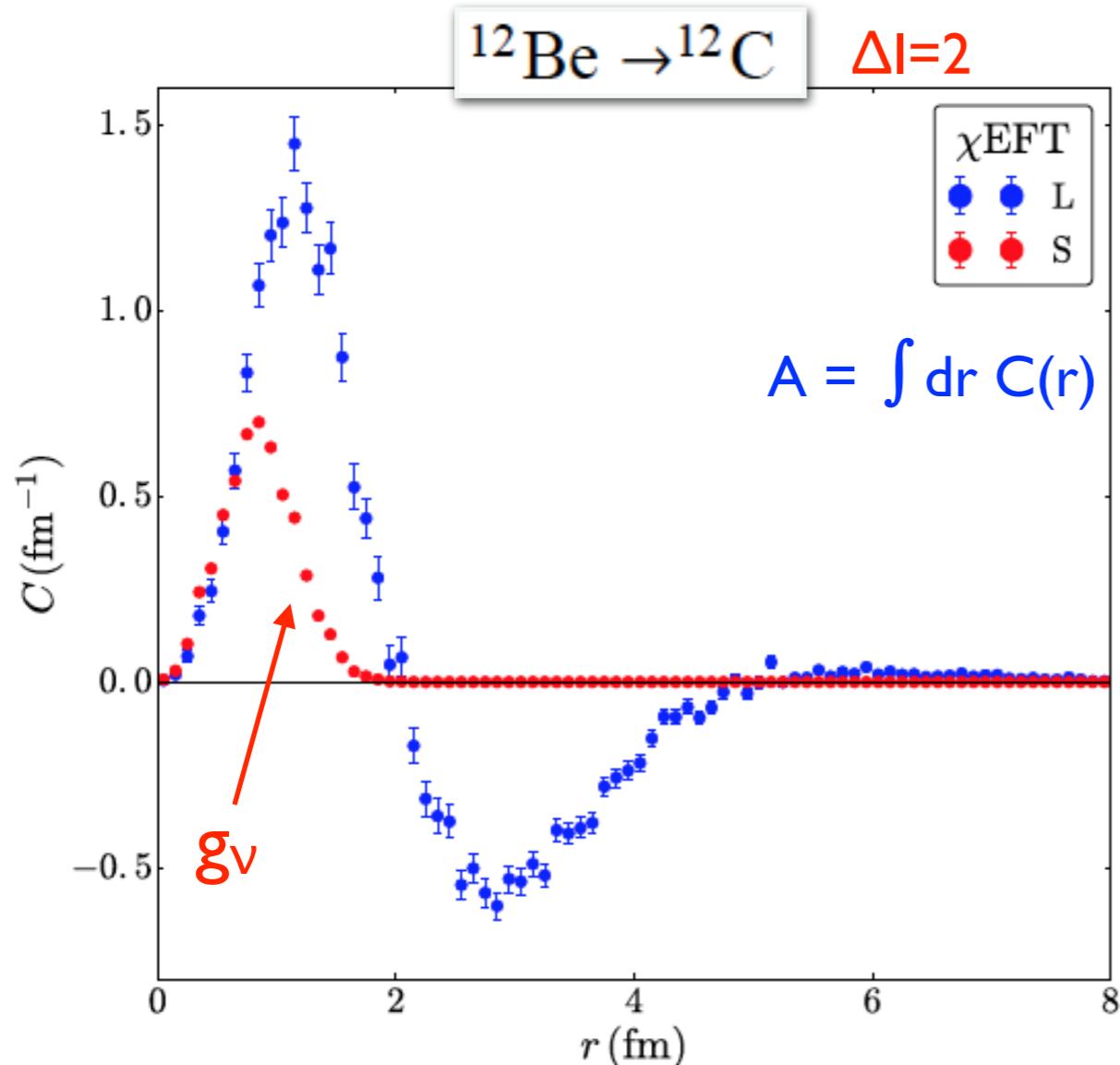
$$T^{(12)} = 3 \tau_3^{(1)} \tau_3^{(2)} - \vec{\tau}^{(1)} \cdot \vec{\tau}^{(2)}$$

Model	Ref.	$R_S$ (fm)	$(\mathcal{C}_1 + \mathcal{C}_2)/2$ (fm $^2$ )	Model	Ref.	$\Lambda$ (MeV)	$(\mathcal{C}_1 + \mathcal{C}_2)/2$ (fm $^2$ )
NV-Ia*	[37]	0.8	-1.03	Entem-Machleidt	[33]	500	-0.47
NV-IIa*	[37]	0.8	-1.44	Entem-Machleidt	[33]	600	-0.14
NV-Ic	[37]	0.6	-1.44	Reinert <i>et al.</i>	[38]	450	-0.67
NV-IIc	[37]	0.6	-0.91	Reinert <i>et al.</i>	[38]	550	-1.01
				NNLO $_{sat}$	[36]	450	-0.39

- The above fits include higher order chiral terms
- Our LO fit gives  $(C_1 + C_2)/2 = 0.71$  fm $^2$  @  $R_S = 0.8$  fm
- All confirm strong violation of Weinberg counting

# Guesstimating numerical impact

V.C , W. Dekens, J. de Vries, M. Graesser, E. Mereghetti, S. Pastore, M. Piarulli, U. van Kolck, R. Wiringa, 1907.11254



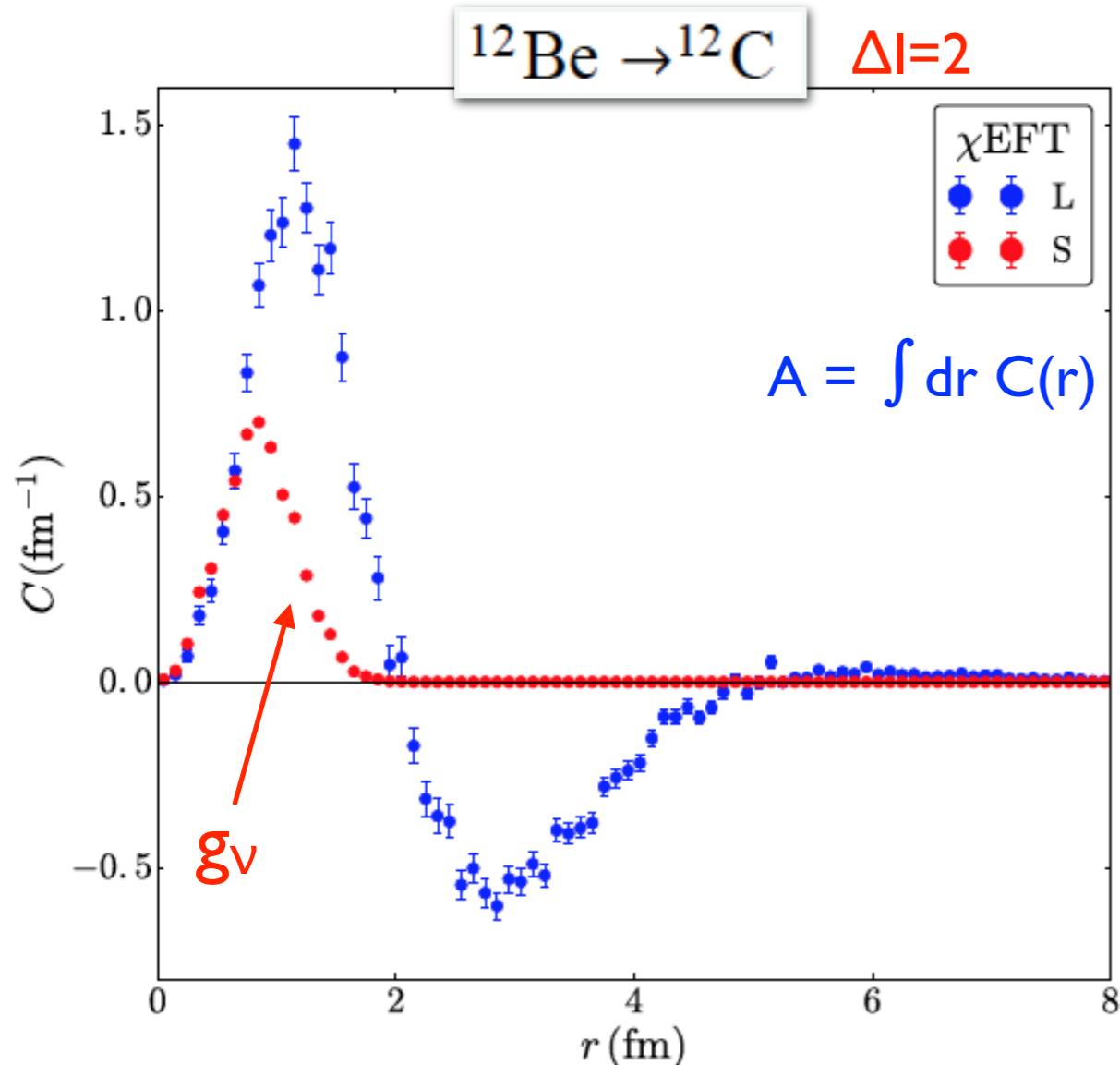
Assume  $g_V \sim (C_1 + C_2)/2$  with  $(C_1 + C_2)/2$  taken from fit to NN data

Evaluate impact in light nuclei using Variational Monte Carlo, with wavefunctions corresponding to the Norfolk chiral potential NV-Ia\* [1606.06335]

$g_V$  contribution sizable in  $\Delta I=2$  transition (due to node):  
for  $A=12$ ,  $A_S/A_L = 0.75$

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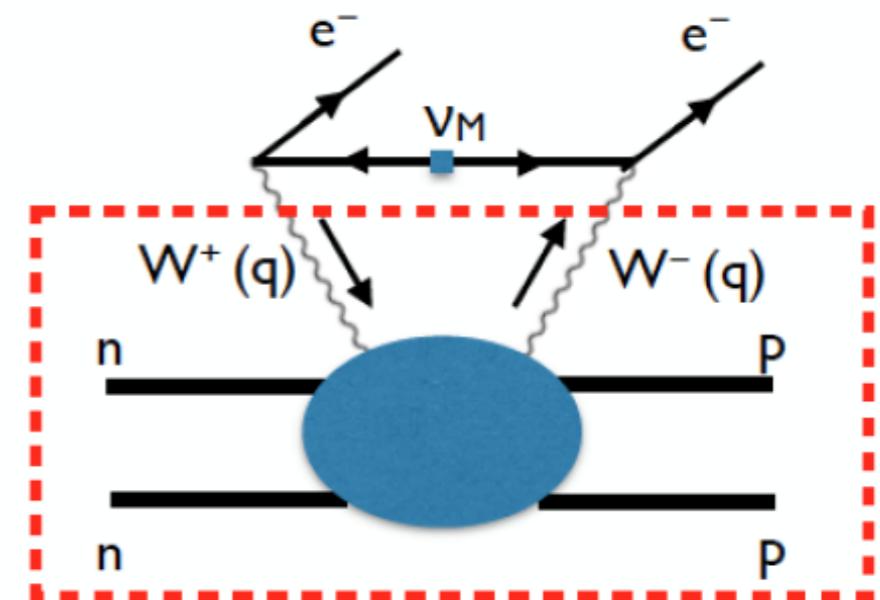
Transitions of experimental interest ( $^{76}\text{Ge} \rightarrow ^{76}\text{Se}, \dots$ ) have  $\Delta I=2$  (and node)  $\Rightarrow$  expect significant effect!

# Non-LQCD strategy to determine $g_V$

- Write full amplitude as convolution of a known kernel with forward amplitude

$$W^+(q)nn \rightarrow W^-(q)PP$$

- Evaluate  $W^+(q)nn \rightarrow W^-(q)PP$  in various momentum regions ( $q$ ) using different techniques:



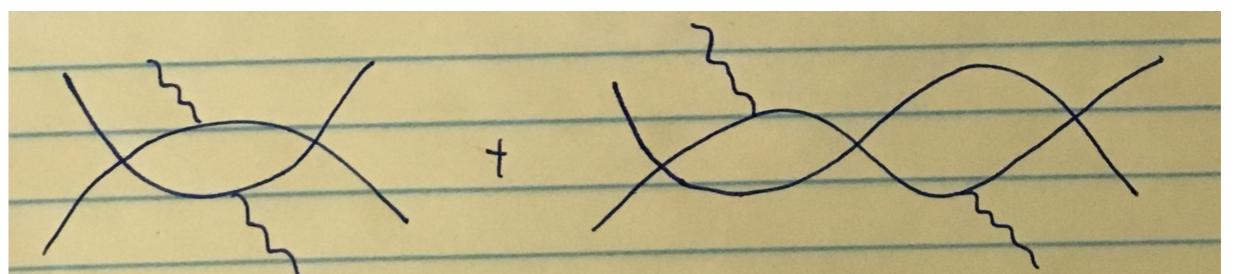
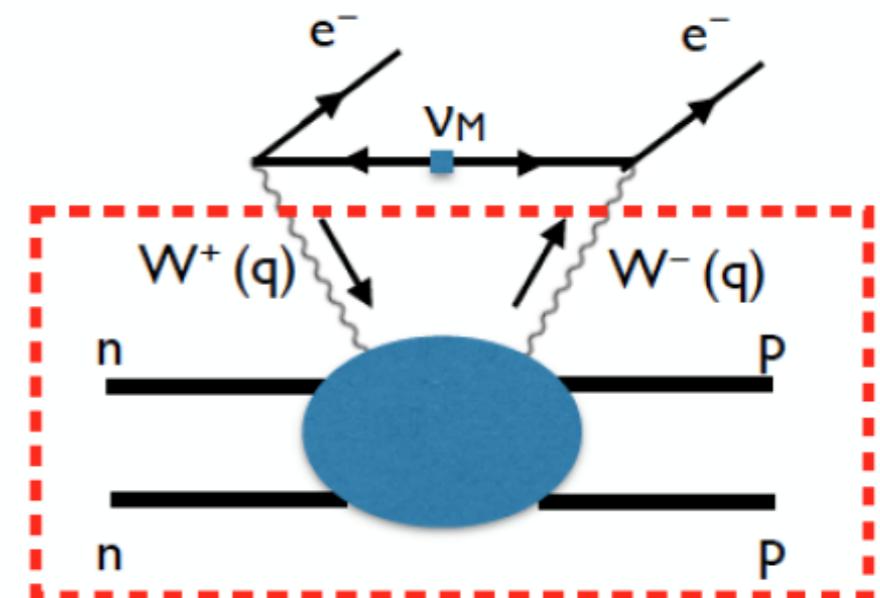
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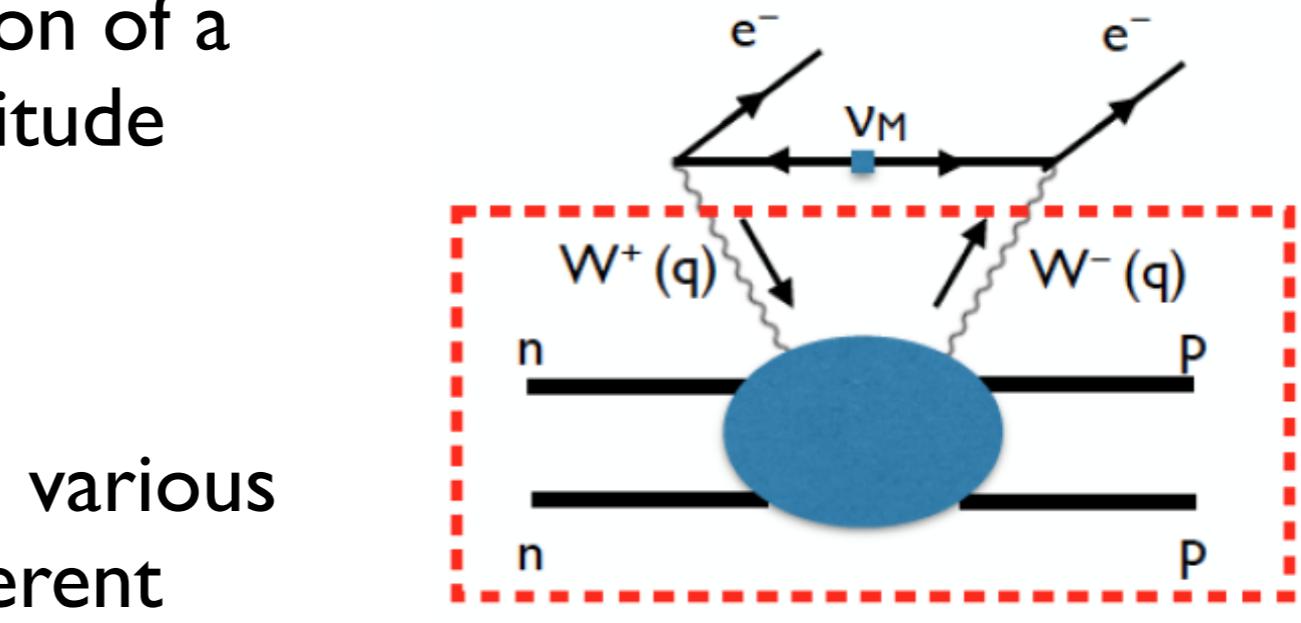
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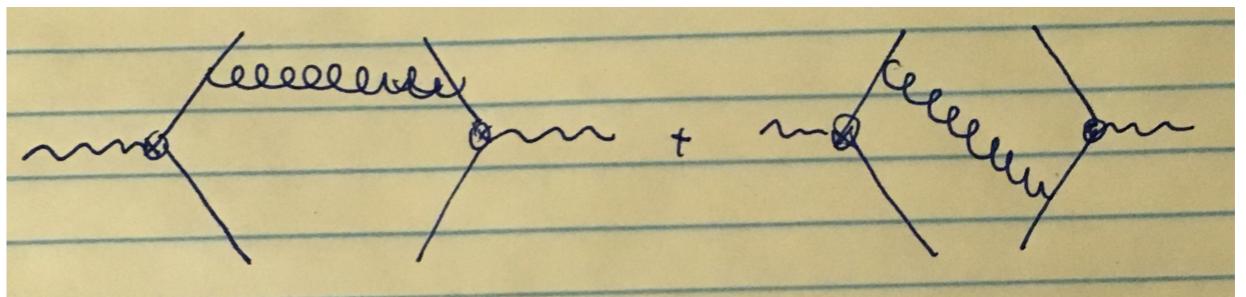
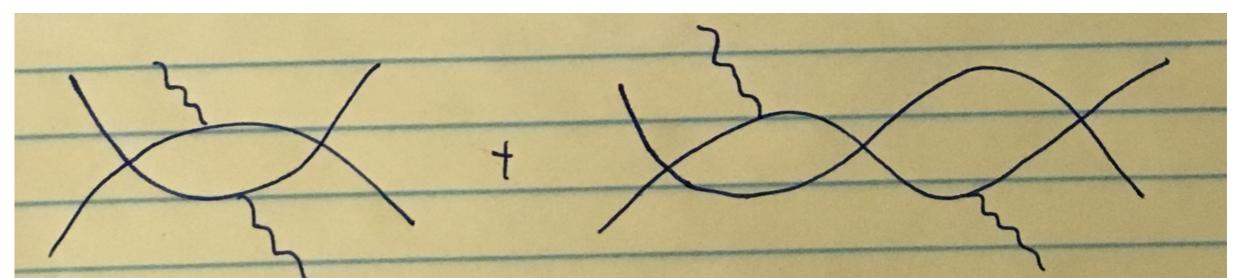
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- pQCD + local matrix element at  $q_E^2 \gg \text{GeV}^2$



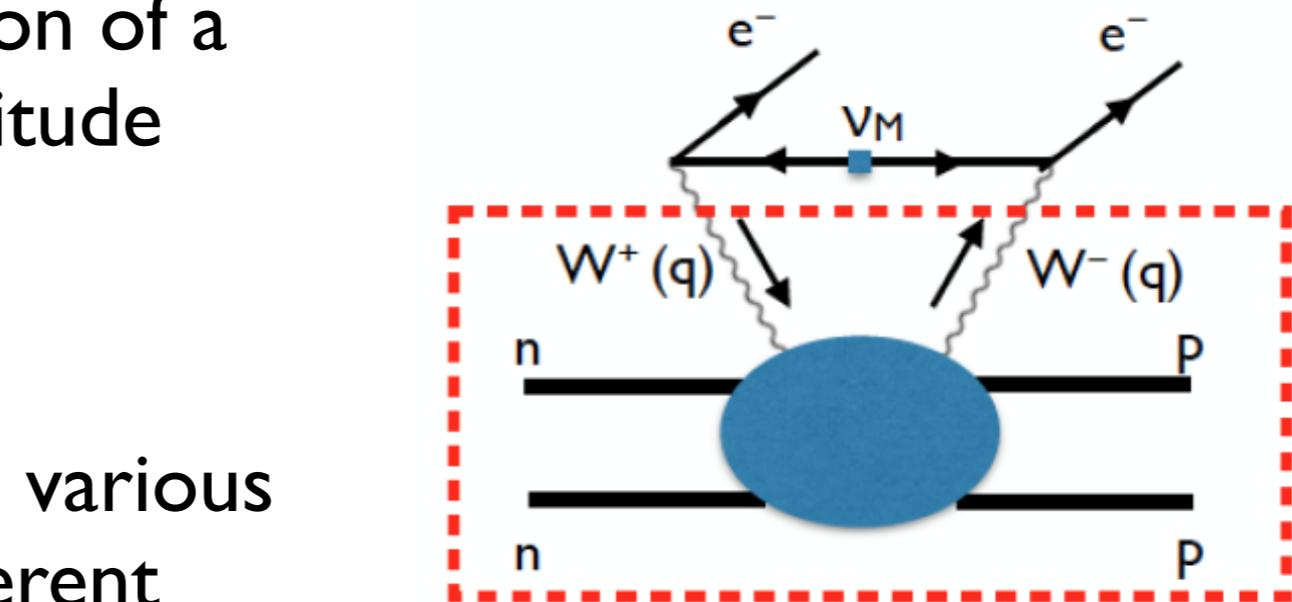
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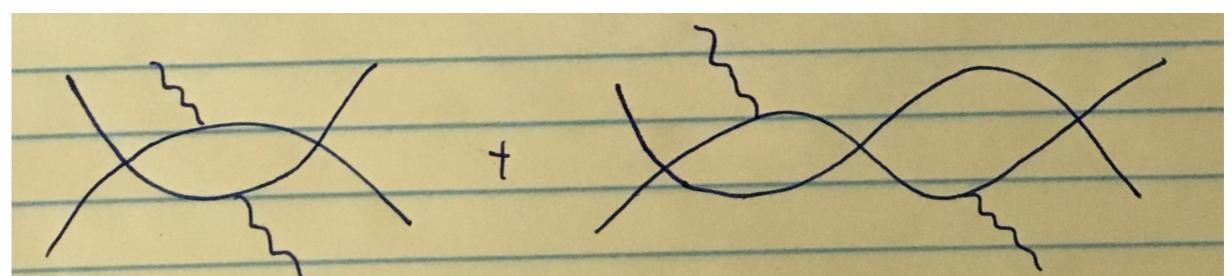
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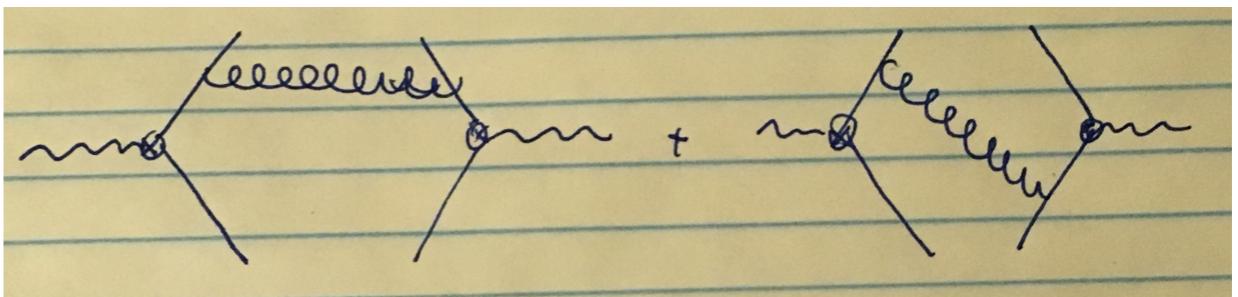
- Pionless EFT at  $q \rightarrow 0$



- pQCD + local matrix element at  $q_E^2 \gg \text{GeV}^2$



- Use electromagnetic I=2 contact term as a sanity check



# Conclusions

- Chiral EFT analysis of light  $V_M$  exchange contribution to  $0\nu\beta\beta$ 
  - Contact  $nn \rightarrow pp$  operator appears at leading order in the  $^1S_0$  channel. Related to  $I=2$  NN electromagnetic contact term.
  - First rough estimate of the coupling implies  $O(1)$  correction to matrix element in light nuclei
  - No new contact needed at NLO
  - At N2LO
    - new non-factorizable potential ( $O(10\%)$  correction in light nuclei, same as form factor effects)
    - corrections to closure approximation due to “ultrasoft”  $v$ 's (not discussed today)
    - 2body  $\times$  1body currents

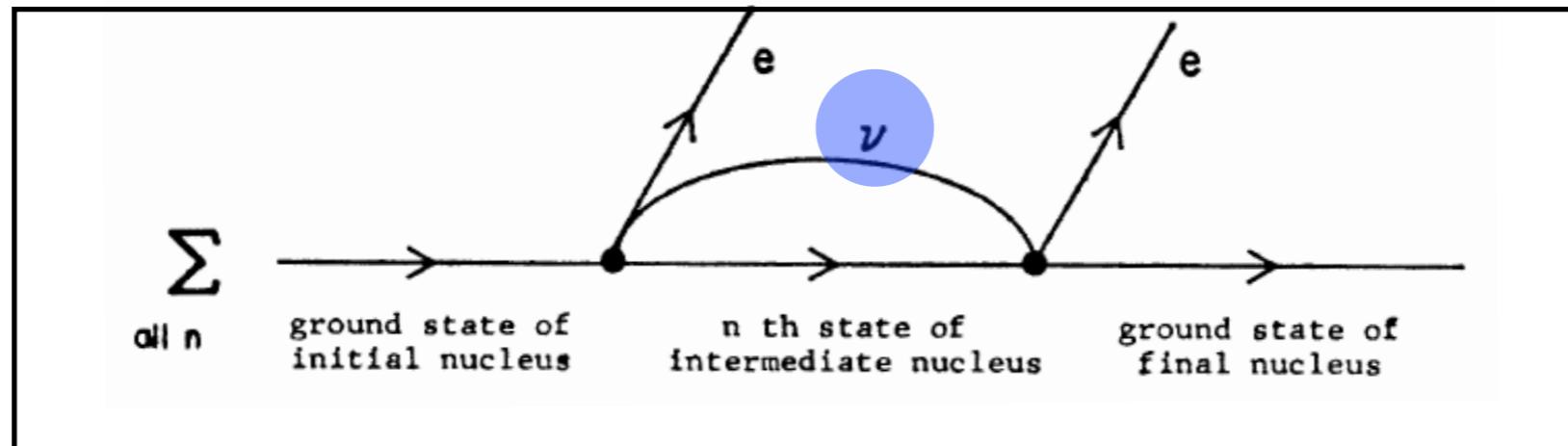
# Backup

# Details to be found in 1907.11254

- Analysis in both in pion-less and chiral EFT
- Chiral EFT with 3 regularizations / renormalization schemes
  - Dim reg (with scale  $\mu$ ) + minimal subtraction
  - Momentum-space cutoff  $\Lambda$  (in Lippmann-Schwinger eqs.)
  - Coordinate space gaussian regulator ( $R_s$ )
- Study of higher partial waves & NLO derivative operator
- Connection to electromagnetic  $I=2$  operators worked out in detail
- Matrix elements in light nuclei using “Norfolk” chiral potentials

# Ultrasoft neutrino contributions

Figure adapted from Primakoff-Rosen 1969



$$T_{\text{usoft}}(\mu_{\text{us}}) = \frac{T_{\text{lept}}}{8\pi^2} \sum_n \langle f | J_\mu | n \rangle \langle n | J^\mu | i \rangle \left\{ (E_2 + E_n - E_i) \left( \log \frac{\mu_{\text{us}}}{2(E_2 + E_n - E_i)} + 1 \right) + 1 \leftrightarrow 2 \right\}$$

V. Cirigliano, W. Dekens, E. Mereghetti, A. Walker-Loud, 1710.01729

- Ultrasoft ν's couple to *nuclear states*: sensitivity to  $E_n - E_i$  and  $\langle f | J_\mu | n \rangle \langle n | J^\mu | i \rangle$  that also determine  $2\nu\beta\beta$  amplitude
- $T_{\text{usoft}}/T_0 \sim (E_n - E_i)/(4\pi k_F)$   $\rightarrow$  N2LO contribution
- $\mu_{\text{us}}$  dependence cancels with  $V_{\nu,2.}$ : consistency check

# $T_{\text{usoft}}$ and double commutator

$$\begin{aligned} \frac{dT_{\text{usoft}}}{d \log \mu_{\text{us}}} &= -T_{\text{lept}} \times \frac{1}{8\pi^2} \langle f | [J_\mu, [J^\mu, H_0]] | i \rangle = T_{\text{lept}} \times \frac{1}{8\pi^2} \langle f | [\mathbf{A}, [\mathbf{A}, H_0]] | i \rangle \\ &= -2 T_{\text{lept}} \times \sum_{a,b} \langle f | \tau^{(a)+} \tau^{(b)+} \tilde{\mathcal{V}}_{AA}^{(a,b)} | i \rangle , \end{aligned}$$

$$-T_{\text{lept}} \frac{d(V_{\nu,2})_{fi}}{d \log \mu_{\text{us}}} = +2 T_{\text{lept}} \times \sum_{a,b} \langle f | \tau^{(a)+} \tau^{(b)+} \tilde{\mathcal{V}}_{AA}^{(a,b)} | i \rangle ,$$

$$\tilde{\mathcal{V}}_{AA}^{(a,b)} = 2 \frac{g_A^4}{(4\pi F_\pi)^2} \frac{\sigma^{(a)} \cdot \mathbf{q} \sigma^{(b)} \cdot \mathbf{q} + \mathbf{q}^2 \mathbf{1}^{(a)} \times \mathbf{1}^{(b)}}{\mathbf{q}^2 + m_\pi^2} - \frac{g_A^2}{(4\pi)^2} 48 C_T \mathbf{1}^{(a)} \times \mathbf{1}^{(b)}$$