The 0νββ-DGT correlation and wave function factorization

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From J. Menendez INT July 2019 talk…

Double Gamow-Teller strength distribution

Measurement of Double Gamow-Teller (DGT) resonance in double charge-exchange reactions ^{48}Ca(pp,nn)^{48}Ti proposed in 80's

Auerbach, Muto, Vogel... 1980's, 90's

Recent experimental plans in RCNP, RIKEN (^{48}Ca), INFN Catania

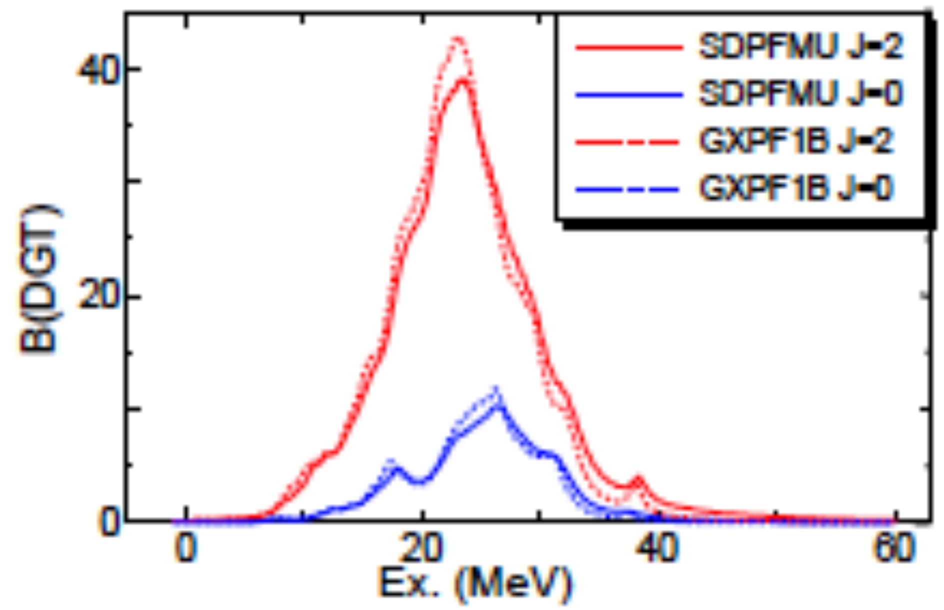
Takaki et al. JPS Conf. Proc. 6 020038 (2015)

Capuzzello et al. EPJA 51 145 (2015), Takahisa, Ejiri et al. arXiv:1703.08264

Promising connection to $\beta\beta$ decay, two-particle-exchange process, especially the (tiny) transition to ground state of final state

Shell model calculation

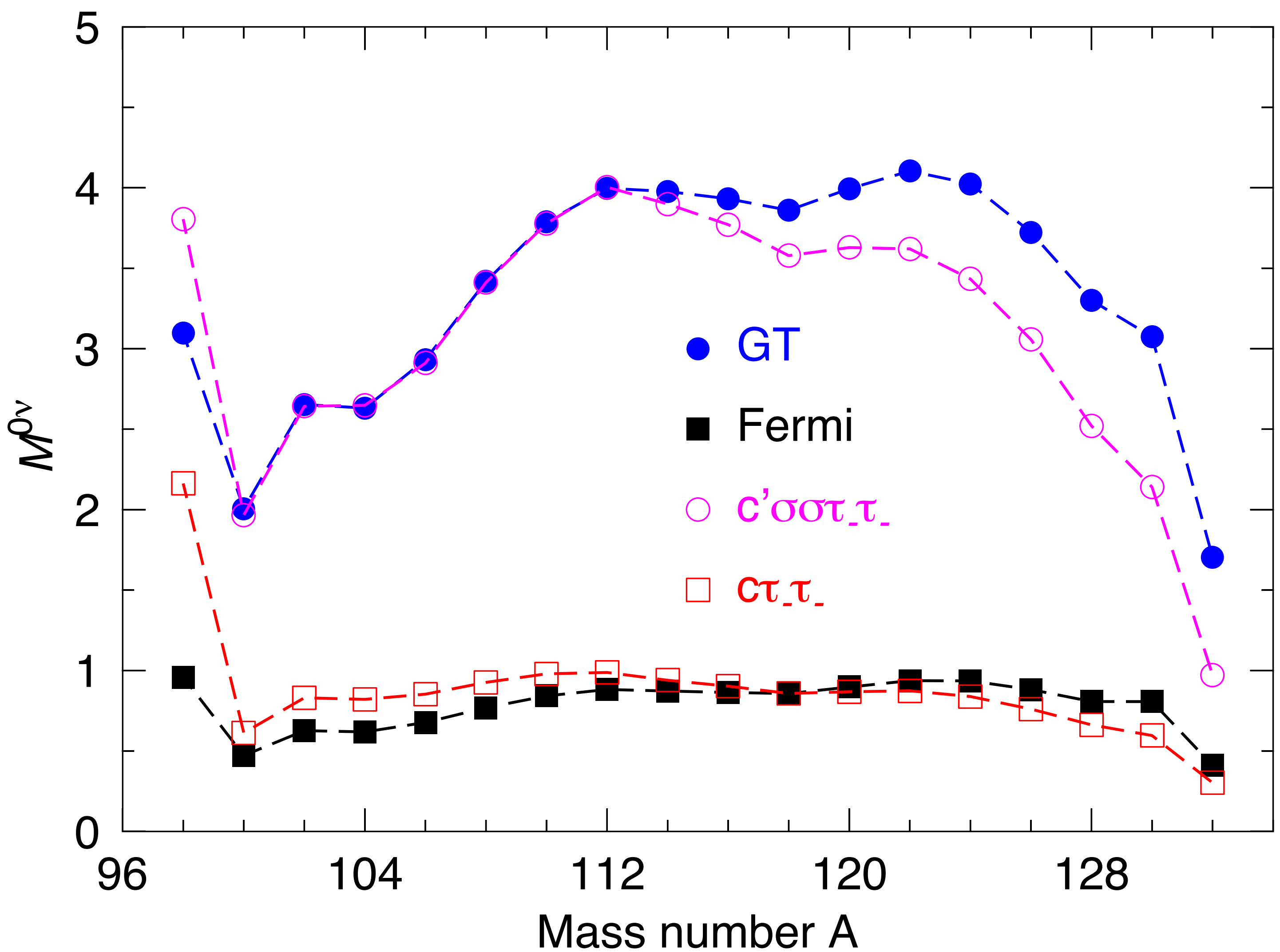
Shimizu, JM, Yako, PRL120 142502 (2018)



$$B(DGT^-;\lambda;i\to f)=\frac{1}{2J_i+1}\left|\left\langle {}^{48}\mathrm{Ti}\left|\left|\left[\sum_i \sigma_i\tau_i^- \times \sum_j \sigma_j\tau_j^-\right]^{(\lambda)}\right|\right| {}^{48}\mathrm{Ca}_{gs}\right\rangle\right|^2$$

0νββ-DGT correlation

T. Rodriguez and G. Martinez-Pinedo PLB 719 (2013)

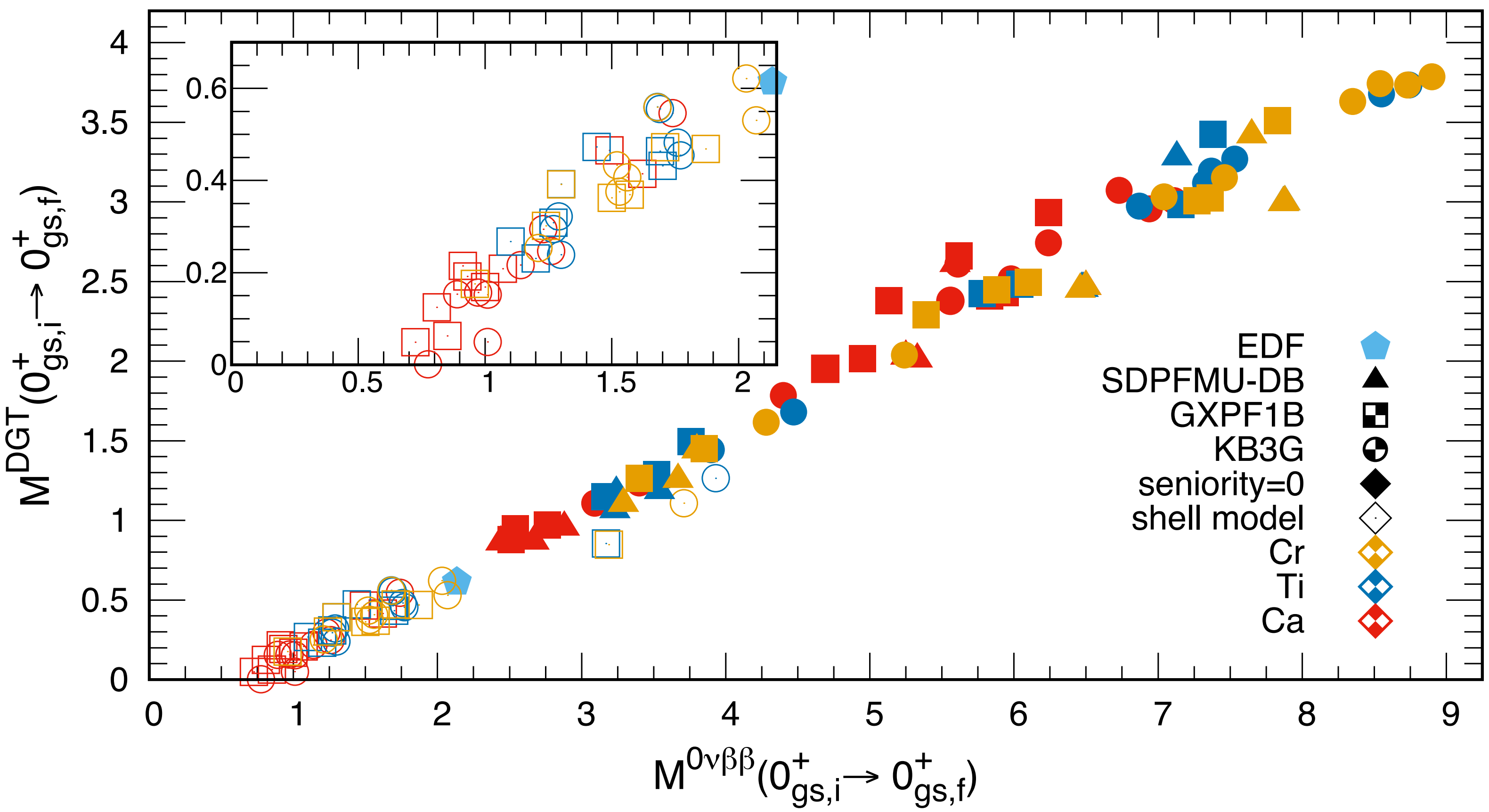


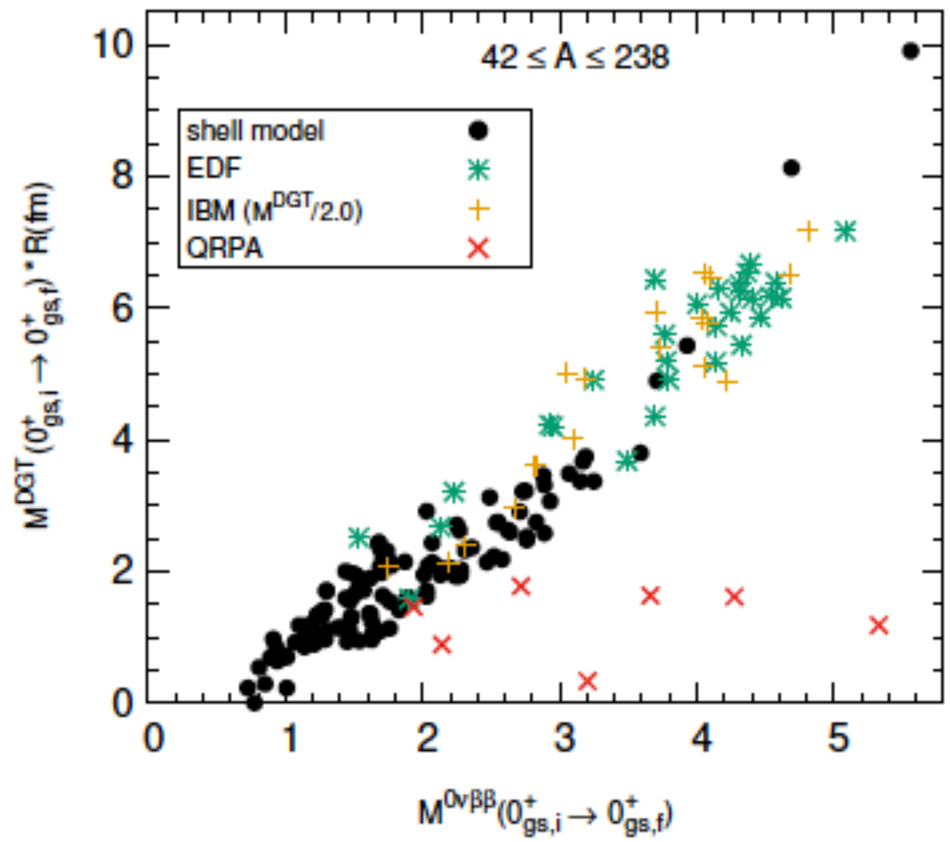
GCM calculations w/Gogny EDF

Match 0νββ matrix elements reasonably well across different A by replacing $H_{GT,F}(r)$ with $c = c' = 2.0$

0νββ-DGT correlation

Shimizu, Menendez, Yako PRL 120 (2018)

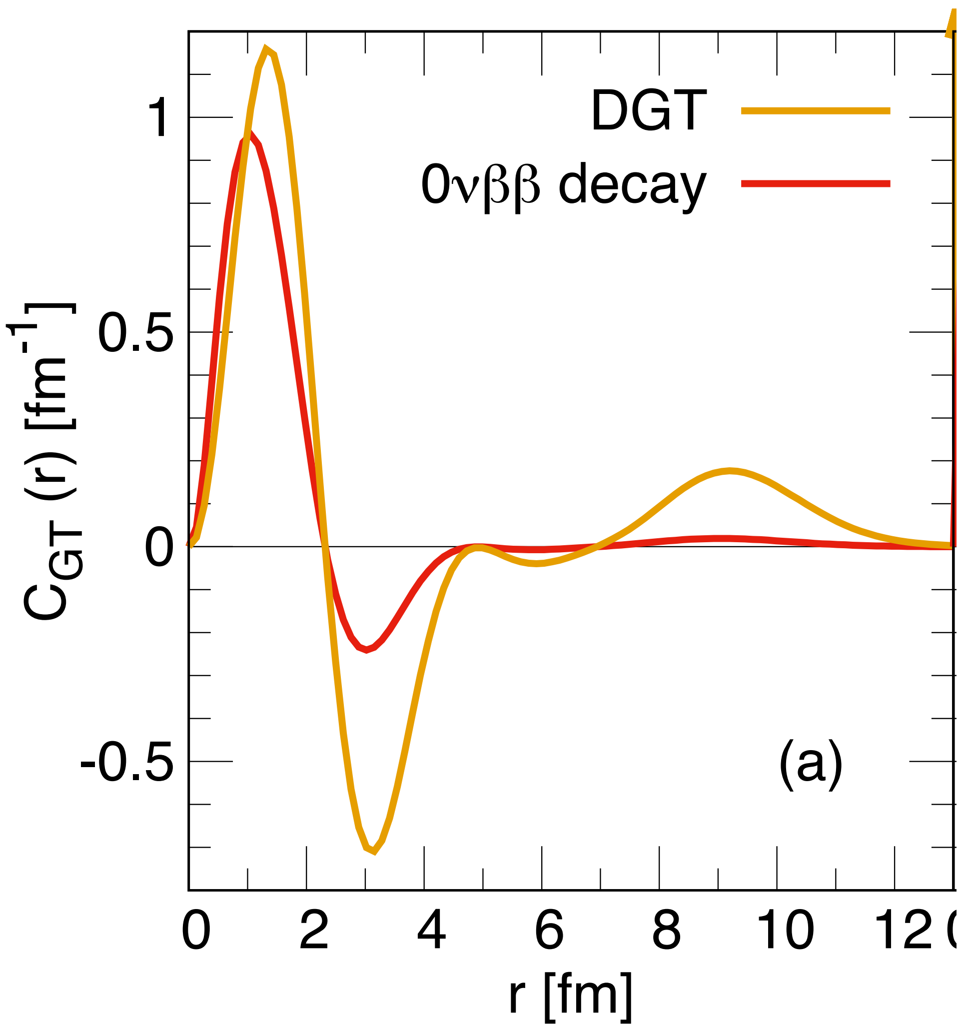




Correlation robust across the chart ($42 \leq A \leq 238$) for a host of different methods (shell model, IBM, EDF-GCM)

Doesn't hold for QRPA calculations

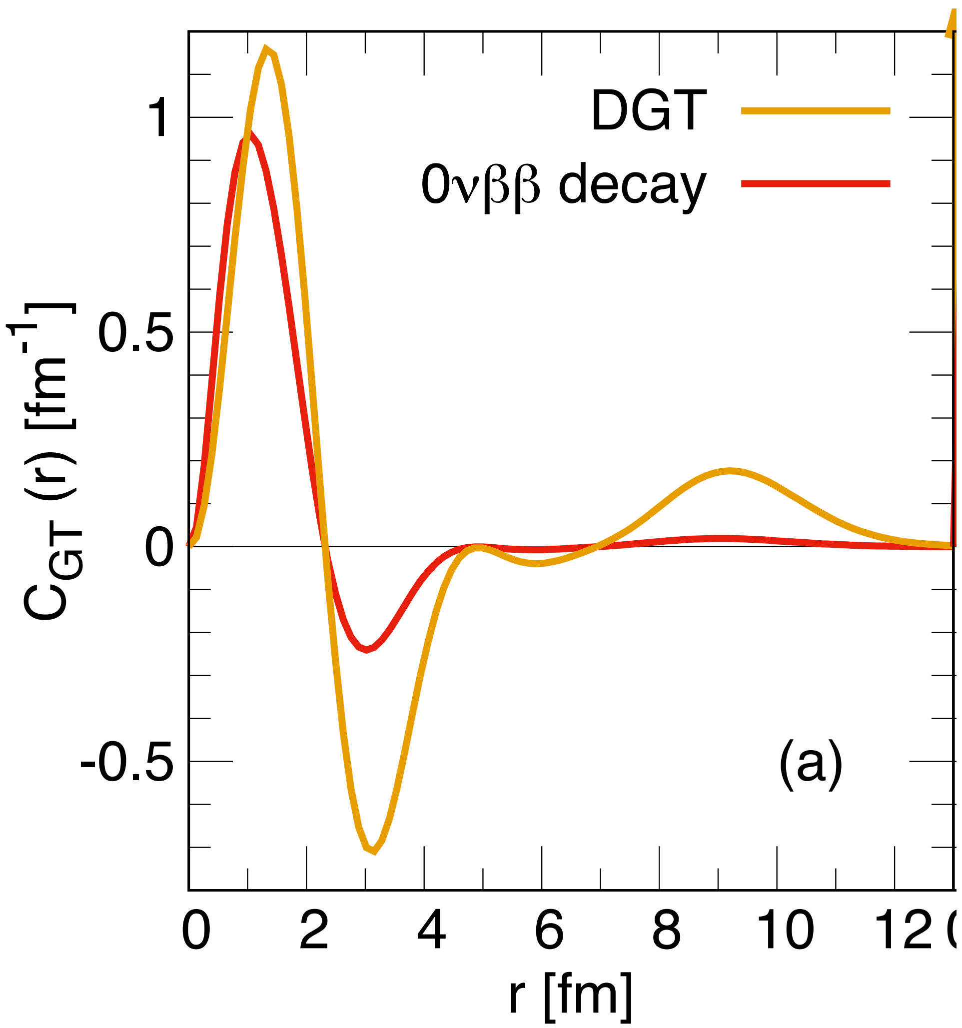
0νββ-DGT correlation



Dominant contributions to both 0νββ and DGT come from r ≲ 2 fm.

Robust cancellation in DGT at large distances, making it "effectively" same range as 0νββ.

0νββ-DGT correlation



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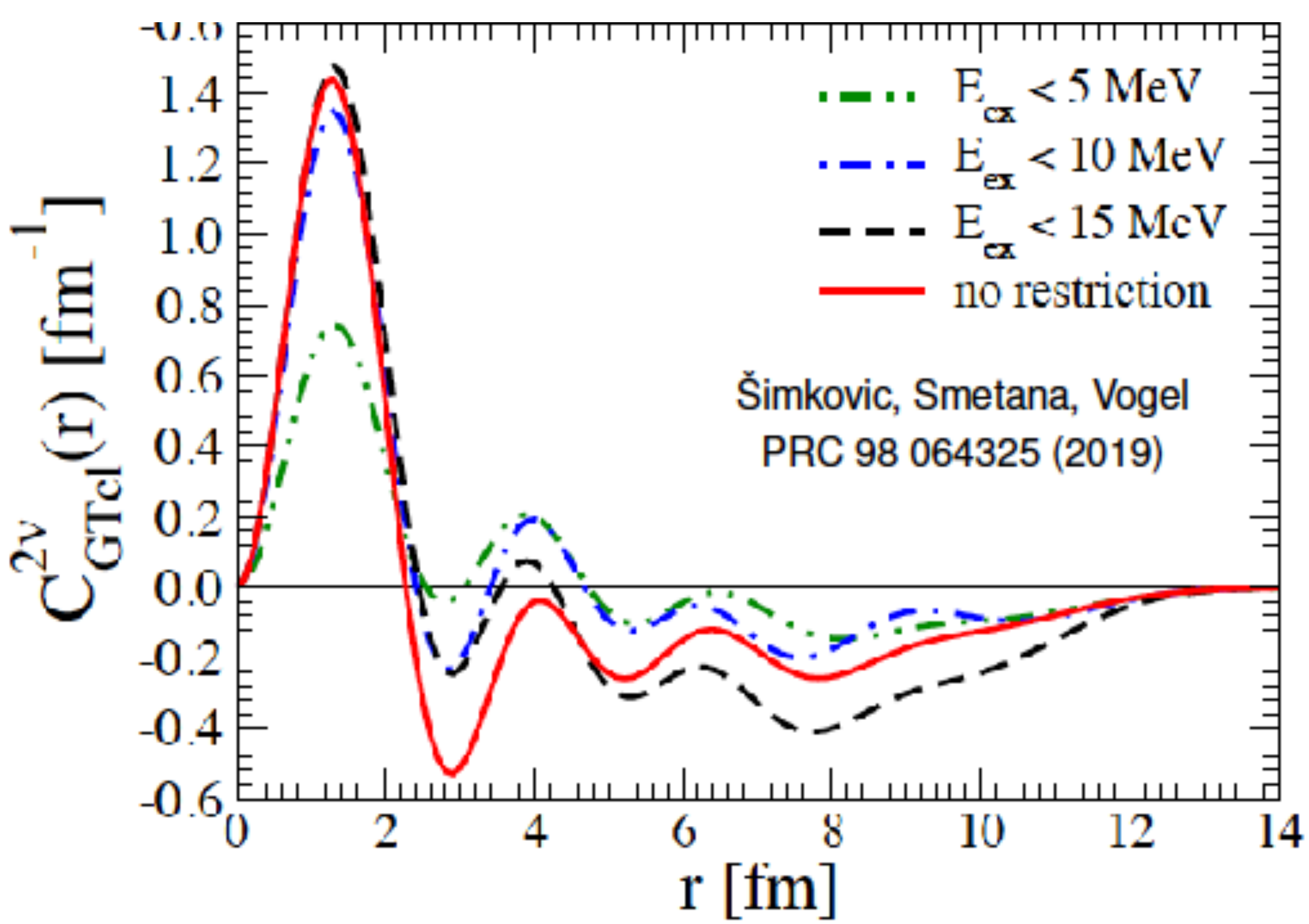
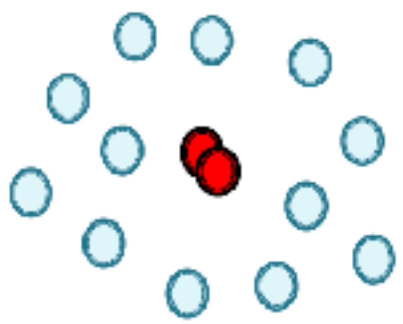


Reduction (QRPA) or not (SM, etc.) of DGT with respect to E_{ex} translates to very different radial dependence

Wave function factorization

Wave function factorization

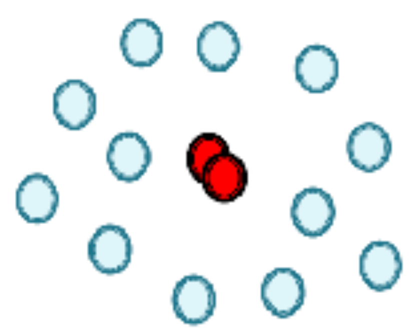


$\mathbf{r}_{12} \to 0$

$$\Psi_n^A(\mathbf{r}_1, \mathbf{r}_2, \ldots, \mathbf{r}_A) \sim \phi(\mathbf{r}_{12})\, \chi_n^A(\mathbf{R}_{12}, \mathbf{r}_3, \ldots \mathbf{r}_A)$$

Universal
2-body physics

state-dependent
many-body physics



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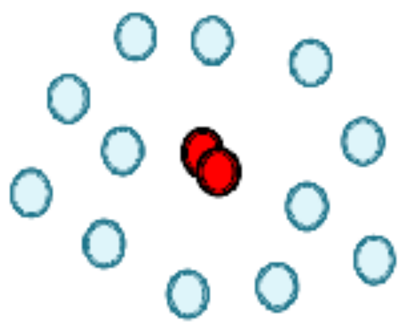
Consider high q (>> Λ) components of low E (<< Λ²) A =2 wf's

$$Q|\psi_n\rangle = \frac{1}{E_n - QH} QVP|\psi_n\rangle \qquad \psi_n(q) = \int_\Lambda^\infty d^3q \int_0^\Lambda d^3p \; G_Q(q, q'; E_n)\, V(q', p)\, \psi_n(p)$$

Wave function factorization



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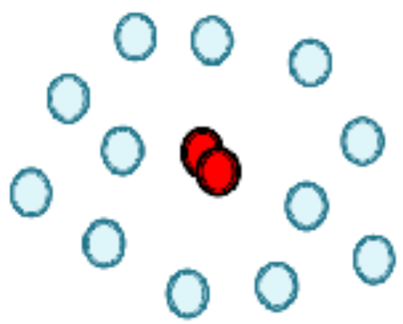
scale separation: $G_Q(q, q'; E_n) \approx G_Q(q, q'; 0) + \ldots \qquad V(q', p) \approx V(q', 0) + \ldots$

Wave function factorization



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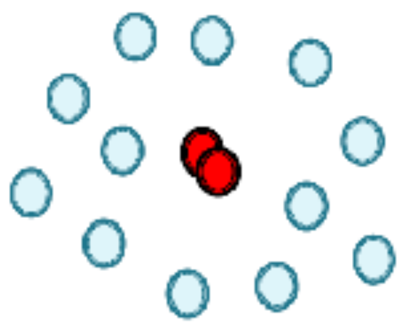
scale separation: $\Rightarrow$

$$\psi_n(q) \approx \gamma(q;\Lambda) \int_0^\Lambda d^3p\, \psi_n(p) \;+\; \eta(q;\Lambda) \int_0^\Lambda d^3p\, p^2 \psi_n(p) \;+ \ldots$$

Anderson et al., PRC **82** (2010)

SKB and Roscher, PRC **86** (2012)

Wave function factorization



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Operator **P**roduct **E**xpansion

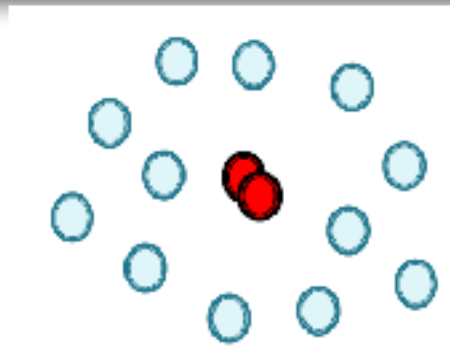
Wilson coefficients

ME's of (smeared) local operators

Anderson et al., PRC **82** (2010)

SKB and Roscher, PRC **86** (2012)

Wave function factorization



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Cruz-Torres et al. arXiv:1907.03658

$$\mathbf{r}_{12} \to 0$$

E.g., pair density

$$\rho_{2b}(\mathbf{r}) = \langle \Psi \,|\, \sum_{i<j} \delta(\mathbf{r} - \mathbf{r}_{ij}) \,|\, \Psi \rangle$$

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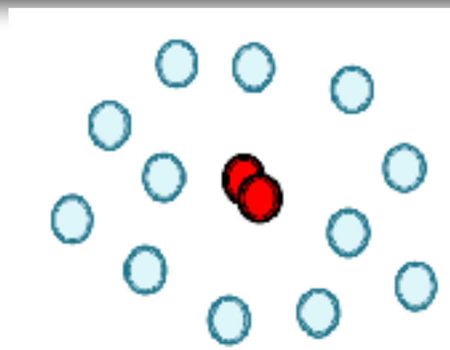

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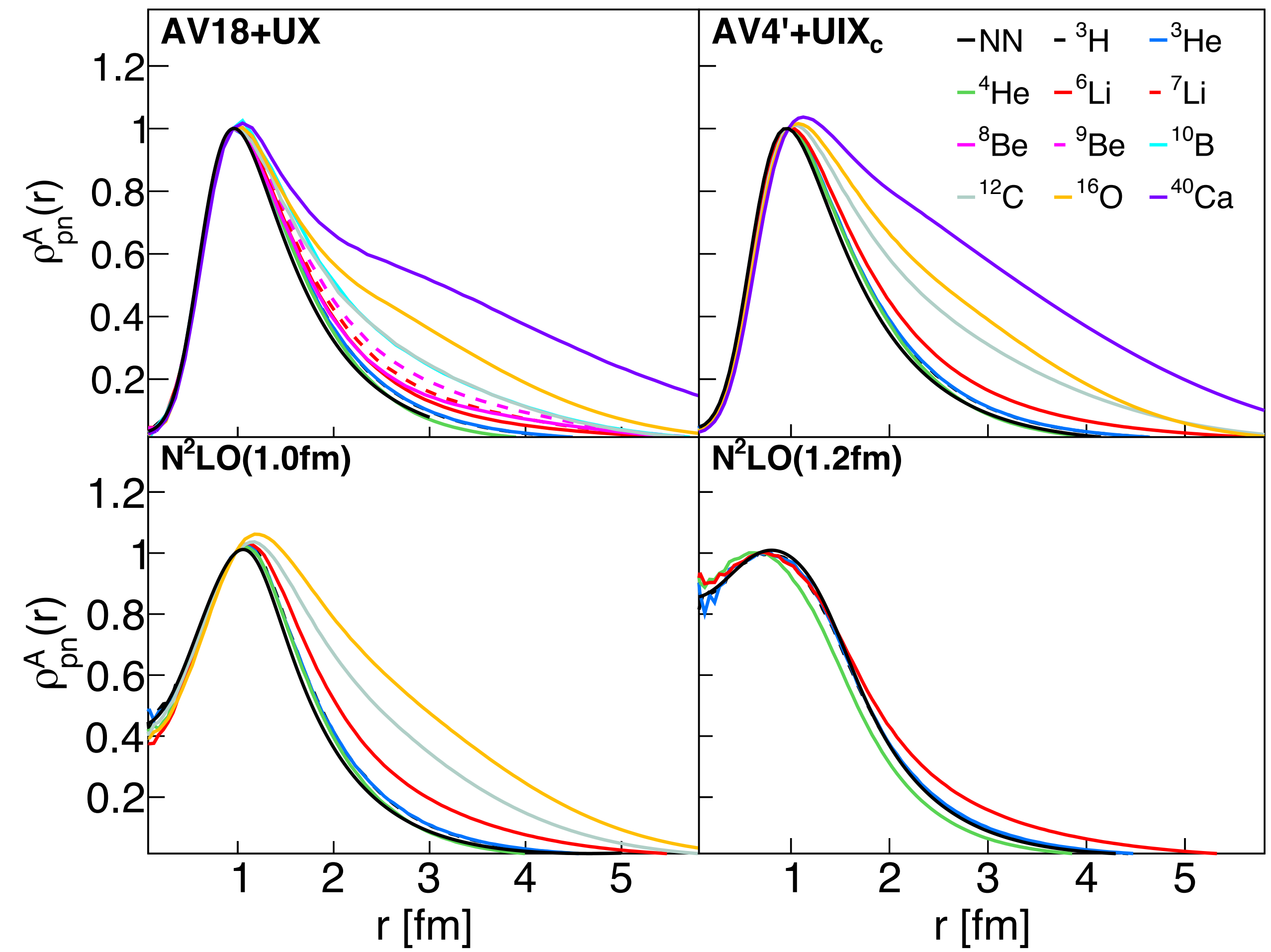

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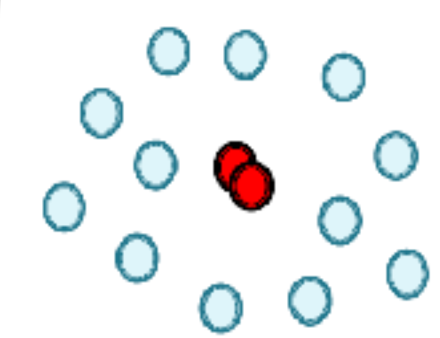

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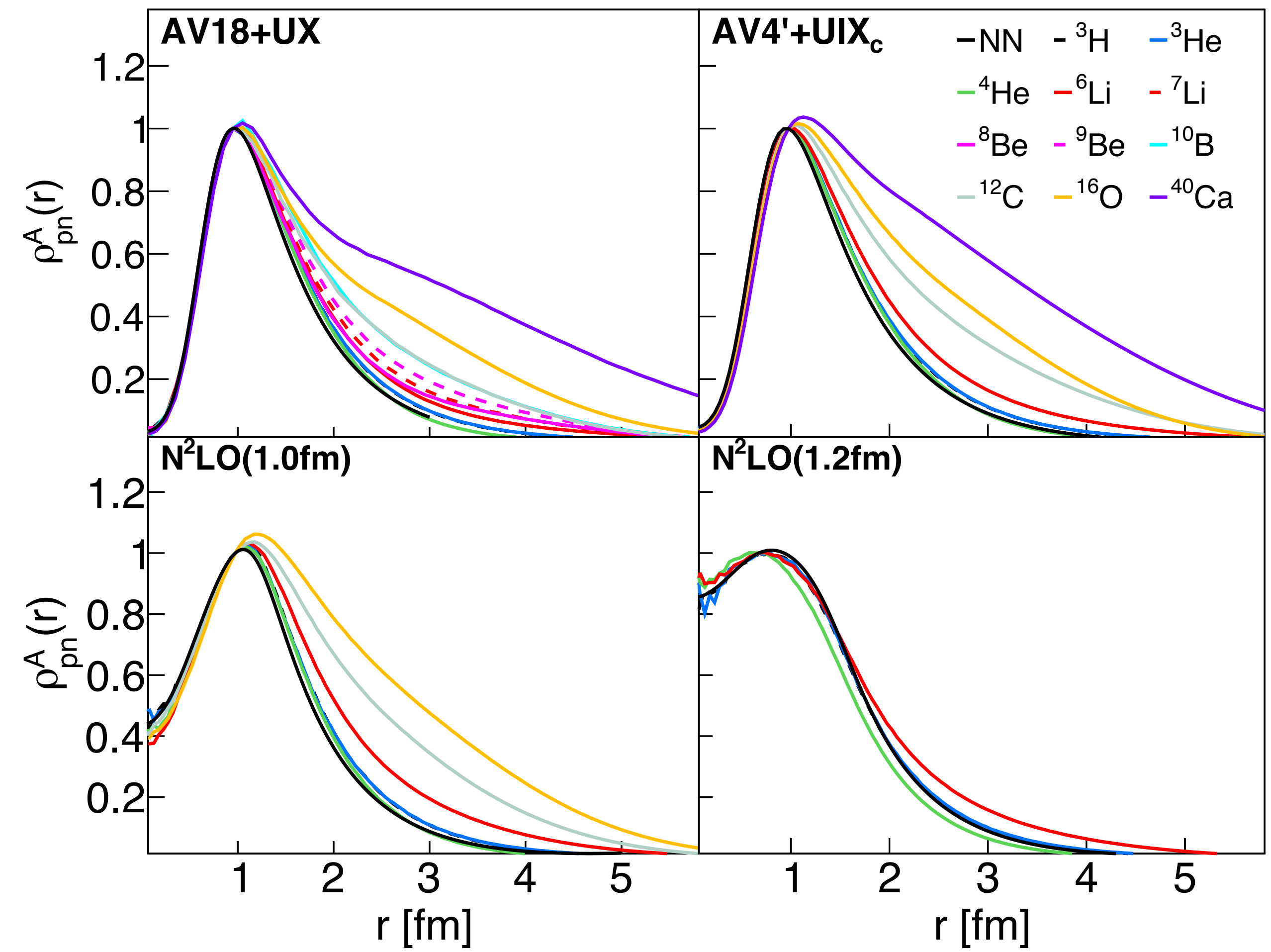

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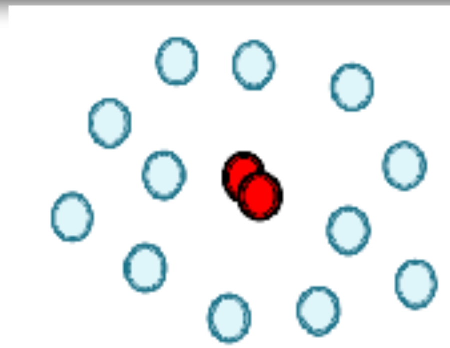
Universal short-distance shape from wilson coeff $\phi(\mathrm{r})$

A-dependent scale factor controlled by χ

Wave function factorization



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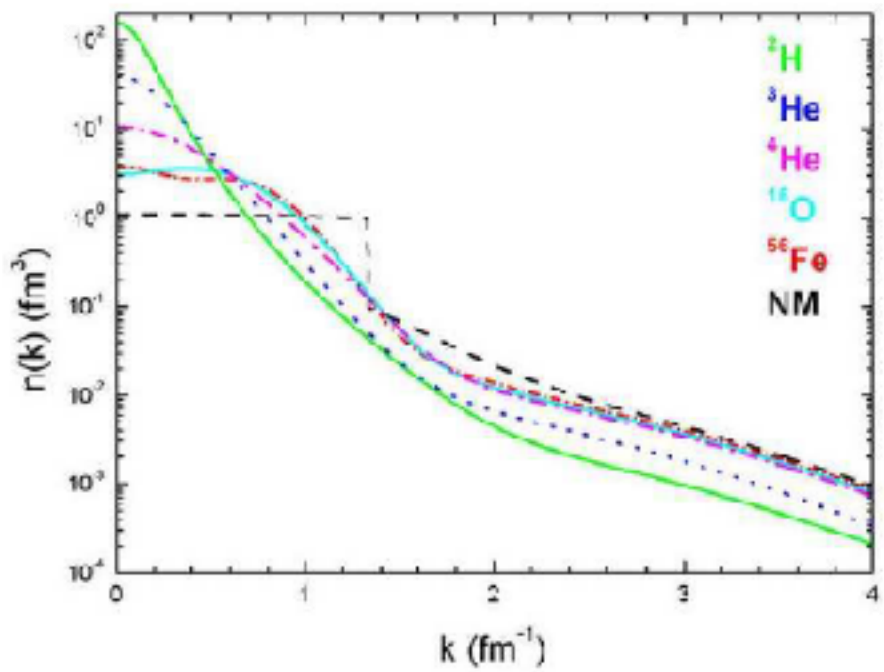
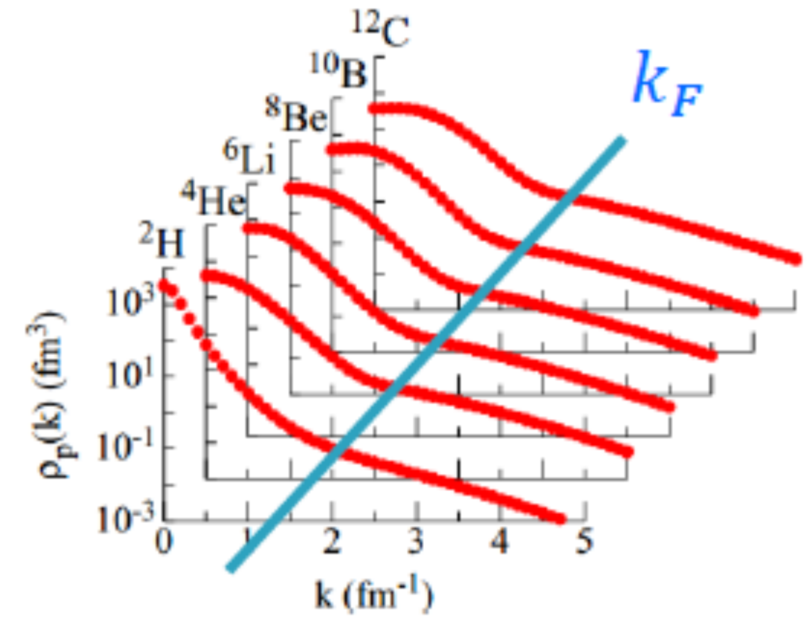
e.g., 1- and 2-body momentum distributions

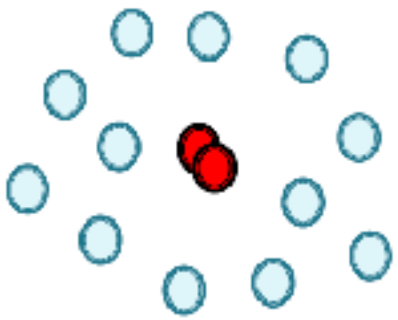


[From C. Ciofi degli Atti and S. Simula]



R.B Wiringa et. al., Phys. Rev. C 89, 024305 (2014)

Wave function factorization



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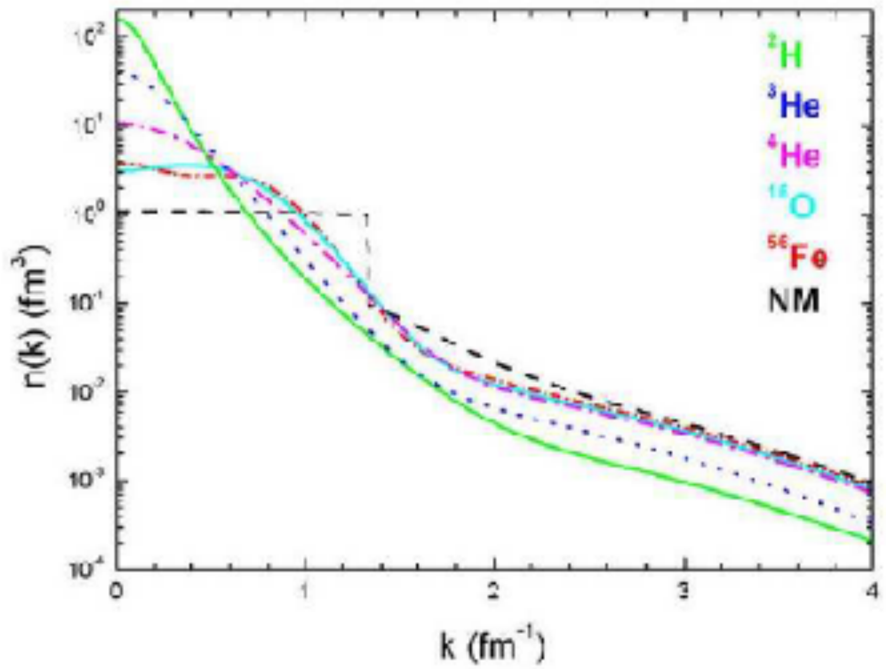

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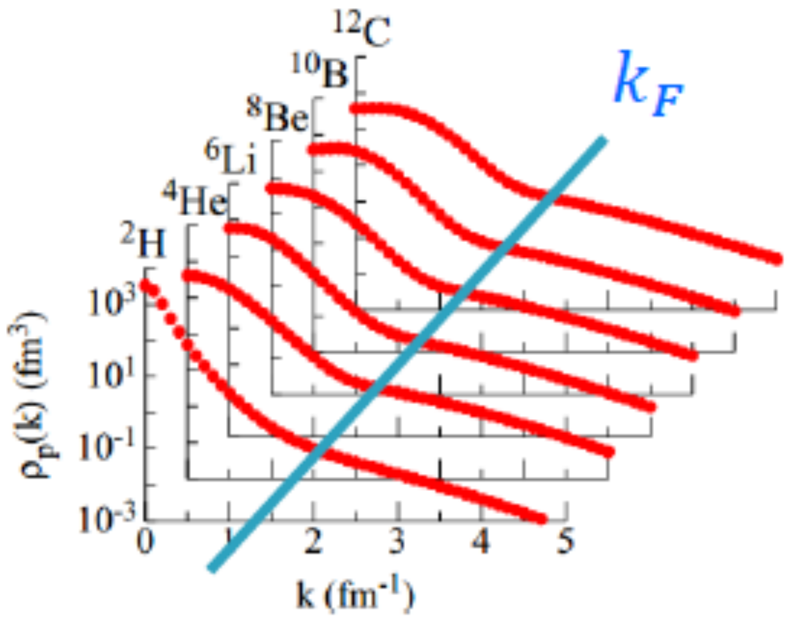

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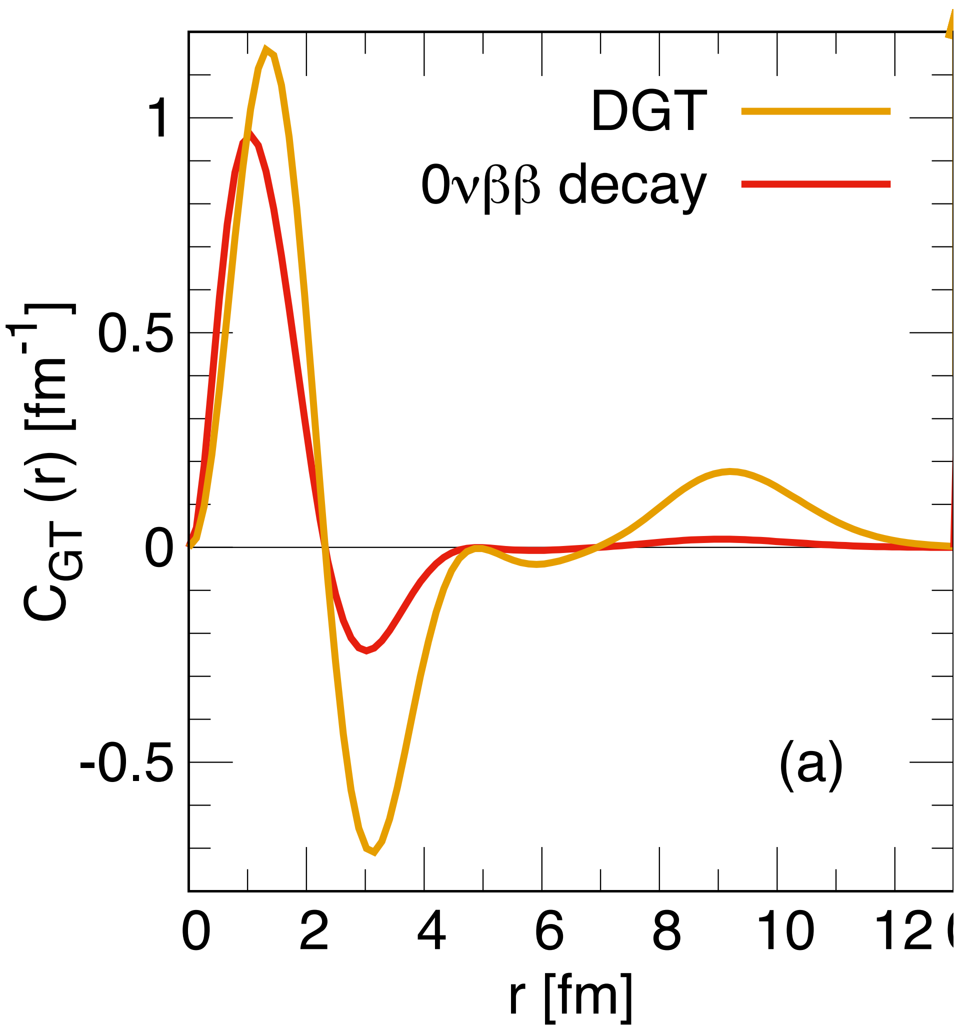
Universal high-q tails, controlled by wilson coeff $\phi(r)$

A-dependent (q-independent) scale factor controlled by χ

Connection with $0\nu\beta\beta$-DGT correlation?



$$\mathscr{M}_{0\nu,GT} = \frac{N(N-1)}{2} \int d1 \cdots dA \, \Psi_f^\dagger(1,2,\ldots,A) \, H_{GT}(r_{12}) \sigma_1 \sigma_2 \tau_1 \tau_2 \, \Psi_i(1,2,\ldots,A)$$

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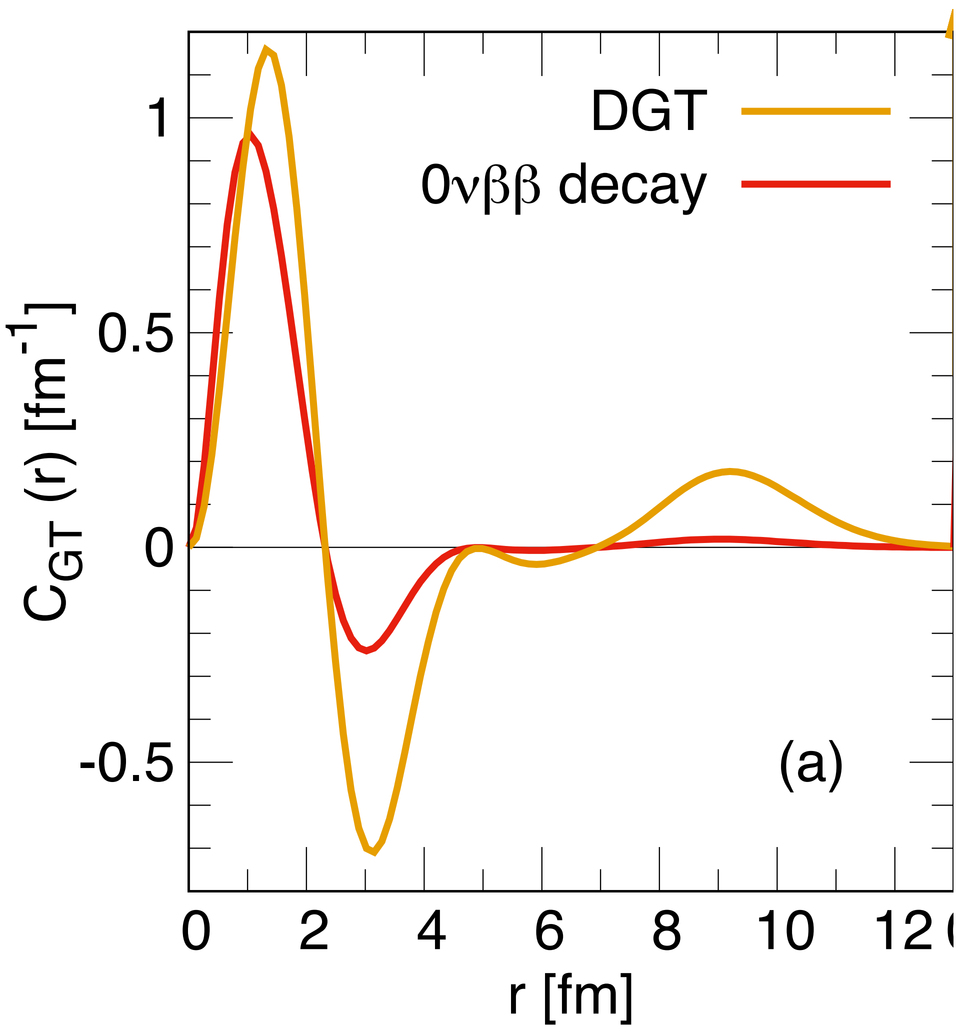

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Dominant contributions to both 0νββ and DGT come from $r \lesssim r_c \sim$ 2-2.5 fm.

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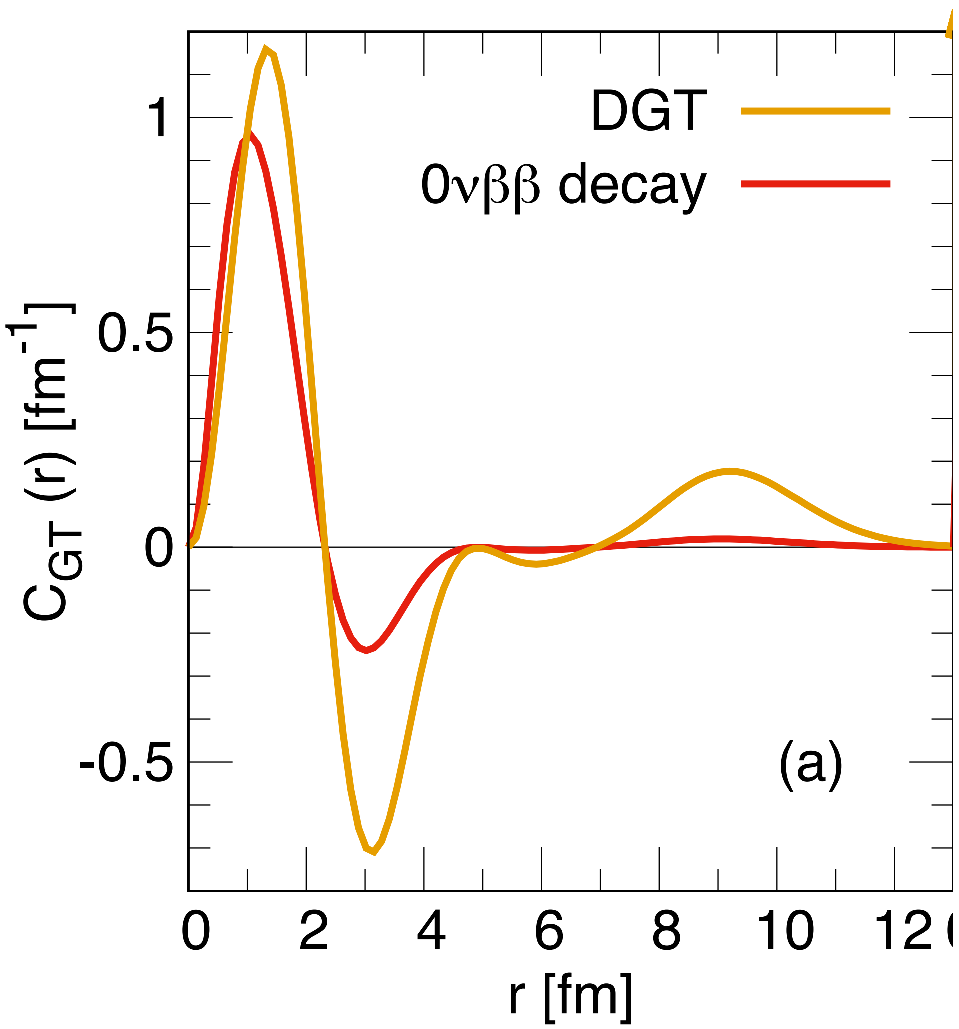

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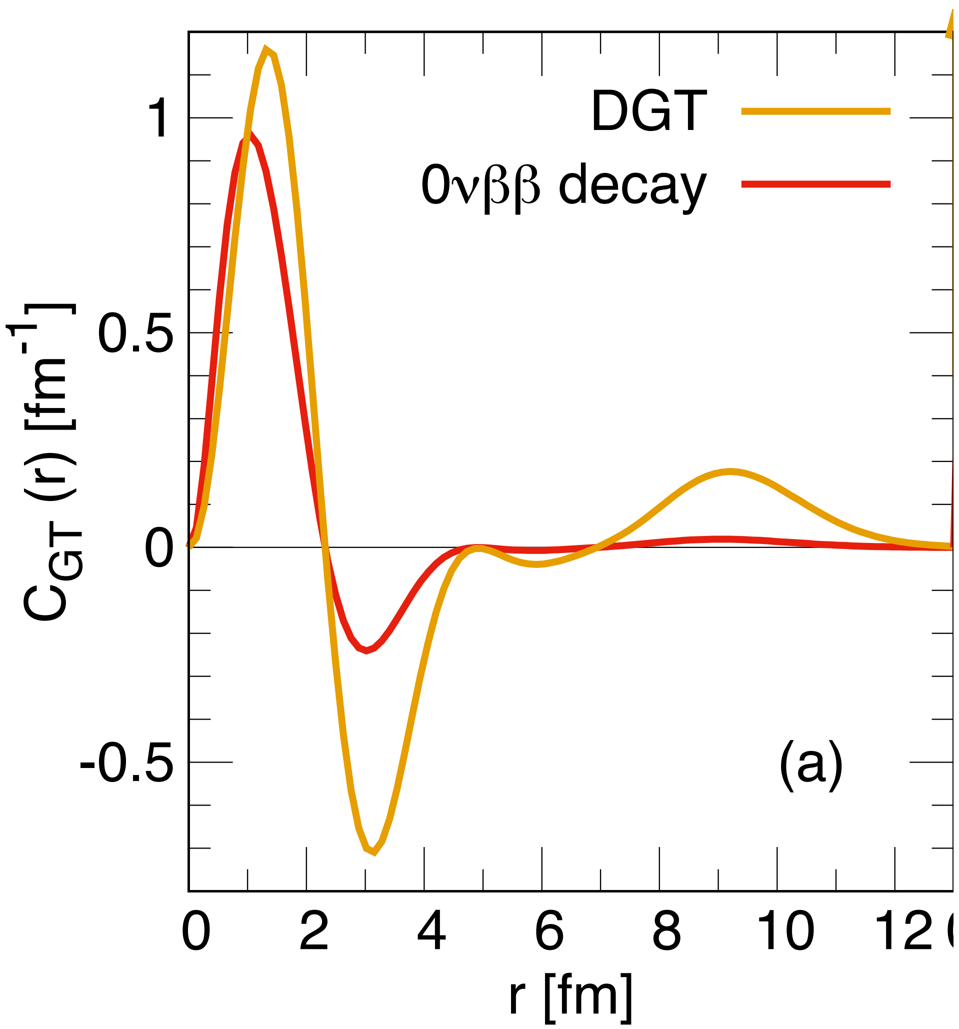
linear correlation follows
many-body physics cancels in ratio

Connection with $0\nu\beta\beta$-DGT correlation?



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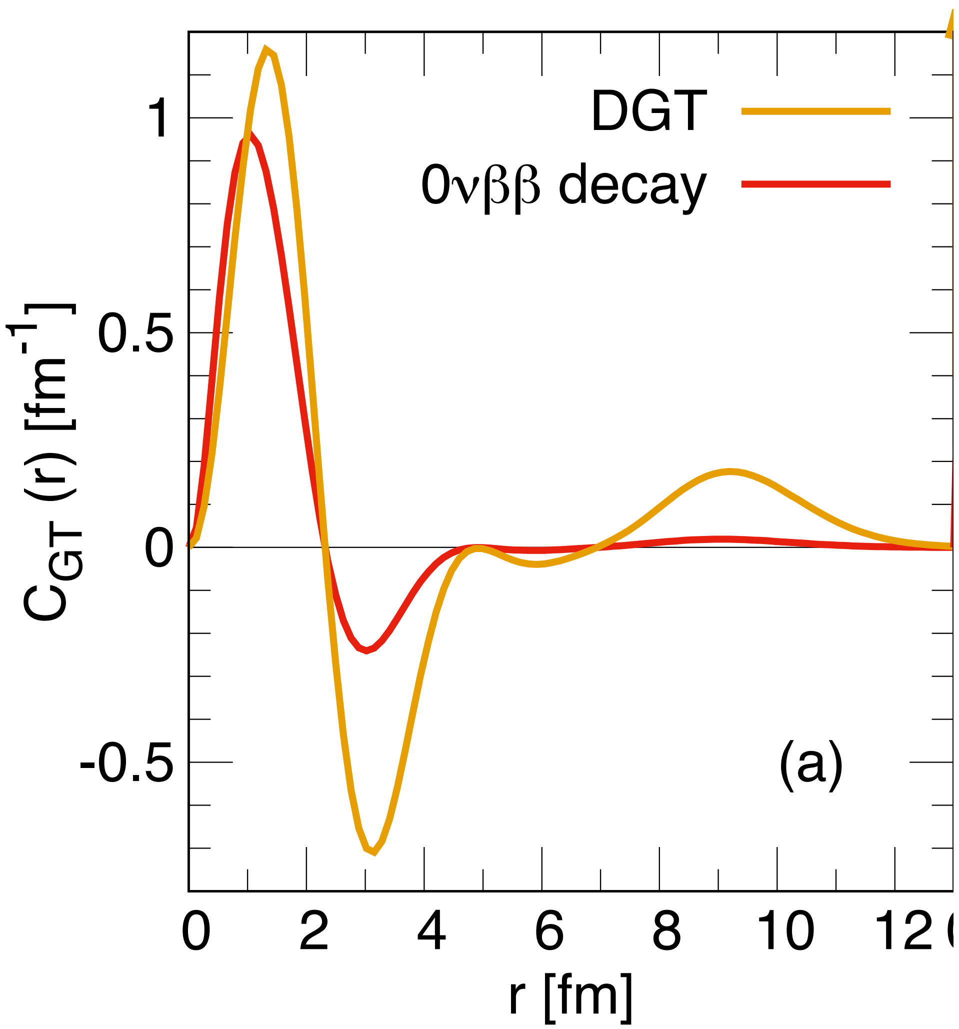

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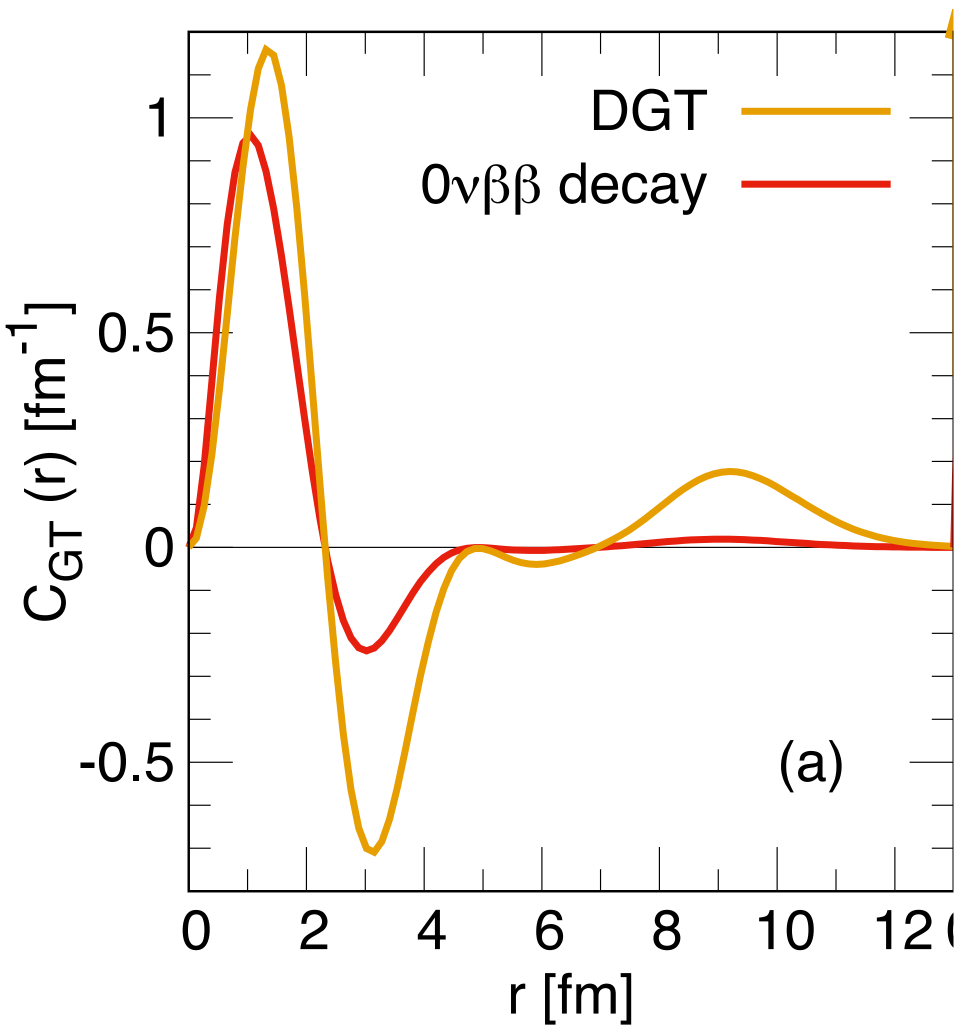

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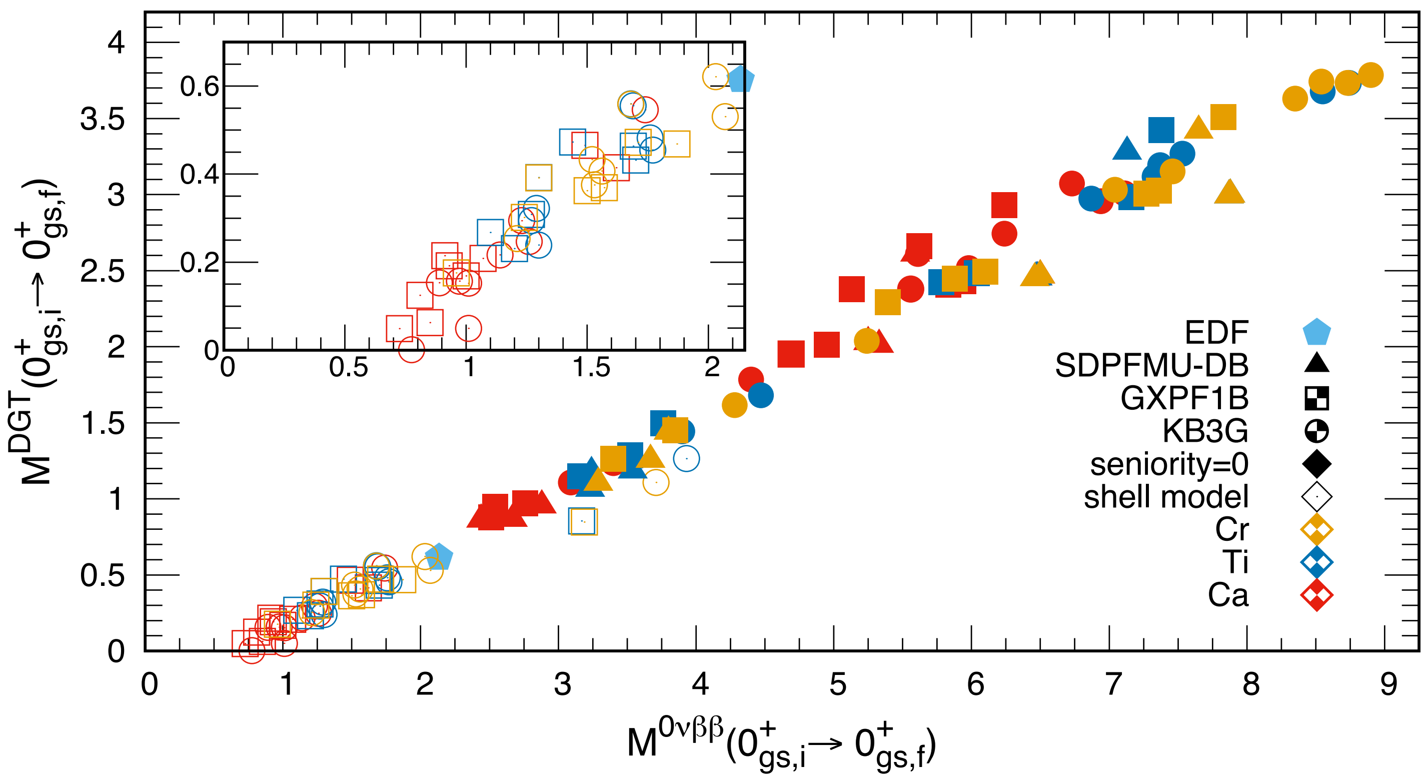
Simple estimates from factorization



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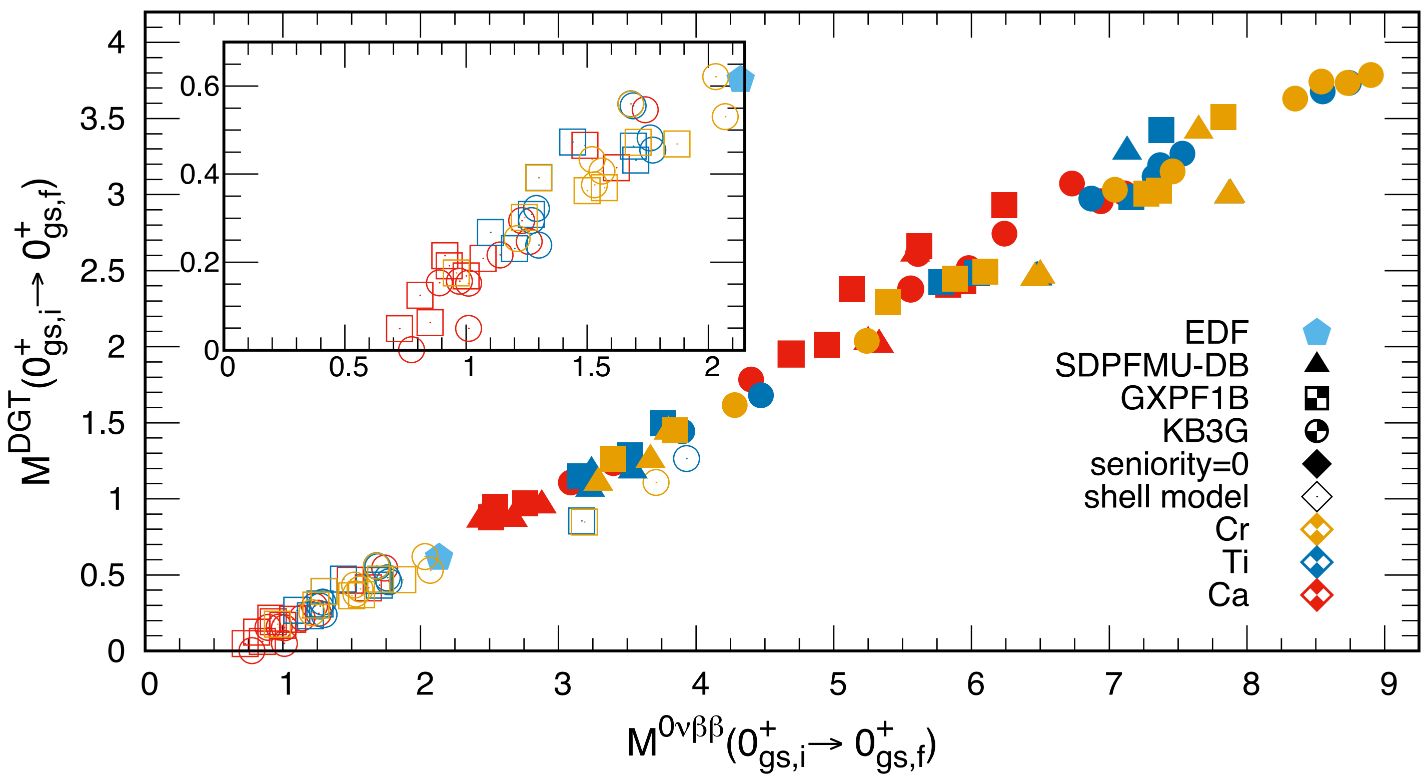
Universal Φ formally given by E=0 2-body w.f. (c.f. Weiss et al. PRC 92 (2015))

Scale and scheme-dependent

Shell model, EDF, IBM, etc. are intrinsically "low resolution" pictures. Try some simple s-wave Φ's with low-resolution scales

Simple estimates from factorization



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observed slope ~.59

First attempt:

Λ MeV	$\frac{1-e^{-r\Lambda}}{r}$	$j_0(\Lambda r)$
200	0.47	0.44
100	0.51	0.53
50	0.53	0.55
20	0.54	0.56
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Calculations missed factor of g_A^2 = 1.59. Numbers now span between .28 - .36

Summary

- Factorization/OPE ideas explains(?) (or is at least consistent with) the linear 0νββ-DGT correlation
- First leading order OPE estimates were shockingly good (5-20% level) , until the damn extraneous g_A^2 factor was noticed.
- Still, the trend is not horrible. Slopes ~ .28 - .36 using “reasonable” universal Φ appropriate to low-resolution SM/ EDF/IBM
- Can we improve this, e.g., subleading OPE terms => deviations from linear scaling?
- Do the ab-initio (IMSRG/IM-GCM/CC) results likewise scale with DGT?

Wave function factorization

LO: $\psi_\alpha^{\Lambda_0}(\mathbf{q}) \approx \gamma(\mathbf{q};\Lambda) \int_0^\Lambda d^3p\, Z_\Lambda \psi_\alpha^\Lambda(\mathbf{p})$ → state-independent ratio for well-separated scales

$$\frac{\psi_\alpha^{\Lambda_0}(\mathbf{q})}{\psi_\alpha^\Lambda(\mathbf{r}=0)} \sim \gamma(\mathbf{q};\Lambda)$$

$$|E_\alpha| \lesssim \Lambda^2 \quad |\mathbf{q}| \gtrsim \Lambda$$

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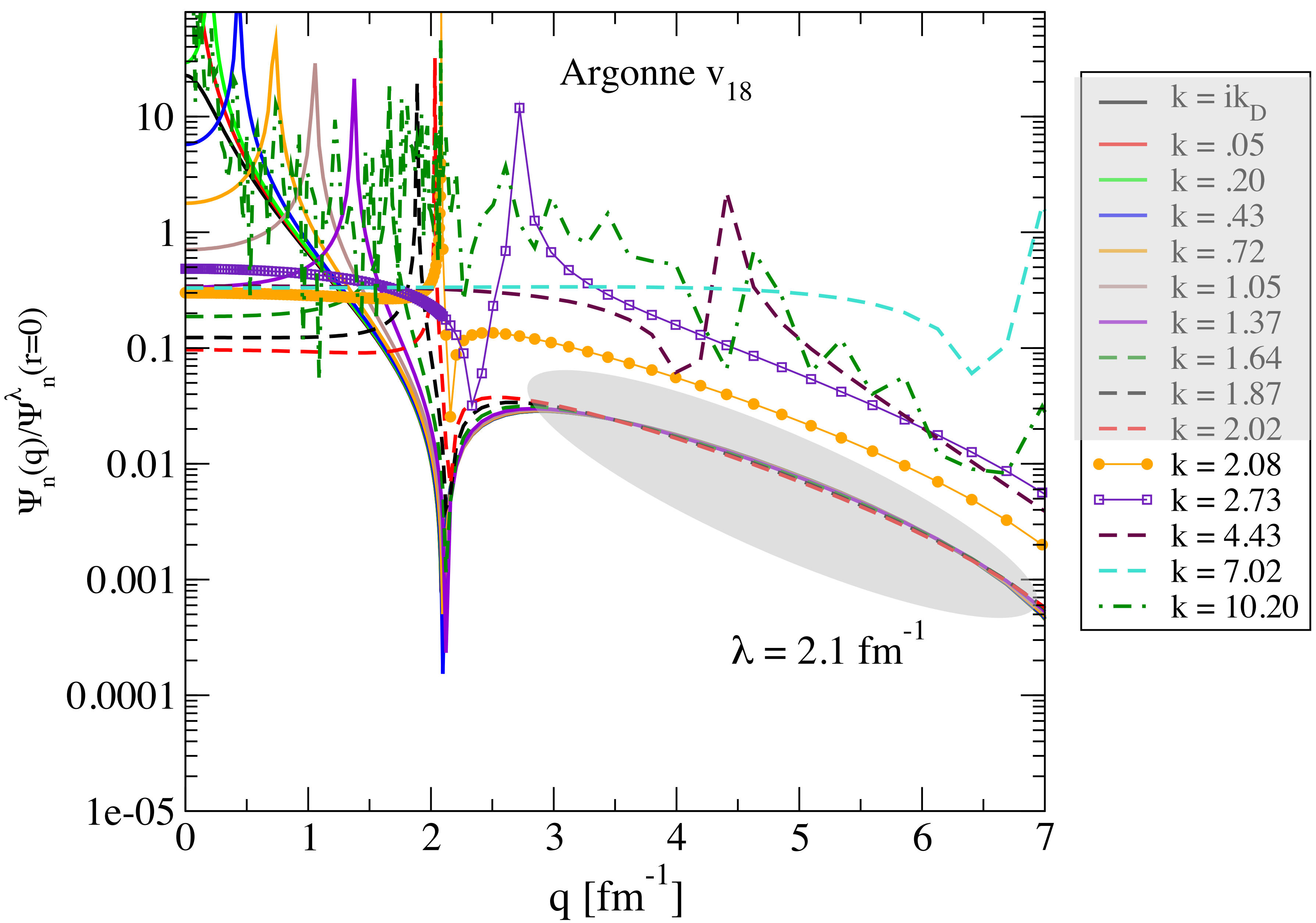
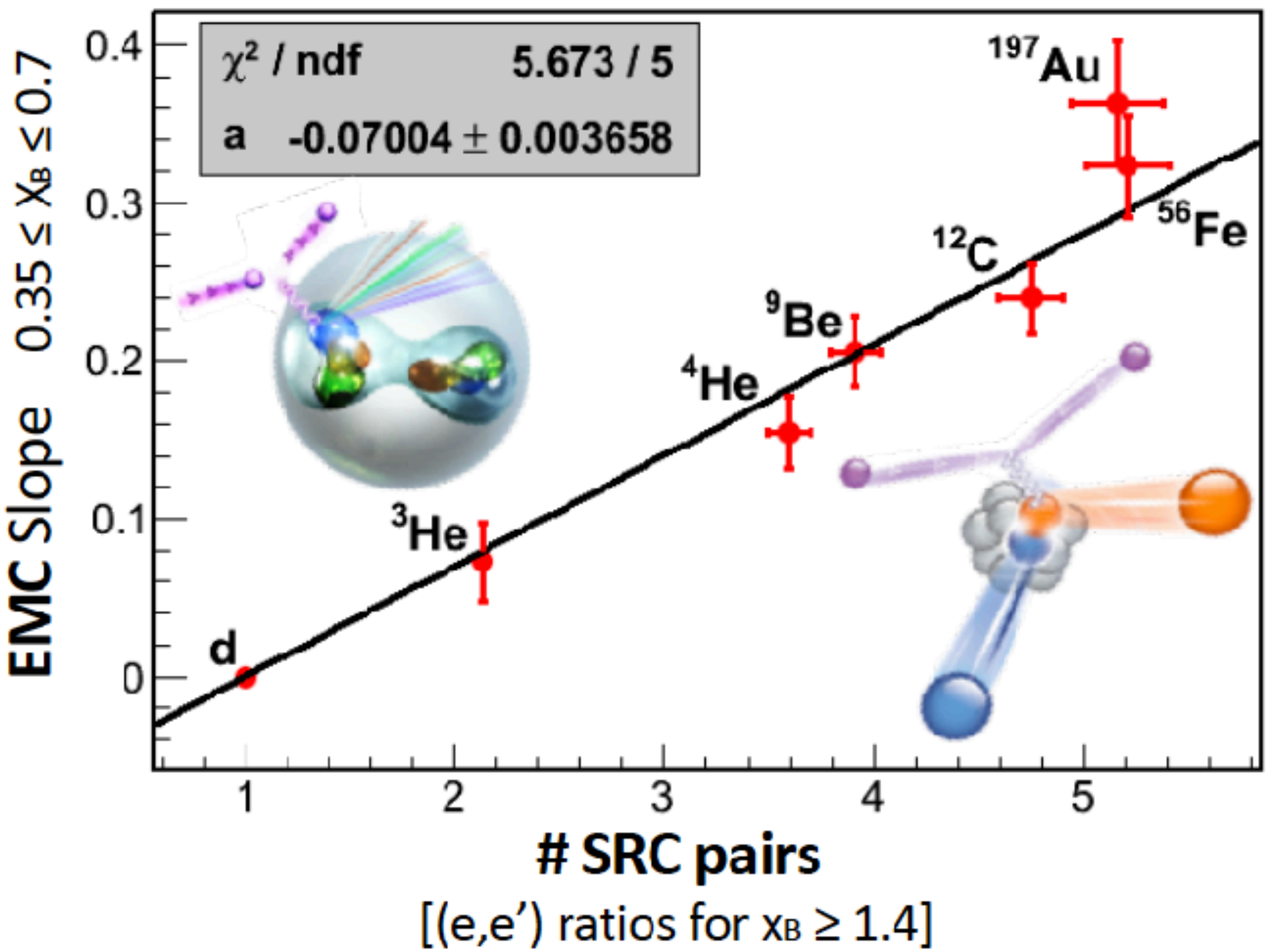


Empirical correlation of EMC effect

Hen et al., RMP (2017); Hen et al., IJMPE (2013); Hen et al., PRC (2012);
Weinstein, Piasetzky, Higinbotham, Gomez, Hen, and Shneor, PRL (2011).



Why should 2 seemingly unrelated processes be linearly related?